Oxford Economic Papers 36 (1984), 268–284

"THE VERDOORN LAW CONTROVERSY": SOME NEW EMPIRICAL EVIDENCE USING U.S. STATE DATA*

By J. S. L. McCOMBIE *and* J. R. DE RIDDER

EVER since Lord Kaldor (1966) in his inaugural lecture first adduced the Verdoorn Law (the statistical relationship between the growth of manufacturing productivity and output) as evidence of substantial economies of scale, it has been surrounded by considerable controversy. The importance of the Verdoorn Law is that it forms the basis of the cumulative causation model of economic growth. This, with the introduction of a balance of payment constraint, has been advanced to explain the persistent disparities in the growth rates of the advanced countries. The law is also an important, though not indispensable, component of the demand oriented approach to economic growth. (See Kaldor, 1978, especially the introduction and Thirlwall and Dixon, 1979.)

The criticisms have been largely directed at the plausibility or otherwise of the assumptions implicit in Kaldor's interpretation of the law. In spite of a number of attempts either to test these assumptions or to specify alternative models that make allowances for these objections, there has not yet been any general agreement as to the seriousness of these criticisms.

For example, a recent study of Verdoorn's Law using time-series data for the U.K. over the post-war period came to the following conclusion:

> 'First, viewed as a structural relationship, Verdoorn's Law does not hold for UK manufacturing industries in the long run. This result is consistent with the findings of Rowthorn, Parikh and Stoneman. Second, manufacturing industry is characterised by a short run version of Verdoorn's Law, but this can be interpreted as Okun's Law.' (Chatterji and Wickens, 1982, p. 36.)

While this quotation is contentious, at least it demonstrates that the inferences drawn by Kaldor from the Verdoorn Law concerning the degree of returns to scale are by no means universally accepted. The debate over the Verdoorn Law has been extensively surveyed in the 'Symposium on Kaldor's Growth Laws' in the Spring, 1983, edition of the *Journal of Post Keynesian Economics* and so we shall only briefly touch on some of the major issues here. (See, in particular, the surveys by McCombie (1983a) and Thirlwall (1983).)

A number of studies have estimated a similar relationship between productivity and output growth using cross-industry (as opposed to cross-country manufacturing) data. See, *inter alios*, Salter (1966), Kennedy (1971) and Wraggs and Robertson (1978). In spite of the similarity of the estimating equation and the fact that the results have also been interpreted as

* We are grateful to Lord Kaldor, Kieran Kennedy, Bob Rowthorn, Tony Thirlwall and anonymous referees for their helpful comments. McCombie retains final responsibility for the arguments advanced in this paper.

measures of returns to scale, this approach is logically independent of the cross-country studies. Kaldor has argued that an important element of increasing returns is derived from the increasing *inter-industry* specialisation of manufacturing. Hence the Verdoorn Law, he argues, should be estimated at the level of total manufacturing. Even if the cross-industry relationship between productivity and output growth was a correctly specified technological relationship, the estimates would abstract from this potentially important component. However, and perhaps more importantly, it is by no means clear that the law using cross-industry data is correctly specified. Two necessary conditions for the estimates to be interpreted as a measure of returns to scale are (a) all the various industries experience the same rate of growth of exogenous technical progress and (b) all industries display the same degree of returns to scale. Both these assumptions may be violated using cross-industry data although the first may not hold also in the case of total manufacturing for the OECD countries. It is also found that in many of the cross-industry studies problems arise similar to those found in the cross-country estimates. For example, the estimates are often sensitive to the choice of regressor and there is the problem of simultaneous equation bias.

In this paper we are concerned with Kaldor's procedure of estimating the law using cross-sectional data for total manufacturing. The approach adopted here is based on the contention that the use of international data in estimating the Verdoorn Law has been exhausted and the estimation of more complex models using this data is unlikely to be very illuminating. Instead, it is argued that the controversies may be resolved through the choice of a new data set that controls for many of these factors which, it has been argued, may bias or vitiate the law. Hence, the law is estimated using state data drawn from the United States, and the advantages of this procedure are discussed below. The results obtained, it is argued, provides fresh evidence relevant to the debate over the Verdoorn Law.

The outline of this paper is as follows. In Sections 1, 2 and 3 the controversy is briefly reviewed and further reservations are expressed concerning some recent attempts to settle the issue. The data and the rationale and advantages of our approach are discussed and the results of the estimation of the Verdoorn Law presented. Section 4 presents the results of the estimation of what has been termed the 'static' Verdoorn Law and tests the importance of external economies of scale. (The 'static' Law is the estimation of the Verdoorn Law using the logarithmic values of the levels of the various variables, rather than their growth rates.) Possible explanations for the divergence that is found in the estimates of the 'dynamic' (traditional) and 'static' laws are also considered. In the conclusions, we assess the implications of our results for the debate concerning the Verdoorn Law.

1. Specification problems and the regional Verdoorn Law

The Verdoorn Law is traditionally specified as

$$p = a_1 + b_1 q \tag{1a}$$

or, since $p \equiv q - e$, equivalently as

$$e = -a_1 + (1 - b_1)q \tag{1b}$$

where p, q and e are the growth rates of (manufacturing) productivity, output and employment, respectively. Equation (1b) is preferable since it avoids the spurious correlation inherent in (1a) resulting from the identity $p \equiv q - e$. The t statistics and estimated values of a_1 and b_1 are identical in the two equations. When equation (1b) is estimated using cross-country data consisting of observations over the period 1950 to 1965 for twelve OECD countries, it is found that the value of the Verdoorn coefficient $(1 - b_1)$ is approximately one half. Kaldor interprets the Verdoorn Law as a production relationship, equivalent to the technical progress function, and argues that this estimate demonstrates the presence of substantial economies of scale.[1]

The debate over the interpretation of the Verdoorn Law really stems from Rowthorn's paper (1975a). Rowthorn argued that implicit in Kaldor's explanation of 'why growth rates differ' was the argument that employment growth was a constraining factor. Thus, the correct procedure should have been to regress productivity growth directly on that of employment. (Henceforth we shall refer to the model with employment growth as a regressor as Rowthorn's specification.) When this is done, and Japan is omitted from the sample as an outlier, there is no significant relationship between the two variables—a result that suggests constant returns to scale. Furthermore, it is perhaps more plausible to assume that both productivity and output growth are jointly determined as the former is likely to have a reciprocal effect on the latter through the price mechanism (Rowthorn 1975b). In this case, the Verdoorn coefficient may be subject to simultaneous equation bias.

A further criticism concerns the necessary assumption that all the advanced countries experience the same rate of growth of exogenous technical progress (Rowthorn 1975a, Gomulka 1971, 1979). If there is significant diffusion of technology from the relatively advanced to less developed countries in the sample, the Verdoorn Law will be mis-specified. Indeed, the law may be simply the result of those countries which derive the greatest benefit from diffusion also having the fastest growth of output. Thus, in this case, the law cannot be taken to reflect the degree of returns to scale. Moreover, there are additional reasons why productivity growth may vary between countries. For example, entrepreneural attitudes to risk and inno-

[1] The Verdoorn coefficient, *per se*, cannot be interpreted as a measure of returns to scale unless the contribution of the capital stock growth (k) is explicitly included in the Verdoorn equation or there is some evidence that its omission does not bias the coefficient. Subsequent to his inaugural lecture, Kaldor (1978) included the gross investment-output ratio as a proxy for k in the law. It was found not to significantly influence the Verdoorn coefficient. Moreover, for the advanced countries there is evidence that the growth of the capital-output ratio has been negligible over the post-war period. If this is correct, the Verdoorn coefficient may be regarded as an unbiased estimate of $(1-\alpha)/\beta$ where α and β are the output elasticities of capital and labour, respectively. If α equals β, a Verdoorn coefficient of one half implies returns to scale of 1.33 (See also Thirlwall, 1980b.)

vation as well as trade union attitudes to restrictive practices and the introduction of new techniques may differ significantly between countries (Kilpatrick and Lawson, 1980).

The procedure adopted in this paper is, rather than specifying a model that allows for the various objections outlined above, to choose a sample that is relatively free of these problems. Our method has been largely anticipated by Kaldor himself:

> 'The primary question that needs to be considered is what *causes* these differences in 'regional' growth rates—whether the term 'regional' is applied to different countries (or even groups of countries) or different areas within the same country. The two questions are not, of course, identical; but up to a point it is illuminating to consider them as if they were, and to apply the same analytical technique to both.' (1970)

The Verdoorn Law was therefore estimated on the basis of state data for the United States over the period 1963–1973.

At this point, it is worth mentioning that while regional data has a number of decided advantages over international data there is one important difference between the regional and cross-country Verdoorn Law. It is likely that the estimate of the Verdoorn coefficient in the case of regional statistics will represent a lower bound of the 'true' magnitude obtained using international data in a correctly specified model. This arises because economies of scale at the national level are likely to be captured in the intercept rather than slope coefficient of the regional Verdoorn Law as they are a function of national output growth rather than state variations in growth. It follows that this will impart a bias in favour of accepting the hypothesis of constant returns to scale and so the regional Verdoorn Law provides a stringent and conservative test of the Kaldorian thesis.

A further advantage of state data is that differences in socio-economic factors are likely to be much smaller between states than between countries. There are also sufficient degrees of freedom to allow for the testing of any regional differences in the various coefficients through the use of dummy variables.

There are few, if any, economic barriers to the long run mobility of investment and, especially, labour between states. Thirlwall (1980a) has gone so far as to argue that it is plausible to assume that the growth of state output is essentially demand rather than supply constrained.[2] This view would suggest that Kaldor's procedure of assuming output to be exogenous is correct, at least for regional data (although it is always useful to see

[2] To quote Thirlwall on this point:
"(. . .) regional growth is demand determined for the obvious reason that no region's growth rate can be constrained by supply when factors of production are freely mobile. For a region in which capital and labour are highly mobile in and out, growth must be demand determined. If the demand for a region's output is strong, labour and capital will migrate to the region to the benefit of that region and to the detriment of others. Supply adjusts to demand. We cannot return to the pre-Keynesian view that demand adjusts to supply". (1980a)

whether Rowthorn's specification gives a markedly different result). Nevertheless, given the importance of the debate over the specification of the law and the possibility of bias in the estimated coefficients noted above due to the problem of simultaneity, it becomes crucial to determine how far this occurs with regional data and whether the use of an instrumental variable procedure is a more appropriate method of estimation. We return to this problem in Section 3.

2. The data

The data for the 49 states used (which exclude Alaska and Hawaii) is taken from various issues of the Census of Manufacturers and the Annual Survey of Manufactures (U.S. Bureau of the Census), and, as noted above, covers 1963–73.[3]

Employment is defined as the total number of wage and salary earners. An alternative which is readily available from the Census is the total number of manhours worked but this relates only to production workers. (In practice use of the latter makes a negligible difference to the estimates.)

Output is value added and the reported data, in current prices, were deflated to constant prices at the 2 digit Standard Industrial Classification level and the statistics summed to derive total manufacturing output.

Three separate proxies were constructed for the growth of the non-labour inputs as there are no estimates for state capital stock at constant prices. The first is the traditional gross investment-output ratio, but a necessary condition for this to be a reasonable proxy is that the state capital-output ratios are uniform, a condition which our other proxies suggested was not found in practice. (The use of the investment-output ratio also gave implausible results which are not reported here.) The second estimate for the growth of capital (k_1) was calculated by us and is based on a form of the perpetual inventory model and the use of cumulative gross investment at constant prices. Statistics are also available for the gross book value of depreciable assets which is the historic cost of capital equipment. The third proxy (k_2) was the growth of this historic cost valuation which we adjusted to constant prices using national weights and price deflators. It is reassuring to note that there is a close association between k_1 and k_2 with a coefficient of determination of over 0.8. As the choice of capital proxy does not make a significant difference to the results, those obtained using only k_1 will be reported here.[4]

[3] 1962 is the earliest year for which it was possible to construct a series for the state capital stock. 1973 was chosen as the terminal year being the last peak year before the world recession. The Verdoorn Law was also estimated for the period 1947 to 1963 by regressing employment on output growth. Given the absence of data on the growth of capital, little reliance can be placed on the estimates and the results are not reported here.

[4] A further description of the method by which these estimates were constructed may be obtained on request from the authors.

3. The specification and estimation of the Verdoorn Law

The Verdoorn Law was first estimated by ordinary least-squares regression analysis and is specified in the traditional form as

$$e = a_1 + b_1 q + b_2 k \tag{2}$$

The degree of returns to scale (v) is given by $(1-b_2)/b_1$ and its standard error determined by the use of Taylor's expansion in the normal manner.[5]

Rowthorn's specification is given by

$$q = a_1' + b_1' e + b_2' k \tag{3}$$

with the degree of returns to scale given by $b_1' + b_2'$.

Kaldor, following Allyn Young (1928), argues that, although economies of scale are undoubtedly important at the firm or plant level, the major gains are to be found at the macro level where they are derived from the inter-industry division of labour and specialisation. Consequently, the Verdoorn Law should be specified using the growth rates for total manufacturing and this paper is concerned with the law at this level of aggregation.[6] The results of the estimation of the Verdoorn Law, together with Rowthorn's specification, are reported in Table 1. Dummy variables were introduced to test for differences in the estimated coefficients resulting from regional influences. The four regional groupings used are those delineated by the U.S. Bureau of Economic Affairs and are the North East, the North Central, the South and the Far West. These dummies proved to be significant in the case of the intercept (i.e. exogenous technical progress) and hence are included in the regression equation.[7] Dummies allowing for shifts in the regression coefficients proved to be insignificant.

As may be seen from Table 1, the estimate of the size of returns to scale given by the Verdoorn Law is 1.45 while Rowthorn's specification with the growth of employment (rather than output) as a regressor gives a value of 1.33. Both these estimates prove to be significantly greater than unity at the 0.99 confidence level. Initially, these results would seem to provide strong confirmation of the hypothesis that manufacturing industry is subject to substantial economies of scale.

[5] Equation (2) is the estimating equation of

$$e = \pi + \frac{1}{\beta} q - \frac{\alpha}{\beta} k$$

where α and β are the output elasticities of capital and labour respectively. Hence $(1-b_2)/b_1$ provides an estimate of the sum of α and β, the degree of returns to scale.

[6] McCombie (1983b) considers the results for eighteen individual industries based on the 2 digit S.I.C. This paper involves a discussion of the possible trade-offs between aggregation and specification errors together with those due to measurement errors.

[7] This leads to the suggestion that the spatial diffusion of innovations may be an important factor in determining interstate variations in productivity growth. A number of specifications of this hypothesis were tested and in each case the importance of diffusion was found to be insignificant. (McCombie, 1982a.)

TABLE 1

The Verdoorn Law: U.S. state data. TOTAL manufacturing, 1963–1973

$(n=49)$

$$(1)\ e=-2.813+1.536d_1+0.915d_2+1.874d_3+0.770q-0.117k$$
$$(-9.07)\ (5.32)\ (2.89)\ (6.21)\ (12.56)\ (-1.38)$$
$$\bar{R}^2=0.887$$
$$\text{SEE}=0.649$$

$$(2)\ q=3.179-1.212d_1-0.466d_2-1.653d_3+1.020e+0.305k$$
$$(8.63)(-3.14)\ (-1.19)\ (-4.07)\ (12.56)\ (3.45)$$
$$\bar{R}^2=0.876$$
$$\text{SEE}=0.746$$

Estimates of returns to scale (v)

Equation:	(1)	(2)
v	1.45	1.33

(Both estimates are significantly greater than unity at the 0.99 confidence level).

Variables: e, q and k are the exponential growth rates of employment, output and the capital stock, respectively. The ds are regional dummies.

Sources: See text.

Note: Figures in parentheses are t values. SEE is the standard error of the equation.

It may be, though, that both estimates of returns to scale are subject to bias due to the problem of simultaneity. As we have noted above, one possible source of bias when international data is used occurs because an increase in productivity will increase a country's relative price competitiveness. This, in turn, will increase the demand for that nation's output. Consequently, neither employment nor output growth should be deemed exogenous since they are jointly determined. This is perhaps not so serious as far as regional data is concerned. Since most prices are set in the national market, there is likely to be little inter-state variation in their growth and hence no significant systematic relationship between the growth of relative prices and output. But if this is correct and it is also true that wages do not absorb all the gains of above average productivity growth, then it will be more profitable to invest in these faster growing states. It has in fact been implicit in the demand oriented approach to economic growth that the level of investment is not a long run constraint on the growth of output. Capital accumulation is primarily a function of the anticipated growth of output and, in the context of long run growth, both employment and capital growth are determined by that of output. (This may explain the relatively large standard error of the coefficient of the capital stock growth of the Verdoorn Law since the latter will be mis-specified.)

Kennedy and Foley (1978) have suggested that in the light of this criticism a better specification of the Verdoorn Law is to use the growth of total

TABLE 2

The Verdoorn Law using total factor input. U.S. Manufacturing: state data 1963–1973

Ordinary-least squares estimates

$n=49$

(1) $f=-1.412+0.972d_1+0.321d_2+1.161d_3+0.606q$ $\bar{R}^2=0.866$

(−5.52) (3.86) (1.21) (4.41) (14.44) SEE=0.568

(2) $q=2.640-0.945d_1-0.082d_2-1.319d_3+1.363f$ $\bar{R}^2=0.838$

(8.75) (−2.29) (−0.20) (−3.07) (14.44) SEE=0.853

Instrumental variable estimates

(A) *Bartlett's grouping method*

$n=32$

(3) $f=-1.659+1.042d_1-0.039d_2+1.053d_3+0.644q$ $\bar{R}^2=0.900$

(−5.69) (3.46) (−1.05) (3.37) (13.09) SEE=0.580

(4) $q=2.666-1.514d_1+0.427d_2-1.760d_3+1.489f$ $\bar{R}^2=0.882$

(8.26) (−3.13) (0.69) (−3.42) (13.46) See=0.867

(B) *Durbin's ranking method*

$n=49$

(5) $f=-1.534+0.907d_1+0.260d_2+1.117d_3+0.635q$ $\bar{R}^2=0.864$

(−5.87) (3.57) (0.97) (4.21) (14.61) SEE=0.572

(6) $q=2.646-0.930d_1-0.072d_2-1.307d_3+1.357f$ $\bar{R}^2=0.838$

(8.76) (−2.24) (−0.18) (−3.02) (14.21) SEE=0.853

(C) *Lagged variables as instruments*

$n=49$

(7) $f=-1.775+0.779d_1+0.140d_2+1.028d_3+0.695q$ $\bar{R}^2=0.852$

(−3.61) (2.27) (0.41) (3.26) (6.32) SEE=0.596

(8) $q=2.594-1.042d_1-0.148d_2-1.408d_3+1.406f$ $\bar{R}^2=0.837$

(7.91) (−2.11) (−0.33) (−2.84) (9.32) SEE=0.854

Estimates of returns to scale (v)

Equation:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
v:	1.65	1.36	1.55	1.49	1.57	1.36	1.43	1.41

(All estimates are significantly greater than unity at the 0.99 confidence level)

Variables: f, q are the growth of total factor inputs and output respectively.
Sources: See text.

inputs rather than employment on the dependent variable so both employment and capital growth are functions of the growth of capital. They specify the Verdoorn Law as

$$f=a_1+b_1q \tag{4}$$

where $f=(\alpha' k+\beta' e)$ and $\alpha'+\beta'=1$.

f is the growth of the sum of the factor inputs, suitably weighted, and b_1 is an estimate of the reciprocal of the degree of returns to scale. In order to weight the inputs the "growth accounting approach" was followed and the relevant factor shares were used.[8]

The results together with Rowthorn's specification (with f as the regressor) are reported in Table 2, equations (1) and (2). The estimates are slightly

[8] A variety of other values for α' and β' were used but the results proved insensitive to the exact magnitude chosen.

larger than those of the traditional specification. They are also significantly greater than unity at the 0.99 confidence level.

It may well be that the assumptions of ordinary-least squares are still not fulfilled. For example, the Verdoorn Law is a production or technological relationship and so firms choose both the output and the inputs simultaneously. While a change in demand conditions may induce a change in output it will, *pari passu*, alter the employment of labour and capital. Another issue is that there may be measurement errors in the construction of estimates of the growth in the volume of output and capital. This may be due to, for example, the need to use national rather than regional price deflators because of absence of information on the latter. (The errors in variables problem has been discussed in greater detail in McCombie, 1981.)

Given the potential importance of the simultaneity and measurement error problems, the total factor input specification of the Verdoorn Law was estimated using instrumental variable methods. Three procedures were used, namely, Bartlett's grouping method, Durbin's ranking method and the use of lagged variables as instruments. In the last approach, the instruments used for the Verdoorn Law were output growth over the periods 1947–1958 and 1958–1963. (In the case of Rowthorn's specification employment growth is used as the instrument.)

It is reassuring to note that these procedures confirm the results of the ordinary-least square estimation. (Table 2, equations (3) to (8).) We may, therefore have a high degree of confidence in accepting that there are substantial economies of scale in manufacturing as Kaldor originally argued in his inaugural lecture (1966).

4. The static Verdoorn Law and external economies of scale.

The most plausible interpretation of the Verdoorn Law is that it is a specification of a linear technical progress function. However, just as Black (1962) has shown that the latter may be derived from a conventional Cobb-Douglas production function, so one possible underlying structure of the Verdoorn law is

$$\ln E = a_1 + b_1 \ln Q + b_2 K \tag{5a}$$

or

$$\ln F = a_2 + b_3 \ln Q \tag{5b}$$

(where the upper case letters denote the levels of the appropriate variables). If this is the correct underlying specification, *a priori*, it is to be expected that both the 'dynamic' Verdoorn Law (estimated using exponential growth rates) and the 'static' Verdoorn Law (using logarithmic values of the levels) should demonstrate the same degree of returns to scale.

Nevertheless, when the static Verdoorn Law is estimated using international data a paradox is found. The Verdoorn coefficient does not differ significantly from unity which suggests the presence of constant returns to scale. (Rowthorn's specification also produces this result. See McCombie 1982b.) This is sufficient to reject Kaldor's thesis of the prevalence of large increasing returns to scale. It becomes of crucial importance, therefore, to determine which of the two specifications is to be preferred.[9]

The explanation of this divergence may be purely statistical, being the result of differing bias due, for example, to measurement errors. A more plausible reason is theoretical and involves the contention that in fact the technical progress function is not derived from the static Cobb-Douglas production function. This argument has been dealt with at length in McCombie (1982b) and so we shall only briefly recapitulate the argument here. The discrepancy may be due to a 'second order identification problem'. This arises because there are numerous underlying structures of the 'dynamic' Verdoorn Law depending upon the constant of integration. It may well be that the orthodox Cobb-Douglas is not the correct structure underlying the Verdoorn Law and so the static law will be mis-specified yielding biased estimates of the returns to scale. Kaldor himself long ago pointed out this difference between the linear technical progress function and the Cobb-Douglas. Nevertheless, this argument seems to have been largely ignored in the literature. For example, a recent textbook on economic growth argues as follows:

> 'Black has shown that if the technical progress function is *linear* then the underlying technology *must* be representable by a Cobb-Douglas production function.' (Jones, 1975, p. 197)

(The second italicised word is our emphasis.)

The results of the estimation of the traditional Verdoorn Law (together with Rowthorn's specification) are reported in Table 3, equations (1) and (2). The large standard error of the coefficient of the capital stock in the Verdoorn Law (equation (1)) is the result of severe multicollinearity (unlike in the case of the 'dynamic' law). It is still possible to test for the hypothesis of increasing returns since the sum of the coefficients b_1 and b_2 equals $(1-\alpha)/\beta$. If the sum of b_1 and b_2 does not differ significantly from unity the hypothesis must be rejected. This proves to be the case and the result is confirmed by Rowthorn's specification.

When total factor input is used as a variable the estimate of returns to scale is, using an instrumental variable procedure, 1.02 in both specifications of the Law.[10] This value is greater than unity at the 0.95 confidence level. The instruments used are the 1947 and 1958 levels of output (equation 3) and employment (equation 4). The use of the instrumental variable ap-

[9] The static Verdoorn Law is akin to a conventional production function except that output is deemed to be exogenous and employment is specified as the dependent variable. The importance of the paradox partly results from the fact that most conventional estimates of production functions demonstrate either very small increasing or constant returns to scale—although there are exceptions. See, for example, Douglas (1976).

[10] The weights used in calculating the total factor input were the average values of the relevant factor shares.

TABLE 3

The Static Verdoorn Law. U.S. Manufacturing: Pooled state data 1973 and 1963

Ordinary-least squares estimates
$n=98$

(1) $\ln E = a + t + d_1 + d_2 + d_3 + 1.087 \ln Q - 0.109 \ln K$ $\quad \bar{R}^2 = 0.991$
$\qquad (16.78) \qquad (-1.65) \qquad$ SEE = 0.136

(2) $\ln Q = a + t + d_1 + d_2 + d_3 + 0.695 \ln E + 0.319 \ln K$ $\quad \bar{R}^2 = 0.995$
$\qquad (16.78) \qquad (7.72) \qquad$ SEE = 0.108

Instrumental variable estimates
$n=98$

(3) $\ln F = a + t + d_1 + d_2 + d_3 + 0.981 \ln Q$ $\quad \bar{R}^2 = 0.997$
$\qquad (113.21) \qquad$ SEE = 0.111

(4) $\ln Q = a + t + d_1 + d_2 + d_3 + 1.016 \ln F$ $\quad \bar{R}^2 = 0.994$
$\qquad (113.20) \qquad$ SEE = 0.113

Estimates of returns to scale (v)

Equation:	(1)	(2)	(3)	(4)
v	1.02	1.01	1.02*	1.02*

(* denotes value is significantly greater than unity at the 0.95 confidence level).

Variables: E, Q, K and F are the levels of employment, output, capital and total factor inputs respectively. t and the ds are time and regional dummies, respectively, the coefficients of which are not reported.

Sources: See text.

proach in fact makes very little difference to the results compared with the ordinary-least squares estimates (not reported). This is not surprising since the variance of the error is small relative to the variance of systematic component. Even though the correlations between the error term and the explanatory variables may be high, Wold's (1957) 'proximity theorem' suggests there will be only a very small bias in the ordinary-least squares estimators.[11]

The conclusion to be drawn is that there are either constant returns to scale or a degree of increasing returns that is small compared to the estimates of the 'dynamic' laws. The paradox is thus also found with the use of state data and it is necessary to next consider any possible explanation for this difference.

Our possible explanation of the paradox is that *both* the traditional dynamic and static laws are mis-specified. This possibility arises from a consideration of the specification and estimation of orthodox neoclassical production functions. The latter have been widely estimated using state data at the 2 digit S.I.C. level of aggregation though not for total manufacturing. (See, for example, Griliches (1967), Hildebrand and Liu (1965) and Moroney (1972).)

[11] Zellner et al (1966) have, in fact, argued that under the neoclassical assumption of the maximisation of expected profit when output is stochastic there will be no simultaneity bias in the use of ordinary-least squares to estimate the production function.

Moreover, the use of state data has a further advantage over international data. A central tenet of Kaldor's thesis is that a substantial part of the returns to scale arise through externalities—namely the benefits occurring through increasing division of labour which in turn is limited by the extent of the market. Apart from trying to reconcile the static dynamic paradox, the regional data allows us to test a form of the externalities hypothesis. This will also have implications for the determinants of national as well as regional productivity levels.

Commonly, neoclassical studies of the production function use 'per establishment' or 'mean data' (state aggregate data divided by the number of firms in the state) in preference to the use of simply the state total values. (The latter statistics are, of course, the values that have been used in the estimation of the dynamic and static laws.)

The rationale for the use of per establishment data is that the usual neoclassical approach views the production function as representing a microeconomic phenomenon. Consequently, since data on plant output, employment and capital are not readily available, this is proxied by the use of state per establishment data. The state is viewed analogously to the representative firm. Dividing the state total values by the number of establishments gives the average value for a firm in that state. It is with these data that the production functions are estimated.

The use of per establishment data and state aggregate data will only yield equivalent results if constant returns to scale prevail. Consequently, both specifications of the static and dynamic laws were estimated using per establishment data to determine what, if any, difference this specification makes to the estimates of the degree of returns to scale.

We have already noted that only small returns to scale were found with the earlier specifications of the static laws and the use of per establishment data makes no significant difference to the estimates of the degree of increasing returns. Perhaps more surprisingly, the results of the dynamic laws estimated using per establishment data show substantial increasing returns of the same magnitude as those obtained using state total values. (The per establishment results are not reported here.) Thus, this explanation of the paradox must be ruled out.

Related to the above discussion is a further consideration that must be taken into account when the suitability of state aggregate values (rather than the per establishment values) are considered. Kaldor places emphasis on the dictum that the division of labour is limited by the extent of the market. This has a spatial counterpart. If inputs have to be shipped over long distances, then the incurred transport costs may offset the benefits of increasing returns. This suggests that the use of the state as the basic unit of observation in the manner above may not be the most appropriate procedure. In order to capture this spatial dimension it would be preferable to use some function of the density of the values of the variables (i.e. the value per square mile) rather than the aggregate values. Ideally, the state urban area

TABLE 4

The Static Verdoorn Law, externalities and the density of production, 1958

(Number of Observations: 48)

The Verdoorn Law

(1) $\ln E = a + d_1 + d_2 + d_3 + 1.147 \ln Q - 0.162 \ln K$	$\bar{R}^2 = 0.997$
(13.07) (−1.86)	SEE = 0.151
(2) $\ln E = a + d_1 + d_2 + d_3 + 1.172 \ln Q - 0.145 \ln K - 0.0005\ IP$	$\bar{R}^2 = 0.997$
(14.61) (−1.83) (3.12)	SEE = 0.138
(3) $\ln E = a + d_1 + d_2 + d_3 + 1.182 \ln Q - 0.157 \ln K - 0.312 \ln IP$	$\bar{R}^2 = 0.997$
(14.15) (−1.92) (−2.58)	SEE = 0.142
Rowthorn's Specification	
(4) $\ln Q = a' + d_1' + d_2' + d_3' + 0.700 \ln E + 0.307 \ln K$	$\bar{R}^2 = 0.996$
(13.07) (5.87)	SEE = 0.118
(5) $\ln Q = a' + d_1' + d_2' + d_3' + 0.715 \ln E + 0.259 \ln K + 0.0004\ IP$	$\bar{R}^2 = 0.998$
(14.60) (5.16) (3.11)	SEE = 0.108
(6) $\ln Q = a' + d_1' + d_2' + d_3' + 0.702 \ln E + 0.271 \ln K + 0.259 \ln IP$	$\bar{R}^2 = 0.998$
(14.15) (5.41) (2.82)	SEE = 0.109

Note: $\ln Q$, $\ln E$ and $\ln K$ are the logarithms of aggregate values of the relevant variables divided by state area. The values are for 1958, while the values of the income potential (IP) are for 1956. The ds are again the regional dummies.

should be used to allow for differences in urbanisation. However, as this is not readily available, it was necessary to use the total land area of the state. Since, of course, this area does not alter over time, the specifications of the dynamic laws remains unaltered. In practice, as may be seen from Table 4, (equations (1) and (4)) this specification of the static law does not make any significant difference to the estimate of returns to scale.

While the density formulation represents a sounder theoretical specification than the use of aggregate values, it still ignores the potentially important external economies of scale that may result from the scale of production in the neighbouring states. For example, it is plausible that part of the gains in productivity from inter-industry specialisation may be due, for example, to a Boston firm being located not only in Massachusetts but being in close proximity to the whole of the North Eastern Industrial Belt. Nevertheless, it is to be expected that the benefits due to the level of output in a nearby state will decline with distance of the firm from that state, if only for the reason of increasing transport costs.

In order to test this hypothesis, the income potential was introduced as a proxy for this externality into the specification of the static laws. The income potential is derived from the gravity model and the latter is defined as

$$I_{ij} = \frac{KP_iP_j}{d_{ij}^{\phi}} \qquad (6)$$

where I_{ij} is the degree of interaction (whether it be of, for example, flows of commodities, people or externalities) between the spatial locations i and j. P_i

and P_j are the populations or some other variable such as the level of income or output. d_{ij} is the physical or economic distance between i and j. β is a parameter that influences the impact of distance on the degree of interaction. (In practice, ϕ is often set equal to unity.)

Equation (6) may be manipulated to derive the population potential as (setting ϕ equal to unity),

$$_iV_j = \frac{I_{ij}}{P_i} = \frac{KP_j}{d_{ij}} \tag{7}$$

A measure of the interaction between i and all other states in the union is required. Moreover, rather than population a measure of output potential is to be preferred. In the absence of data on output potentials, income had to be used as a proxy and in order to take account of the interaction between all the states. The income potential is defined as

$$_iIP = K \sum_j (Y_j d_{ij}^{-1}) \tag{8}$$

(see Richardson, 1974).

Warntz (1965) has calculated the income potential for 48 states (which excludes the District of Columbia as well as Alaska and Hawaii) for the year 1956, which makes it possible to test the externalities hypothesis although, of course, we are committed to accepting the conceptual and statistical problems inherent in his procedure. The hypothesis to be tested is that the level of state productivity is significantly influenced by the level of output in neighbouring states. The two equations to be estimated are similar to the static laws but with the income potential (*IP*) included as a regressor. As it is not clear precisely how the income potential should enter into the equations, the logarithmic values of the income potential were also included in an alternative specification. It is to be expected that the coefficient of the income potential will be negative in the Verdoorn Law and positive in Rowthorn's specification.

The results of the estimations of these equations are reported in Table 4, equations (2), (3), (5) and (6) and do indeed tend to confirm the hypothesis of the importance of external economies of scale.[12] The coefficients of IP and ln *IP* take the expected sign and are significantly different from zero. Given the close fit, the proximity theorem again suggests any bias due to simultaneity is likely to be small. The results imply that merely relating the state level of output to the value of the state inputs ignores an important external determinant level of productivity. On the other hand, it is found that the sum of the output elasticities α and β do not differ significantly from unity. Considering equations (3) and (6), where the income potential enters multiplicatively, the sum of α, β and γ (where γ is the output

[12] The specification using total factor input was also estimated but severe multicollinearity exists making any interpretation of the results difficult.

elasticity of the income potential) are 1.24 and 1.23 respectively. Although these values are not a conventional measure of returns to scale, they nevertheless reflect the combined magnitude of internal and external economies of scale.[13]

These results would tend to confirm Adam Smith's dictum (so strongly emphasised by Allyn Young and later by Kaldor) that "the division of labour is limited by the extent of the market" although in this case it is the spatial aspect that the model is capturing. (Allyn Young's views on the importance of external economies of scale and increasing returns have recently been discussed by Blitch (1983).)

5. Conclusions

There are two related controversies concerning the disparate rates of growth of productivity and output experienced by the advanced countries over the post-war period. The first is the extent to which the long term growth rates are demand constrained (through the impact of the balance of payments) or are limited by the growth of factor inputs (most notably the labour supply). Secondly, there is the dispute over the degree of increasing returns that are present in the manufacturing sector and whether economic growth proceeds in a cumulative causation manner.

The concern of this paper has been with the second problem and we have attempted to provide estimates of the degree of returns to scale not subject to the objections inherent in the use of international data. To this end, the Verdoorn Law was estimated by both ordinary-least squares and by instrumental variable approaches using state data drawn from the United States for the period from 1963 to 1973. The estimates of the degree of returns to scale obtained range from 1.33 to 1.65 depending upon the exact specification of the regression equation. These results provide strong confirmation for Kaldor's argument about the importance of increasing returns in the growth of the advanced countries.

A further anomaly that is found with the use of international data is that when the logarithmic values of the levels of the various variables are used, rather than exponential growth rates, either constant or comparatively very small returns to scale are found. This result persists with the use of regional data. Kaldor has placed considerable emphasis on the importance of external economies of scale following Adam Smith's dictum that the degree of division of labour is limited by the extent of the market. U.S. state data allows us to test a specification of this hypothesis through the use of the income potential as a measure of the degree of externality. While it is only possible to do this for the 'static' specifications, the income potential proves to be statistically significant, which suggests that external economies of scale are an important determinant of the level of productivity in manufacturing.

[13] The absence of data for state income potentials for any other year but 1956 precluded testing the externality hypothesis using the dynamic Verdoorn Law and hence determining whether this may be responsible for the paradox.

The most probable explanation of the divergence of results between the dynamic and static laws is that the conventional Cobb-Douglas production function is not the correct underlying structure of the Verdoorn Law. Consequently, the estimate of increasing returns derived from the static law will be biased. Unfortunately, it has not been possible to identify the correct functional form underlying the dynamic Verdoorn Law. Notwithstanding this, the main conclusion of this paper is that dynamic increasing returns to scale are an important factor in determining productivity growth.

Hull University,
I.C.I., Melbourne

REFERENCES

BLACK, J. (1962), "The Technical Progress Functions and the Production Function" *Economica*, 29, 166–170.

BLITCH, C. P. (1983), "Allyn Young on Increasing Returns" *Journal of Post Keynesian Economics*, V. 359–372.

CHATTERJI, M. and WICKENS, M. R. (1982), "Productivity, Factor Transfers and Economic Growth" *Economica*, 49, 21–38.

DOUGLAS, P. H. (1976), "The Cobb-Douglas Production Function Once Again: Its History, Its Testing and Some New Empirical Values" *Journal of Political Economy*. 84, 903–915.

GOMULKA, S. (1971), *Inventive Activity and the Stages of Economic Growth*. Aarhus, Institute of Economics.

GOMULKA, S. (1979), "Britain's Slow Industrial Growth—Increasing Inefficiency versus Low Rate of Technical Change" in W. Beckerman (ed.) *Slow Growth in Britain, Causes and Consequences*. Clarendon Press.

GRILICHES, Z. (1967), "Production Functions in Manufacturing: Some Preliminary Results" in *The Theory and Empirical Analysis of Production*, M. Brown (ed.) New York Bureau of Economic Research.

HILDEBRAND, G. and LIU, T. C. (1965), *Manufacturing Production Functions in the United States, 1957*. Cornell University Press.

JONES, H. G. (1975), *An Introduction to Modern Theories of Economic Growth*, Nelson.

KALDOR, N. (1966), *Causes of the Slow Rate of Growth of the United Kingdom, An Inaugural Lecture*, Cambridge University Press.

KALDOR, N. (1970), "The Case for Regional Policies" *Scottish Journal of Political Economy*, 17, 337–348.

KALDOR, N. (1975), "Economic Growth and Verdoorn's Law: A Comment on Mr. Rowthorn's Article" *Economic Journal*, 85, 891–896.

KALDOR, N. (1978), *Essays in Economic Theory*. Duckworth.

KENNEDY, K. A. (1971), *Productivity and Industrial Growth*. Clarendon Press.

KENNEDY, K. A. and FOLEY, A. (1978), *Industrial Development*. The Economic and Social Research Institute, Dublin.

KILPATRICK, A. and LAWSON, T. (1980), "On the nature of Industrial Decline in the U.K." *Cambridge Journal of Economics*, 4, 85–102.

MCCOMBIE, J. S. L. (1981), "What Still Remains of Kaldor's Laws" *Economic Journal*, 91, 206–216.

MCCOMBIE, J. S. L. (1982a), "How Important is the Spatial Diffusion of Innovations in Explaining Regional Growth Rate Disparities?" *Urban Studies*, 19, 377–382.

MCCOMBIE, J. S. L. (1982b), "Economic Growth, Kaldor's Laws and the Static-Dynamic Verdoorn Law Paradox" *Applied Economics*, 14, 279–294.

McCombie, J. S. L. (1983a), "Kaldor's Laws in Retrospect" *Journal of Post Keynesian Economics*, V, 414–429.

McCombie, J. S. L. (1983b), "Aggregation and the Verdoorn Law" (mimeo)

Moroney, J. R. (1972), *The Structure of Production in American Industry*. University of North Carolina Press.

Parikh, A. (1978), "Differences in Growth and Kaldor's Laws" *Economica*, 45, 83–91.

Richardson, H. W. (1974), "Agglomeration Potential: A Generalisation of the Income Potential Concept" *Journal of Regional Science*, 14, 325–336.

Rowthorn, R. E. (1975a), "What Remains of Kaldor's Laws?" *Economic Journal*, 85, 10–19.

Rowthorn, R. E. (1975b), "A Reply to Lord Kaldor's Comment" *Economic Journal*, 85, 897–901.

Salter, W. E. G. (1966), *Productivity and Technical Change*. Cambridge University Press.

Stoneman, P. (1979), "Kaldor's Law and British Economic Growth 1800–1970" *Applied Economics*, 11, 309–319.

Thirlwall, A. P. and Dixon, R. J. (1979), "An Export-Led Growth Model with a Balance of Payments Constraint" in J. Bowers (ed.) *Inflation, Development and Integration: Essays in Honour of A. J. Brown*, University of Leeds Press.

Thirlwall, A. P. (1978), "The U.K.'s Economic Problem: A Balance of Payments Constraint" *National Westminster Bank Review*.

Thirlwall, A. P. (1980a), "Regional Problems are Balance of Payments' Problems" *Regional Studies*, 14, 419–425.

Thirlwall, A. P. (1980b), "Rowthorn's Interpretation of Verdoorn's Law" *Economic Journal*, 90, 386–388.

Thirlwall, A. P. (1983), "A Plain Man's Guide to Kaldor's Laws" *Journal of Post Keynesian Economics*, V, 345–358.

Verdoorn, P. J. (1949), "Fattori Che Regolano lo Sviluppo Della Producttività del Lavoro" *L'Industria*.

Warntz, W. (1965), *Macrogeography and Income Fronts*. Philadelphia: Regional Science Research Institute Monograph Series No. 3.

Wold, H. and Faxer, R. (1957), "On the Specification Error in Regression Analysis. *Annals of Mathematical Statistics*, 28, 265–267.

Wraggs, W. and Robertson, J. (1978), "Post-War Trends in Employment, Productivity, Output, Labour Costs and Prices by Industry in the United Kingdom" *Department of Employment, Research Paper No.* 3.

Young, A. A., (1928), "Increasing Returns and Economic Progress" *Economic Journal*, 38, 527–542.

Zellner, A., Kmenta, J. and Dreze, J. (1966), "Specification and Estimation of Cobb-Douglas Production Function Models" *Econometrica*, 34, 784–795.