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Factors that Determine the Growth of Labour Productivity

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[Translated by A.P. Thirlwall from the original 1949 article in Italian.[1]]

1. One of the difficulties in long-term planning is to estimate the future level of labour productivity. Unless this is known, one does not know the relation between output and employment.

Since it cannot be assumed that the annual rate of growth of labour productivity will be constant, and the production function cannot be used, an alternative method of estimating the future level of labour productivity is suggested.

2. The statistics available for the periods 1870 to 1914 and 1914 to 1930 for various countries suggest the existence of a fairly constant relation over a long period between the growth of labour productivity and the volume of industrial production.

From analysing the historical series for industry as a whole (Table 2.1) and for individual industrial sectors, for the two time periods, it is found that the average value of the elasticity of productivity with respect to output is approximately 0.45 (with limits of 0.41 and 0.57). This means that over the long period a change in the volume of production, say of about 10 per cent, tends to be associated with an average increase in labour productivity of 4.5 per cent.

3. In fact, one could have expected *a priori* to find a correlation between labour productivity and output, given that the division of labour only comes about through increases in the volume of production; therefore the expansion of production creates the possibility of further rationalisation which has the same effects as mechanisation.

This interdependence of a purely theoretical character does not by itself imply that the elasticity will be constant because in practice it will be influenced by various economic factors; none the less, it can be demonstrated (see the Appendix) that under the normal assumptions of long-period analysis the elasticity assumes a mathematical form that tends to make it – within reasonable limits – fairly independent of variations in such economic factors.

Moreover, it is found that when the economic conditions of the various countries and different periods of time are taken into account, the values of

Table 2.1 Annual increases in volume of production and labour productivity in industry

Period	*Country*	*Annual change* Production %	Labour productivity %		*Elasticity*
1913–1930	Switzerland	2.40	1.03	0.43	
1841–1907	UK	2.40	0.98	0.41	0.43
1907–1930		1.28	0.605	0.47	
1869–1899	USA	5.61	2.31	0.42	0.42
1899–1939		3.35	1.91	0.57	
1882–1907	Germany	4.38	2.14	0.49	(1859–1939)
		Period between the wars			
1924–1938	Switzerland	5.0	5.3	1.06	Regression equation $d \log \frac{x}{a} = 0.573\, d \log, x + 0.00239$
1926–1938	Japan	6.7	3.4	0.51	
1924–1938	Finland	5.1	3.2	0.63	
1927–1938	Hungary	3.4	2.8	0.82	
1924–1938	Holland	2.3	2.6	1.13	
1924–1938	Norway	2.6	2.5	0.96	
1924–1938	Denmark	3.5	1.9	0.54	
1927–1938	Poland	1.6	1.9	1.18	
1924–1938	UK	1.4	1.5	1.07	
1924–1939	USA	0.6	1.0	1.67	
1924–1938	Canada	1.6	1.0	0.63	
1924–1938	Czechoslovakia	0.4	0.7	—	
1927–1938	Estonia	0.8	0.4	0.50	
1924–1938	Italy	0.8	0.2	0.25	

the elasticities calculated theoretically are of the same size as those found empirically.

4. While the hypothesis of constant elasticity is not in practice very suitable for making forecasts, it can nevertheless be used profitably as one criterion for making a judgement, on the basis of past experience, about the realisation of long-term plans.

(a) If in a plan we have the data available on labour requirements and the data on production, and the value of the elasticity falls within the limits that have been found empirically, then we can say that the plan under study, from the point of view of labour productivity alone, is technically possible and economically plausible.

(b) If instead data are only available on labour productivity, labour requirements can be forecast on the basis of historical values of the elasticity,

Table 2.2 Comparison of the productivity elasticities based on the Monnet and Saraceno plans and their historical values

Industrial sector	*Italy (Saraceno)*	*France (Monnet)*	*Historical value of the elasticity*		
			Value	*Country*	*Period*
1. Automobiles	—	0.65	0.70	USA	1919–1929
2. Rubber	0.52	—	0.60	Holland	1922–1939
3. Food	0.51	—	0.51	USA	1899–1937
4. Wood	0.52	—	0.46	USA	1899–1937
5. Construction Material	0.42	0.32	—	—	—
6. Paper	0.35	—	0.44	USA	1899–1937
7. Chemicals	0.35	—	0.29	USA	1899–1937
8. Public Utilities	0.11	—	—	—	—
9. Metals	0.29	0.60	0.52	France	1890/94–1924/29
Blast Furnaces	—	—	1.52	USA	1899–1937
Iron Products	—	—	0.31	USA	1899–1937
10. Textiles	0.77	0.45	0.44	USA	1899–1937
Cotton	—	—	0.46	France	1873/79–1926/36
	—	—	0.51	USA	1899–1937
Artificial Silk	—	—	0.68	USA	1899–1937
	—	—	0.87	Holland	1922–1939
11. Clothing	0.42	—	—	—	—
Average	0.52*	0.51	0.57	USA	1899–1937

Note:
*Including mining.
Sources: Italy: *Elementi per un piano* ecc., September 1947 (n. 7) p. 125.
France: *Premier plan de Modernisation* ecc., November 1946–January 1947, p. 78.

and the soundness of the plan can be judged on the basis of the availability of labour.

(c) On the other hand, in the cases in which a plan does not exist, the value of the elasticity of productivity gives a rough idea of how much industrial production must expand to absorb a certain availability of labour.

5. Finally, this method allows us to make separate calculations for individual industrial sectors. If the historical elasticities are calculated for sectors instead of for industry as a whole, one takes into account differences in technical and economic conditions existing between industries (for example, differences in the production function and in the elasticity of labour supply).
6. Until now[2] only the Monnet and Saraceno Plans have given data relating to both labour and production. In Table 2.2 a comparison is made between

the value of the elasticity calculated on the basis of these two plans and some historical values.

In this table it is evident that on the whole there exists a rather close relation between the three series of values; considerable divergencies are found in the case of textiles and metallurgy, but here a more detailed subdivision of the data of the plans would be necessary because of the heterogeneity of the technical production relations employed in the same principal branches of the two industrial sectors (rayon in comparison with cotton, blast furnaces in comparison with rolling mills).

The lack of precise data on investment does not allow us to establish how much of the divergencies found in these two sectors, and other smaller divergencies, have been influenced by differences in investment policy.

7. As a general rule to follow, if new plans are available in the future, it is suggested that the years 1937 or 1938 should be taken as a starting point for analysis rather than 1947 or 1948. These latter years are still influenced by the consequences of the war. If 1952/53 or 1960 are the final years of the plan the important characteristics of the period of reconstruction can be considered to have disappeared.

In comparing 1938 with 1952/53 normal conditions can be considered to prevail, bearing in mind permanent changes due to the war.

If a close correspondence is found to exist in each individual industry in the three[3] countries between the increase in production and capital and labour requirements, it is possible to obtain a number of normal [elasticity] values for the different industries.

In the case of wide divergencies an analysis of a more general character is suggested. Taking into account other variables (such as the development of production techniques; the amount of unused capacity in 1938; the relation between total labour and capital requirements and so on) an attempt can be made to find some less rigid relations between labour and capital requirements in the industries under examination; for this purpose a method is outlined in the Appendix which, although it cannot be applied in practice, serves to establish some starting points for research along these lines.

The choice of the most efficient and practical method will depend on the quality and quantity of the statistical material available. However, leaving aside the method that may be chosen it is clear that, proceeding in this way, concrete and quantitative criteria can be obtained to judge the compatibility of the labour market compared with other aspects of the plan.

Appendix

1. Conditions for a stable relation between labour productivity and output. If we let:
   a be the quantity of labour[4]
   $\dot{a}$ be the first derivative with respect to time
   x be the volume of production

$\dot{x}$ be the first derivative with respect to time
the elasticity of labour productivity with respect to output can be written:

$$K = \frac{\frac{d}{dt}\left(\frac{x}{a}\right)}{\frac{x}{a}} \Bigg/ \frac{\dot{x}}{x} = \frac{a}{\dot{x}} \cdot \frac{a\dot{x} - x\dot{a}}{a^2}$$

or

$$\dot{K} = 1 - \frac{\dot{a}/a}{\dot{x}/x}$$

Assuming the production function is Cobb–Douglas[5]:

$$x = a^{\alpha} b^{\beta} \qquad (b \text{ is capital})$$

and differentiating with respect to time:

$$\begin{aligned} \dot{x} &= \alpha a^{\alpha-1} b^{\beta} \dot{a} + \beta b^{\beta-1} a^{\alpha} \dot{b} \\ &= \alpha x \frac{\dot{a}}{a} + \beta x \frac{\dot{b}}{b} \end{aligned}$$

one obtains:

$$\begin{aligned} \frac{\dot{x}}{x} &= \alpha \frac{\dot{a}}{a} + \beta \frac{\dot{b}}{b} \\ \frac{\dot{x}/x}{\dot{a}/a} &= \alpha + \beta\left(\frac{\dot{b}/b}{\dot{a}/a}\right) \end{aligned}$$

from which:

$$K = 1 - \frac{1}{\alpha + \beta\left(\frac{\dot{b}/b}{\dot{a}/a}\right)}. \qquad \text{(I)}$$

If α and β are assumed to be constant, the constancy of *K* evidently depends on the constancy of the relation $\dot{b}/b{:}\dot{a}/a$.
2. The constancy of the elasticity of capital with respect to labour can be proved using a system of equations similar to that developed by Tinbergen.[6]
For our purposes the following equations will be sufficient:

3. – I: System of equations

Production equation: $x = a^{\alpha} b^{\beta}$ (1)

Labour demand: $$v = \alpha \cdot \frac{x}{a} \tag{2}$$

Labour supply: $$v = \alpha\left(\frac{a}{p}\right)^{\rho} e^{\lambda t} \tag{3}$$

Capital supply: $$\dot{b} = \gamma x \tag{4}$$
Population: $$p = e^{\pi t} \tag{5}$$

In equation (2): the demand for labour: the average wage (v) is equal to the marginal product of labour.

In equation (3): the supply of labour: this equation can also be written:

$$\frac{a}{p} = \left(\frac{v}{\ell}\right)^{\rho 1} \text{ where,}$$

- a is the number of people employed in industry,
- p is the total active population,
- ℓ is average wage in non-industrial production,
- ρ^1 is essentially an elasticity of competition: in fact the percentage of labour supply in industry is determined by the relation between the average wage in industry and that in other branches of production. In equation (3) it is assumed that the average wage increases at the constant annual rate e^{λ}. According to Tinbergen, the factor $e^{\lambda t}$ in equation (3) may be considered as indicating the increased demands of trade unions for higher wages.

If for the initial value ($t = 0$) of a, p and b we assume the number 1, the constant in equation (3) is α.

In equation (4): γ is the average propensity to invest.

In equation (5): a constant annual increase is assumed (e^{π}).

3. – II: $\dot{a}/a$.

From equations (3) and (5):

$$\begin{aligned} v &= \alpha\left(\frac{a}{p}\right)^{\rho} e^{\lambda t} \\ &= \alpha \cdot a^{\rho} \cdot e^{-\pi t p} \cdot e^{\lambda t} \\ &= \alpha a^{\rho} e^{(\lambda - \pi p)^t} \\ &= \alpha \cdot a^{\rho} e^{\mu t} \end{aligned}$$

(where $\mu = \lambda - \pi p$).

From equation (6) and (2) we have:

$$\alpha a^{\rho} e^{\mu t} = \alpha \frac{x}{a}$$

but from equation (1) $x = a^{\alpha} \cdot b^{\beta}$, therefore:

$$a^{\rho} e^{\mu t} = \frac{a^{\alpha} b^{\beta}}{a}. \tag{6}$$

It follows that:

$$a^{\rho} e^{\mu t} = a^{\alpha-1} b^{\beta}, \text{ from which}$$
$$a = b^{\beta/w} e^{-\mu t/w} \tag{7}$$
(where $w = 1 + \rho - \alpha$).

Differentiating equation (7) with respect to time gives:

$$\dot{a} = \frac{\beta}{w} \cdot b^{\frac{\beta}{w}-1} e^{-\frac{\mu}{w}t} \dot{b} - \frac{\mu}{w} b^{\beta/w} e^{-\mu t/w}, \text{ and therefore}$$
$$\frac{\dot{a}}{a} = \frac{\beta}{w} \cdot \frac{\dot{b}}{b} - \frac{\mu}{w}.$$

Equation (11) gives a relation between $\dot{a}/a$ and $\dot{b}/b$. However, it only considers the equations (1) (2) and (3) of 3. – I and therefore neglects the dependence of b on the other variables in the system as given by equation (4).

3. – III: $\dot{b}/b$

From equation (4) we can write: $\dfrac{\dot{b}}{b} = \gamma \dfrac{x}{b}$ (8)

Since we can choose freely the instant for which $t = 0$, we take $t = 0$ for the year for which the elasticity is to be calculated. However, in such a case, we are tied by the initial values for the variables considered, as assumed in 3. – I; for example:

$$a_0 = b_0 = p_0 = 1: \; v_0 = \alpha.$$

Therefore, it follows from equation (1) that $x_0 = 1$, and from equation (8):

$$\frac{\dot{b}_0}{b} = \gamma \frac{x_0}{b_0} = \gamma.$$

Dividing equation (II) by $\dfrac{\dot{b}_0}{b_0}$, we find:

$$\frac{\dot{a}_0}{a} \bigg/ \frac{\dot{b}_0}{b_0} = \frac{\beta}{w} - \frac{\mu}{\gamma w}. \qquad \text{(III)}$$

3. – IV: K

Substituting equation (III) into (I) we find:

$$K = 1 - \cfrac{1}{\alpha + \cfrac{\beta}{\cfrac{\beta}{w} - \cfrac{\mu}{\gamma w}}}$$

$$= 1 - \cfrac{1}{\alpha + \cfrac{w}{1 - \cfrac{\mu}{\beta\gamma}}}$$

From which, letting $\mu = \lambda - \pi\rho$
$w = 1 + \rho - \alpha,$

$$K = 1 - \frac{1 - \frac{\mu}{\beta\gamma}}{\alpha - \frac{\alpha\mu}{\gamma\beta} + 1 + \rho - \alpha}$$

$$= \frac{\rho + (1-\alpha)\frac{\mu}{\beta\gamma}}{\rho + 1 - \frac{\alpha\mu}{\beta\gamma}}$$

and finally:

$$K = \frac{\rho\left(1 - \frac{1-\alpha}{\beta}\cdot\frac{\pi}{\gamma}\right) + \frac{1-\alpha}{\beta}\cdot\frac{\lambda}{\gamma}}{\rho\left(1 + \frac{\alpha}{\beta}\cdot\frac{\pi}{\gamma}\right) + 1 - \frac{\alpha}{\beta}\cdot\frac{\lambda}{\gamma}} \qquad \text{(IV)}$$

The stability of K can easily be seen taking different combinations of π and λ (taking as given α, β and γ). It appears therefore that quite considerable modifications would be necessary for K to lie outside certain limits, for example ± 0.15 around an initial value of 0.45.[7] Analogous conclusions would be reached if variations in α, β and γ were taken for fixed values of π and λ.

Notes

1. 'Fattori che Regolano lo Sviluppo della Produttività del Lavoro', *L'Industria*, 1949. This paper is the origin of 'Verdoorn's Law'. Verdoorn died in 1985 and the translation was not authorised by the author prior to his death.
2. *i.e.* up to August 1948.
3. *Translator's note*: it is not clear what three countries the author has in mind.
4. *Translator's note*: Verdoorn defines a as labour productivity. This is clearly a mistake.
5. The Cobb–Douglas production function has been chosen to represent the relation between production, capital and labour because it has been used a long time as a theoretical device. However, it can be proved that also using a more general formulation of the production function the same formula can be obtained as those described below.
6. *Weltwirtschaftliches Archiv*, May 1942, p. 530.
7. *Translator's note*: Letting $\alpha = 0.7$; $\beta = 0.3$; $\rho = 1$; $\pi = 0.01$; $\gamma = 4$; $\lambda = 0.01$ gives $K = 0.5$.