LIQUIDITY PREFERENCE AND THE THEORY OF INTEREST AND MONEY

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PART I

1. INTRODUCTION

THE AIM OF this paper is to reconsider critically some of the most important old and recent theories of the rate of interest and money and to formulate, eventually, a more general theory that will take into account the vital contributions of each analysis as well as the part played by different basic hypotheses.

The analysis will proceed according to the following plan:

I. We start out by briefly re-examining the Keynesian theory. In so doing our principal aim is to determine what is the part played in the Keynesian system by the "liquidity preference," on the one hand, and by the very special assumptions about the supply of labor, on the other. This will permit us to distinguish those results that are due to a real improvement of analysis from conclusions that depend on the difference of basic assumptions.

II. We then proceed to consider the properties of systems in which one or both Keynesian hypotheses are abandoned. We thus check our previous results and test the logical consistency of the "classical" theory of money and the dichotomy of real and monetary economics.

III. From this analysis will gradually emerge our general theory of the rate of interest and money; and we can proceed to use this theory to test critically some recent "Keynesian" theories and more especially those formulated by J. R. Hicks in *Value and Capital*[1] and by A. P. Lerner in several articles.

IV. Finally, to make clear the conclusions that follow from our theory, we take issue in the controversial question as to whether the rate of interest is determined by "real" or by monetary factors.

In order to simplify the task, our analysis proceeds in general, under "static" assumptions; this does not mean that we neglect time but only that we assume the Hicksian (total) "elasticity of expectation" to be always unity. In Hicks's own words this means that "a change in current prices will change expected prices in the same direction and in the same proportion."[2] As shown by Oscar Lange, this implies that we assume the "expectation functions," connecting expected with present prices, to be homogeneous of the first degree.[3]

[1] J. R. Hicks, *Value and Capital*, Oxford University Press, 1939, 331 pp.

[2] *Ibid.*, p. 205.

[3] Cf. O. Lange, "Say's Law: a Restatement and Criticism" in *Studies in Mathematical Economics and Econometrics*, edited by Lange, McIntyre, and Yntema, The University of Chicago Press, 1942, pp. 67–68.

Since all the theories we examine or formulate in this paper are concerned with the determinants of equilibrium and not with the explanation of business cycles, this simplification, although it is serious in some respects, does not seem unwarranted.

2. THREE ALTERNATIVE MACROSTATIC SYSTEMS

As a first step in the analysis, we must set up a system of equations describing the relation between the variables to be analyzed. In doing this we are at once confronted with a difficult choice between rigor and convenience; the only rigorous procedure is to set up a complete "Walrasian" system and to determine the equilibrium prices and quantities of each good: but this system is cumbersome and not well suited to an essentially literary exposition such as we intend to develop here. The alternative is to work with a reduced system: we must then be satisfied with the rather vague notions of "physical output," "investment," "price level," etc. In what follows we have chosen, in principle, the second alternative, but we shall check our conclusions with a more general system whenever necessary.

The equations of our system are:

$$M = L(r, Y), \tag{1}$$

$$I = I(r, Y), \tag{2}$$

$$S = S(r, Y), \tag{3}$$

$$S = I, \tag{4}$$

$$Y \equiv PX, \tag{5}$$

$$X = X(N), \tag{6}$$

$$W = X'(N)P. \tag{7}$$

The symbols have the following meaning: Y, money income; M, quantity of money in the system (regarded as given); r, rate of interest; S and I, saving and investment respectively, all measured in money; P, price level; N, aggregate employment; W, money wage rate; X, an index of physical output.[4] We may also define C, consumption measured in money, by the following identity:

$$C \equiv Y - I. \tag{8}$$

Identity (5) can be regarded as defining money income. There are

[4] This system is partly taken from earlier writings on the subject. See especially O. Lange, "The Rate of Interest and the Optimum Propensity to Consume," *Economica*, Vol. 5 (N. S.), February, 1938, pp. 12–32, and J. R. Hicks, "Mr. Keynes and the 'Classics'; A Suggested Interpretation," ECONOMETRICA, Vol. 5, April, 1937, pp. 147–159.

so far 8 unknowns and only 7 equations; we lack the equation relating the wage rate and the supply of labor. This equation takes a substantially different form in the "Keynesian" system as compared with the "classical" systems.

In the classical systems the suppliers of labor (as well as the suppliers of all other commodities) are supposed to behave "rationally." In the same way as the supply of any commodity depends on the relative price of the commodity so the supply of labor is taken to depend not on the money wage rate, but on the real wage rate. Under the classical hypothesis, therefore, the last equation of the system takes the form:

$$(9a) \quad N = F\left(\frac{W}{P}\right); \text{ or, in the inverse form: } W = F^{-1}(N)P.$$

The function F is a continuous function, although not necessarily monotonically increasing.

The Keynesian assumptions concerning the supply-of-labor schedule are quite different. In the Keynesian system, within certain limits to be specified presently, the supply of labor is assumed to be perfectly elastic at the historically ruling wage rate, say w_0. The limits mentioned above are given by equation (9a). For every value of W and P the corresponding value of N from (9a) gives the maximum amount of labor obtainable in the market. As long as the demand is less than this, the wage rate remains fixed as w_0. But as soon as all those who wanted to be employed at the ruling real wage rate w_0/P have found employment, wages become flexible upward. The supply of labor will not increase unless the money wage rate rises relative to the price level.

In order to write the last equation of the "Keynesian" form of our system, we must express this rather complicated hypothesis in functional form. Taking (9a) as a starting point, we may write:

$$(9) \quad W = \alpha w_0 + \beta F^{-1}(N)P,$$

where α and β are functions of N, W, P, characterized by the following properties:

$$(10) \quad \begin{aligned} &\alpha = 1, \quad \beta = 0, \quad \text{for} \quad N \leqq N_0, \\ &\alpha = 0, \quad \beta = 1, \quad \text{for} \quad N > N_0, \end{aligned}$$

where N_0 is said to be "full employment." Equations and inequalities (10) thus state that, unless there is "full employment" ($N = N_0$), the wage rate is not really a variable of the system but a datum, a result of "history" or of "economic policy" or of both. Equation (9) then reduces to $W = w_0$. But after "full employment" has been reached at wage rate w_0, the supply of labor ceases to be perfectly elastic: W becomes a vari-

able to be determined by the system and (9) becomes a "genuine" equation. We should add that, even in the "Keynesian" system, it is admitted that the wage rate will begin to be flexible downward before employment has reached the zero level: but in order not to complicate equation (9) still further we can, without serious harm, leave the hypothesis in its most stringent form.

For generality we may also use equation (9) as it now stands, as the "supply of labor" function of the "classical" theory. But instead of conditions (10) we have the identities (for all values of N)

$$\alpha \equiv 0, \qquad \beta \equiv 1. \tag{11}$$

Some remarks are also necessary concerning the "demand for money" equation. According to the "quantity theory of money," the demand for money does not depend on the rate of interest but varies directly with money income. Under this hypothesis equation (1) reduces to

$$M = kY. \tag{1a}$$

By properly combining the equations and conditions written above, we obtain three different systems which we will analyze in turn.

I. A "Keynesian" system consisting of equations (1) to (7) and (9) and conditions (10).

II. A "crude classical" system consisting of equations (1a), (2) to (7), and (9), and identities (11).

III. A "generalized classical" system consisting of the equations listed under II but with (1a) replaced by (1).

3. A RECONSIDERATION OF THE KEYNESIAN THEORY

In reconsidering the Keynesian system we shall essentially follow the lines suggested by J. R. Hicks in his fundamental paper, "Mr. Keynes and the 'Classics.'"[5] Our main task will be to clarify and develop his arguments, taking into account later theoretical developments.

Close consideration of the Keynesian system of equations [equations (1) to (7) and (9) to (10)] reveals that the first 4 equations contain only 4 unknowns and form a determinate system: the system of monetary equilibrium. We therefore begin by discussing its equations and its solution.

4. THE TRANSACTION DEMAND FOR MONEY

In a free capitalistic economy, money serves two purposes: (a) it is a medium of exchange, (b) it is a form of holding assets. There are accordingly two sources of demand for money: the transaction demand for money and the demand for money as an asset. This is the fundamental proposition on which the theory of the rate of interest and

[5] ECONOMETRICA, Vol. 5, April, 1937, pp. 147–159.

money rests; it is therefore necessary to analyze closely each source of demand and the factors that determine it.

The transaction demand for money is closely connected with the concept of the income period. We may define the income period as the (typical) time interval elapsing between the dates at which members of the community are paid for services rendered. We shall assume for the moment that this income period is approximately the same for every individual and that it coincides with the expenditure period.[6]

Each individual begins the income period with a certain income arising out of direct services rendered or out of property and with assets (physical and nonphysical) having a certain market value. In his endeavor to reach the highest level of satisfaction he is confronted with two sets of decisions: (a) he must decide what part of his income he will spend on consumption and what part he will save, (b) he must determine how to dispose of his assets.

The first set of decisions presents no special difficulty of analysis. On the basis of his tastes, his income, and market prices he will make a certain plan of expenditure to be carried out in the course of the income period. The amount of money that is necessary for individuals to carry out their expenditure plans is the *transaction demand for money by consumers*, as of the beginning of the period. The average transaction demand, on the other hand, depends on the rate at which expenditure takes place within the period.[7]

The difference between the individual's money income and the amount he decides to spend in the fashion discussed above is the money value of his savings (dissavings) for the income period. It represents the net increment in the value of his assets.

5. THE DEMAND FOR MONEY AS AN ASSET

Having made his consumption-saving plan, the individual has to make decisions concerning the assets he owns. These assets, let us note, consist of property carried over from the preceding income period *plus current savings*.

There are essentially three forms in which people can keep their assets: (a) money, (b) securities,[8] and (c) physical assets.

[6] This means, for instance, that people are required by custom or contract to pay within the income period for what they have consumed in the period (rent, grocery bill, etc.) or else must rely on "consumers' credit."

[7] Thus if expenditure should proceed at an approximately even rate, it would be one-half the initial demand.

[8] Under the name of securities we include both fixed-income-bearing certificates and common stocks or equities. From the strictly economic point of view, common stocks should perhaps be considered as a form of holding physical assets. For institutional reasons, however, equities have very special properties which make them in many respects more similar to bonds than to physical assets.

We shall for the moment eliminate the third alternative by distinguishing between entrepreneurial and nonentrepreneurial decisions. We consider as entrepreneurs individuals who hold assets in physical form; decisions concerning the acquisition or disposal of physical assets will accordingly be treated as entrepreneurial decisions and will be analyzed in connection with the schedule of the propensity to invest [equation (3)]. An individual's decision to acquire directly physical assets (say a house) or to reinvest profits in his enterprise can be split into two separate decisions, a decision to lend (to himself) and a decision to increase his entrepreneurial risk by borrowing (from himself).

We are therefore concerned here exclusively with decisions concerning nonphysical assets and with those factors that influence the choice between the first two alternatives. Our problem is to determine whether there is any reason for individuals to wish to hold some or all of their assets in the form of money and thus to demand money over and above the quantity they need for transactions.

In this respect there is little to add to the exhaustive treatment that this subject has received in recent literature.[9]

There are two properties that all assets, whether physical or not, share in different degrees: liquidity and risk. Following a criterion particularly stressed by Jacob Marschak, we shall define liquidity of an asset in terms of the perfection of the market in which it is traded. An asset is liquid if this market is perfect, i.e., an individual's decision to buy or sell does not affect the price finitely; it is illiquid in the opposite case. It is riskless if the price at which it sells is constant or practically so; it is risky if the price fluctuates widely.

Securities clearly share with money the property of being highly liquid assets. Where there is an organized market, securities will not be significantly inferior to money in this respect. They have, however, two clear drawbacks in comparison with cash:

(a) They are not a medium of exchange. Assets generally accrue in the form of money through savings, and a separate transaction is necessary to transform them into securities. This transaction involves both subjective and objective costs.

(b) They are more risky than money since their market price is not constant. Even the "safest" type of securities, on which the risk of default can be neglected, fluctuates in price as the rate of interest moves. There are, it is true, some types of loans for which this last risk can be neglected, namely very-short-term loans. Let us assume, for the sake

[9] See, for instance, J. R. Hicks, *Value and Capital*, Chapters XIII and XIV and *passim;* J. M. Keynes, *The General Theory of Employment, Interest and Money*, New York, Harcourt, Brace and Company, 1936, 403 pp.; Mabel Timlin, *Keynesian Economics*, University of Toronto Press, 1942, Chapters V and VI; etc.

of precision, that the money market is open only on the first day of the income period; then the shortest type of loans will be those that mature at the end of said period. These types of assets will not be subject to the risk mentioned under (b) since, by assumption, the rate of interest cannot change while they are outstanding.[10]

It is just for this type of assets, however, that the disadvantage mentioned under (a), namely the cost of investment, weighs more heavily: for the yield they promise for the very short duration of the loan can only be small, so that even a moderate cost is sufficient to wipe it out. If, as is likely, the cost of investment does not rise in proportion to the amount invested, then short loans may be an interesting investment for large sums, but not so for small investors. Thus, if this were the only possible form of investment, we should expect that any fall in the rate of interest, not accompanied by a corresponding fall in the cost of investing, would induce a growing number of potential investors to keep their assets in the form of money, rather than securities; that is to say, we should expect a fall in the rate of interest to increase the demand for money as an asset.

In this respect, securities of longer maturity would appear to be superior, since the yield to be gathered by holding them until maturity is larger, while the cost of acquiring them need not be different. But as the importance of the cost element decreases, the importance of the risk element grows. As is well known, a given change in the rate of interest will affect most the present value of those bonds whose maturity is furthest away. If the only reason for owning assets were to earn the income they produce, these price fluctuations would not be so important. For, as long as the owner is in a position to hold the asset until maturity, there would be only a potential loss, a loss of better opportunities. There can be little doubt, however, that for a large part of the community the main reason for holding assets is as a reserve against contingencies. A form of assets whose value is not certain must be, *ceteris paribus*, inferior to one whose value is certain, namely money.

This very fact, besides, gives an additional reason why bonds of longer maturity should be a less safe form of holding assets. For there is much less certainty about faraway income periods than there is about the near future and the possibility that one will have to realize the assets before their maturity, if any, increases accordingly; while, on the other hand, it becomes increasingly difficult to make reliable forecasts about the level of the rate of interest and the future market value of the assets.

[10] Even if this assumption were relaxed, the possible fluctuations in the rate of interest would be negligible and the extent to which they would affect the present value of the securities mentioned above could be disregarded.

Securities, on the other hand, are clearly superior to money in that they yield an income. The ruling rate of interest measures the remuneration to be obtained by accepting the drawbacks and assuming the risks that are characteristic of securities as compared with money. Or, to look at it from another point of view, it measures the cost of holding money instead of securities in terms of forgone income. Thus a fall in the rate of interest has, in any event, the effect of making cash cheaper and hence more attractive as a form of holding assets.

In addition, several other reasons can be mentioned that cause a low rate of interest to discourage the holding of securities. In the first place, the risk element involved in holding securities becomes more pronounced when the rate of interest is low, for a smaller fall in the capital value of the asset is sufficient to wipe out the income already earned by holding the asset. Thus, for instance, the smaller the rate of interest, the smaller is the *percentage change* in the rate itself necessary to absorb the yield obtained by holding the asset a given length of time. Again, it has been pointed out by some authors that, as the rate of interest becomes lower, there is some ground to expect that possible movements will be predominantly in the direction of an increase and therefore unfavorable to the holders of securities.

In conclusion then, the lower the rate of interest, the larger will be the number of owners of assets who will prefer to hold these assets in the form of money for the income period; the demand for money to hold (as distinguished from money to spend, previously considered) or demand for money as an asset is a decreasing function of the rate of interest. Denoting this demand by D_a, we can write

$$D_a = D_a(r)$$

for the schedule of demand for money to hold.

What can we say about the characteristics of this function? It must clearly be a monotonically decreasing function of the rate of interest; in addition, however, it must have, in the author's opinion, two important properties:

In the first place, there must be some value of r, say r', such that $D_a(r) = 0$ for $r \geqq r'$. For there must be, for every individual, some minimum net yield per income period that will induce him to part entirely with money as an asset. Hence, if he can find some type of securities such that by holding them for a given number of income periods he expects to obtain a net yield equal to or larger than the minimum, his demand for money to hold will fall to zero.[11]

[11] Let i_0 denote the minimum yield (per income period) at which an individual is ready to hold no assets in the form of money during the period. We may also assume, without being unrealistic, that this minimum yield is the same for each

Since this is true for every individual, there must also be some system of interest rates which is sufficient to reduce the aggregate demand to zero.

The second characteristic is more peculiar. Since securities are an "inferior" way of holding assets, it is generally recognized that there must be some minimum rate of interest, say r'', at which nobody will be willing to hold nonphysical assets except in the form of money. When this level is reached, the demand for money to hold becomes "absolute" and the rate of interest cannot fall any lower. Hence, $D_a'(r) = \infty$ for $r \gtrless r''$.

6. THE DEMAND FOR MONEY: CONCLUSION

We have so far discussed the demand for money as an asset and the transaction demand for money by individuals; to complete the analysis we must consider the transaction demand by firms. In principle, the same considerations apply here as were stated in connection with individuals' transaction demand. Firms, as well as individuals, have an institutional expenditure-receipt pattern and, given this pattern, the average demand depends on the volume of transactions. We must however recognize that, in the case of firms, generalizations are less meaningful since their expenditure and receipt flows are generally less certain and uniform than for individuals.

Then, too, we must admit that we may have oversimplified the consumers' transaction demand by assuming that individuals have a rigorously defined plan of expenditure at the beginning of the income period. It may very well be that under more realistic conditions they will de-

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income period. Suppose that the securities which, in his opinion, present the best opportunity are expected by him to produce a net yield (including capital appreciation) i_0', i_1', $\cdots$, i_n' in periods 1, 2, $\cdots$, n. He will be induced to invest provided there is some value of n for which

$$(1 + i_0')(1 + i_1') \cdots (1 + i_n') \geqq (1 + i_0)^n.$$

From M. Timlin's treatment of this subject (*Keynesian Economics*, Chapter III) it would appear that marginal holders should expect any security to yield the same net income, at least during the current period. This however is correct only if the expectations of all dealers about the future short rates of interest agree with the market expectation as shown by the forward rates established in the market. [The forward rate for the nth income period ahead can always be found by comparing the price of riskless securities maturing n periods ahead with those maturing $(n+1)$ periods ahead.] But if an individual believes this forward rate to be too high he may acquire the security at once even though he may expect that it will yield in the current period less than some other security. For, assuming that he is right, he will be able to realize his capital gain as soon as the market recognizes its error and there is no telling when this will occur. If he should wait until the next income period and hold for the current one the asset that promises to pay a higher yield, he may lose his chance of making the expected capital gain.

sire to carry some cash above the amount they plan to spend as a reserve and to avoid ending the period with a zero cash balance. This however does not substantially affect our argument. All we are interested in establishing is that, within an institutional framework, there must be for any given volume (value) of transactions a certain amount of money that is necessary to carry them out. This amount clearly depends on such institutional factors as the length of the income period and the prevailing customs as to the settlement of current purchases by firms and must therefore be substantially independent of the level of the rate of interest. The level of the rate of interest influences decisions concerning the disposition of assets, and *money needed to carry out transactions planned for the coming income period is not an asset.* In particular, there must be some level of the rate of interest that is sufficient to reduce to zero the demand for money to hold, and hence the total demand to its minimum institutional level which depends on the volume of transactions. As the rate of interest rises above this level, the demand for money will be substantially unaffected and will depend exclusively on the level of money income.

On the basis of these considerations we may, in a first approximation, split the total demand for money into two parts: the demand for money to hold, $D_a(r)$, and the demand for money to spend or for transactions, $D_T(Y)$; and write

$$L(r, Y) = D_a(r) + D_T(Y) = M. \tag{12}$$

This is not really necessary for our argument, but is very useful since it will constantly remind us of the two sources of demand for money and it will permit us to analyze more conveniently the part played by each variable.

With this in mind we shall find it useful to consider the functioning of the money market in which decisions concerning the disposition of nonphysical assets are carried out.

7. THE MONEY MARKET AND THE SHORT-RUN EQUILIBRIUM OF THE RATE OF INTEREST

There are two ways of looking at this market: (a) in terms of flows (savings and net borrowing) and (b) in terms of stocks. It is from this latter point of view that we shall consider it at this moment.

The supply in this market consists of the stock that is not needed for transactions. On the basis of our first approximation (12), this supply, denoted by S_a, will be

$$S_a = M - D_T(Y),$$

and is determined for any value of the money income and the fixed supply of money.

A position of equilibrium in the money market is reached when a system of interest rates is established at which dealers are willing to hold for the income period all the available supply. Or, from a different angle, the system of interest rates is determined by the price (in terms of forgone income) that dealers are willing to pay to hold assets in the form of money for the coming income period.

This can easily be translated into the usual Marshallian supply and demand apparatus, provided we replace the system of interest rates by a single rate r, as shown in Figure 1.

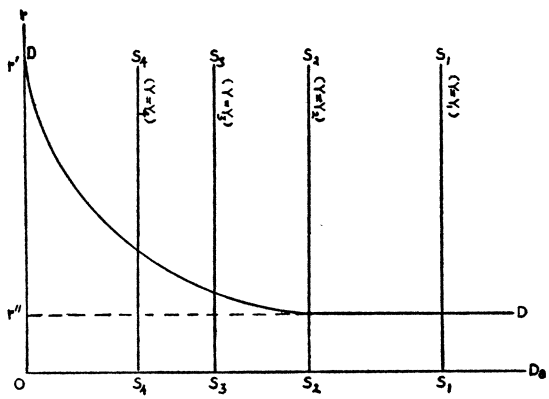

FIGURE 1

DD is the demand curve for money to hold, sloping downward and to the right (when the price, the rate of interest, rises, the demand falls, as in the case of ordinary commodities). The vertical lines are various supply curves corresponding to different values of Y and the fixed value of M. As the income increases, the supply falls: hence

$$Y_4 > Y_3 > Y_2 > \cdots .$$

Since a fall in supply causes a rise in price, the graph shows clearly that equation (1) gives r as an increasing function of Y.

The characteristics of the D_a function described above are shown in in the graph. We noted that, for $r \geqq r'$ the demand falls to zero; hence the graph of DD joins the vertical axis and coincides with it.

On the other hand, when the rate of interest falls to the level r'', the demand for money to hold becomes infinitely elastic. Any increase in the supply of money to hold now fails to affect the rate of interest, for the owners of the extra supply will either desire to hold this in the form of cash; or else they will find some owners of securities, who, being just indifferent as to holding cash or securities, will be willing to sell without any necessity for bidding up the price of securities (lower-

ing the rate of interest). Thus, in Figure 1, when the interest rate r'' is reached, the graph of DD becomes parallel to the D_a axis; the income corresponding to r'' cannot be more than Y_2; but if income should fall below Y_2 it would not change the interest rate.[12] This situation that plays such an important role in Keynes's *General Theory* will be referred to as the "Keynesian case."

In the diagram we have assumed that there is a single rate of interest r, instead of a whole system of rates for loans of different duration. While it may be assumed that in principle all the rates tend to move in the same direction, we must bear in mind that the extent to which a change in the supply of money changes the rates on loans of different maturities depends on the character of interest expectations.

A change in the supply will necessarily affect the short rates (unless the short rate has already reached its minimum). But the extent to which it will affect longer rates depends on the relation between the current spot rate and expected future rates.

To denote the relationship between current and expected rates we may again use the Hicksian elasticity of expectation. If this elasticity is unity, expected short rates move in the same direction and in the same proportion as the spot rate; if it is less than unity, a given percentage change in short rates leads to a smaller percentage change in expected rates; and vice versa for elasticity larger than one.

If the expectations about future short rates are based predominantly on the current shorter rates, then the elasticity of expectation tends toward one and the whole system of rates moves in close conformity. But if dealers have rigid expectations based on different elements, the elasticity of expectation will be low and a change in short rates will affect longer rates only to the extent that some of the discount rates, which determine the present value of the assets, are changed.

In practice we may expect that this elasticity will be larger than zero and smaller than one and that it will be larger for the rates expected in the near future.[13]

To the extent that this is true there will be two reasons why rates on loans of shorter maturity should move in closer agreement with the very short rate: (a) because they are more affected by a change in the current short rate, (b) because the other future short rates (of which they are an average) are more influenced by such a change.

[12] From equation (1) we obtain $dr/dY = -L_Y/L_r$, where the subscripts denote partial derivatives. Hence $dr/dY = 0$ if $|L_r| = \infty$.

[13] Denoting by $r_1, r_2, \cdots, r_n$ the short rate of interest anticipated for periods $1, 2, \cdots, n$, we may expect that

$$\frac{\partial r_1}{\partial r_0} > \frac{\partial r_2}{\partial r_0} > \cdots > \frac{\partial r_n}{\partial r_0}.$$

These necessary qualifications do not alter our previous conclusions concerning the determination of equilibrium in the money market. The equilibrium system of interest rates is determined in each period by the condition that the supply of money to hold, which (given M) depends on the transaction demand for money and hence on income, be equal to the demand for money to hold. We may therefore proceed to draw the graph of equation (1), $M=L(r, Y)$. This is the LL curve of Figure 3. Any point on this curve shows the equilibrium value of r corresponding to a value of Y and the fixed value of M: it shows therefore positions of possible equilibrium in the money market. We must prove next that only one point on this curve is consistent with the long-run equilibrium of the system.

8. SAVING, INVESTMENT, AND THE *IS* FUNCTION

The first part of our system yields a second relationship between interest and income. Making use of equations (2) and (3) and the equilibrium condition (4) we obtain: $I(r, Y)=S(r, Y)$. In order to gain some idea of the shape of this curve we may again make use of a graphical method illustrated in Figure 2.

Figure 2-B is the graph of equation (3). Since $\partial S/\partial r$ is usually considered small and of unknown sign we have simplified the drawing by eliminating r. This curve describes the relationship between money income and the proportion of it that people choose not to consume. Its position depends on the value of the fixed money wage rate w_0: given the wage rate, to any level of money income there corresponds a certain real income and price level and, therefore, a certain level of money saving. In this diagram Y_2 denotes the highest money income that can be reached with the money wage rate w_0, and A is the full employment relationship between saving and income.

The straight line beginning at A gives the relationship between money income and money saving once full employment has been reached and the second part of condition (10) replaces the first.[14] We have then what is usually called inflation: real income cannot change but money income can rise to any level. As all prices rise simultaneously the amount of real income saved is unchanged while its money value rises in the same proportion as the price level and money income.[15] The dotted curved line, on the other hand, gives a potential

[14] This line is the continuation of the radius vector from the origin to A.

[15] This is strictly correct only if inflation does not provoke any permanent redistribution of income; or if the redistribution does not affect the aggregate propensity to save. Since wages rise with prices we can exclude redistributions from working class to nonworking class. But we cannot exclude redistribution from fixed-income receivers (especially owners of securities) to profits. It is difficult to say whether this will change sensibly the aggregate propensity to save; it is probably a good approximation to assume that the effect will be negligible.

relation between S and I if it were possible to raise the real income above the full employment level.

Figure 2-A is the graph of equation (2). Each curve in this graph shows the amount of investment that would be undertaken at different levels of the rate of interest and for a fixed value of the income. To larger values of Y correspond investment curves higher and to the right.

Since the vertical scale is the same in both Figure 2-A and Figure 2-B, we may use the following method to find the shape of $S(Y)=I(r, Y)$: For any value of Y, say Y_1, the corresponding amount of saving, S_1, can be read from the SS curve. But in equilibrium $S=I$, hence we can draw a line parallel to the Y axis at height S_1 and prolong it until it inter-

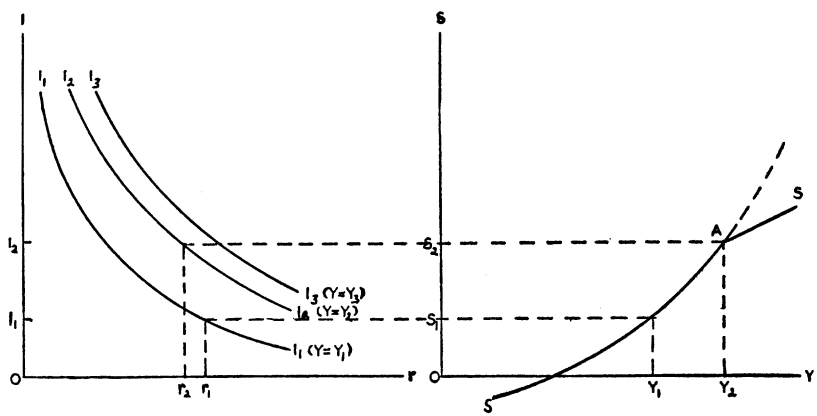

FIGURE 2-A FIGURE 2-B

sects the investment curve of Figure 2-A corresponding to the income Y_1. We may thus find the rate of interest r_1 that corresponds to the given income Y_1.

The character of the relationship between r and Y that emerges from this diagram cannot be established a priori as in the case of the LL curve discussed before. For, as Y increases, S in Figure 2-B increases too, but the corresponding value of r in Figure 2-A may increase or decrease. It all depends on the way the change in income affects the position of the investment curves. If the increase in income tends to raise the desire to save more than the desire to invest, the rate of interest will fall; in the opposite case it will rise.[16] This last possibility is, in our opinion, unlikely to occur, but it may materialize when entrepreneurs are highly optimistic and the existing equipment is already working at capacity.

[16] From $S(r, Y)=I(r, Y)$ we obtain $dr/dY=(S_Y-I_Y)/(I_r-S_r)$, where the subscripts denote partial derivatives. Since I_r-S_r may be expected to be negative, we have $dr/dY \gtreqless 0$ as $S_Y \lesseqgtr I_Y$.

The relationship between r and Y emerging from equations (2) and (3) and the equilibrium condition (4) is shown as the IS curve of Figure 3. In the normal case it will slope downward and to the right as in this diagram, but it is conceivable that, at least in a certain range, it may slope upward to the right. In this case $S_Y < I_Y$ and it is usually assumed that the equilibrium of the system will be unstable (and neutral if $S_Y = I_Y$). We shall see, however, that, with inelastic money supply, the negative slope of the IS curve is a sufficient but not necessary condition for stability.

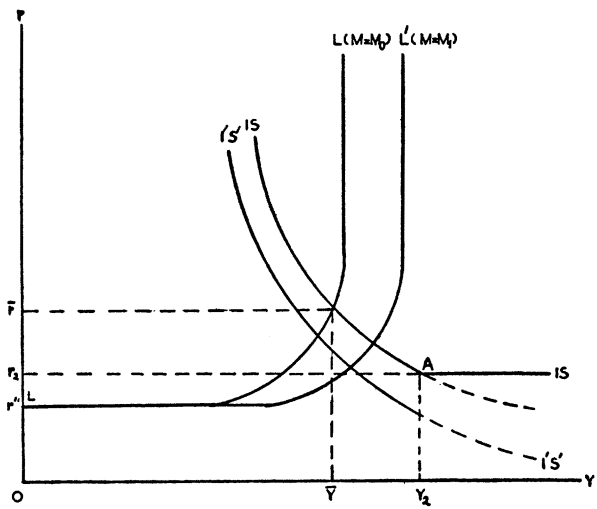

FIGURE 3

The IS curve must also have another important property. In Figure 3, A denotes the equilibrium relationship between full-employment income (Y_2) and rate of interest (r_2). Money income cannot rise above the full-employment level denoted by Y_2 except through inflation, i.e., if wages and prices rise in the same proportion as income. As the stage of inflationary prices and wage increases is reached, the "real" value of investment that it pays to undertake at any interest rate is unchanged since yields and costs change in the same proportion.[17] The

[17] Following the example of Mr. Keynes we may define the marginal efficiency of an asset as the discount rate that makes the sum of the expected marginal discounted yields equal to the marginal cost of the asset. The expected yields need not all be equal since they depend on the expected net physical yield as well as on expected future prices; and neither is necessarily constant in time. But the expected physical yield does not depend on prices; and, owing to our "static assumption" of unit elasticity of expectation, expected prices change in the same proportion as present prices. Therefore the summation of the yields changes in

money value of profitable investments, on the other hand, rises proportionally to prices and money income. As we have seen above, the same will be true of money savings. It follows that inflationary increases in income raise saving and investment in the same proportion and must therefore leave the equilibrium value of the rate of interest unchanged at the full-employment level r_2. It is for this reason that in Figure 3, to the right of A, the IS curve becomes parallel to the income axis. The dotted curved line beyond A is again the hypothetical relationship between r and Y if it were possible to raise real income above the full-employment level (and if the wage rate should remain unchanged at the level w_0).

9. THE MONEY MARKET AND THE DETERMINANTS OF MONETARY EQUILIBRIUM

We may now finally proceed to consider the process by which the equilibrium of the system is established. For this purpose we must once more revert to the money market which we must, this time, consider in terms of flows rather than in terms of stocks.

In Section 5 we have seen that the rate of interest is established in the money market by the condition that supply of and demand for the stock of money to hold must be equal. This condition is sufficient to determine a position of short-run equilibrium, i.e., a position of equilibrium for the income period. We must now consider under what conditions this level of the rate of interest will also represent a position of long-run equilibrium. As in the textbook analysis of demand and supply, a position of long-run equilibrium is characterized by the fact that neither price nor quantity (demanded and supplied) tend to change any further. In the present case a position of long-run equilibrium will be reached only when the rate of interest does not tend to change from one income period to the other and this in turn is possible only if the stock of money to hold remains constant in time.

Now in each income period people increase their assets by current savings; the money thus saved, since it is not needed for transactions, constitutes an increase in the supply of money to hold. Borrowing, on the other hand, automatically decreases the supply of money to hold by taking cash out of the money market and putting it into active circulation again, through expenditure on investments. If net saving exceeds net borrowing then, on balance, the supply of money to hold

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the same proportion as marginal cost and so does the aggregate value of investments having marginal efficiency equal to or larger than r_2. Under unit elasticity of expectation a given change in all present prices does not modify entrepreneurs' production plans.

will increase above the level of the previous period, say $D_{a\cdot 0}$. But at the old rate of interest (r_0) people will not want to hold the extra supply; they will therefore try to purchase securities and thus will lower the rate of interest. If, on the other hand, at the interest rate r_0 borrowers desire to borrow in the period more than the current amount of money savings, they must induce dealers in the money market to reduce the demand for money as an asset below the previous level $D_{a\cdot 0}$; and this is possible only if the rate of interest rises. There are then three possibilities. (The subscripts 0 and 1 denote quantities in periods zero and one, respectively.)

(1) $S_1 > I_1$: then $D_{a\cdot 1} > D_{a\cdot 0}$ and the rate of interest falls.

(2) $S_1 = I_1$: here $D_{a\cdot 1} = D_{a\cdot 0}$ and the rate of interest is unchanged.

(3) $S_1 < I_1$: then $D_{a\cdot 1} < D_{a\cdot 0}$ and the rate of interest rises.

Recalling our definition of long-run equilibrium, we see at once that only situation (2) satisfies it. In equilibrium then, both demand for and supply of the stock of money to hold and demand for and supply of the flow of saving must be equal.[17a] In addition, however, it is necessary that the flows of saving and of borrowing be themselves constant in time. This is possible only if two conditions hold: (a) The borrowing that occurs must be equal to the amount of investment that entrepreneurs wish to undertake at the given rate of interest and income level. The relationship between I_1, r_1, and Y_1 must be described by a point on the corresponding curve of Figure 2-A. (b) The income (and the rate of interest) must be as large as is required to induce people to go on saving an amount S_1. The relationship between Y_1, S_1, and r_1 must be described by a point lying on the curve of Figure 2-B. But if conditions (a) and (b) are satisfied the relationship between Y and r will be described by a point lying on the IS curve of Figure 3. Thus a position of full equilibrium must be represented by a point lying at the same time on the LL curve (denoting equilibrium between demand for and supply of the stock of money to hold) and on the IS curve (denoting equality and constancy in time of the inflow and outflow of cash in the money market); hence it must be given by the intersection of these two curves.

This is shown in Figure 3 where the equilibrium values of r and Y, thus determined, are denoted by $\bar{r}$ and $\overline{Y}$. Analytically this corresponds to the simultaneous solution of the two relationships between the income and the rate of interest obtained from equations (1), (2), (3), and (4): $M = L(r, Y)$ and $S(r, Y) = I(r, Y)$.

[17a] The classical example of the level of water in a reservoir fits this case perfectly. The rate of interest, like the level of the water, can be constant only if inflow and outflow are equal.

10. A DYNAMIC MODEL OF THE KEYNESIAN THEORY AND THE STABILITY OF EQUILIBRIUM

So far our analysis has apparently been "timeless"[18] since it was based on the system of equations of Section 2, in which time does not appear explicitly. A close examination of the last sections, and especially Sections 7 and 9, will reveal, however, that dynamic elements have gradually slipped into our analysis, thanks to the device of "long- and short-run equilibrium," the oldest and simplest device of developing a dynamic theory with a static apparatus. Actually the criterion that distinguishes short- from long-run equilibrium is essentially a dynamic one: namely, the length of time that is required for certain decisions to be carried out, or, more generally, for certain causes to show their effects.

In our case, the equilibrium of the "money market" is a condition of short-run equilibrium (that determines the rate of interest for each period) because it is the result of decisions that can be carried into effect immediately. The condition saving = investment, on the other hand, is a condition of long-run equilibrium because the equality of *ex ante* saving and investment cannot be brought about instantaneously. This is a different way of stating the familiar proposition that the multiplier takes time to work out its full effect. This well-known fact is in turn explained essentially by the existence of a fundamental time lag: the lag between the time when income is earned and the time when it becomes available for expenditure. In the economic systems in which we live, people are usually paid for services already rendered. The income earned (or produced) in a period is the value of services rendered which will be paid for at the end of the normal income period; while the income available for expenditure represents payment for services rendered in the previous period. Decisions as to spending and saving can refer only to the disposable income, and are essentially motivated by it, even though income earned may have some influence.

This explains why the graph of the *IS* curve, unlike the *LL* curve, describes not instantaneous relationships but only possible positions of long-run equilibrium. When the two curves intersect we have a position of full equilibrium since both short- and long-run conditions are satisfied.

It will therefore be useful at this point to give explicit recognition to the dynamic elements that form the basis of our approach. This is the purpose of the following system of difference equations which may be considered as the simplest dynamic model of our theory.

[18] The word "timeless" has been used here to avoid confusion since the word "static" has already been used to denote the assumption of homogeneity of the first degree of the "expectations functions."

$$M = L(r_t, Y_{d \cdot t}), \tag{2.1}$$

$$I_t = I(r_t, Y_{d \cdot t}), \tag{2.2}$$

$$S_t = S(r_t, Y_{d \cdot t}), \tag{2.3}$$

$$Y_{d \cdot t} = C_t + S_t, \tag{2.4}$$

$$Y_t = C_t + I_t, \tag{2.5}$$

$$Y_{d \cdot t} = Y_{t-1}. \tag{2.6}$$

In this system Y denotes income earned and Y_d income disposable. This is a new variable to which corresponds the new equation (2.6). The remaining equations of the system are unchanged.

By repeated substitution the system reduces to the two equations

$$Y_t = Y_{t-1} - S_t + I_t = Y_{t-1} - S(Y_{t-1}, r_t) + I(Y_{t-1}, r_t),$$
$$M = L(r_t, Y_{t-1}).$$

Solving the second equation for r_t and substituting in the first, we obtain a single equation of the form: $Y_t = f(Y_{t-1})$ which determines the time path of the income. By similar procedure we obtain the time sequence of the other variables.

If the system is stable, each variable approaches some definite value which it will maintain in time until there occurs some change in the form of the functional relationship or in some parameter (M or w_0). Equation (2.1) is again the "equation of the money market" that determines the value of r for any period; but we have a position of long-run equilibrium only when $r_t = r_{t-1}$. And this implies $Y_t = Y_{d \cdot t} = Y_{t-1}$ and therefore $S_t = I_t$.

The importance of this system is not limited to the fact that it defines rigorously concepts that were loosely used in our previous analysis. It serves also another important purpose: namely it permits us to determine the conditions of stability for the system.

Following the usual method, we proceed to expand equations (2.1) to (2.3) by Taylor series around the equilibrium values neglecting all terms of degree higher than one. We then obtain:

$$0 = L_r \dot{r}_t + L_Y \dot{Y}_{t-1} + \cdots,$$
$$I_t = I(\bar{r}, \bar{Y}) + I_r \dot{r}_t + I_Y \dot{Y}_{t-1} + \cdots,$$
$$S_t = S(\bar{r}, \bar{Y}) + S_r \dot{r}_t + S_Y \dot{Y}_{t-1}.$$

Subscripts denote partial derivatives taken around the equilibrium values $(\bar{r}, \bar{Y})$ and $\dot{r}_t = r_t - \bar{r}$, $\dot{Y}_t = Y_t - \bar{Y}$. By making use of (4) and (5) and by repeated substitution we obtain the following linear difference equation with constant coefficients:

$$\dot{Y}_t = \dot{Y}_{t-1}\left[1 + \frac{L_Y}{L_r}(S_r - I_r) + I_Y - S_Y\right].$$

The solution of this equation takes the form: $\dot{Y} = \kappa\lambda^t$ or $Y = (Y_0 - \overline{Y})\lambda^t$, since $\dot{Y}_0 = Y_0 - \overline{Y} = \kappa$. Y_0 is determined by the initial conditions and

$$\lambda = 1 + \frac{L_Y}{L_r}(S_r - I_r) + I_Y - S_Y.$$

The stability condition is $|\lambda| < 1$; in the present case this reduces to

$$(2.7) \qquad -\frac{L_Y}{L_r} - \frac{r}{S_r - I_r} < \frac{I_Y - S_Y}{S_r - I_r} < -\frac{L_Y}{L_r}.$$

Since the middle term is the slope of the IS curve and the right-hand term is the slope of the LL curve, the right-hand condition has a very clear graphical meaning. Stability requires that the slope of the IS curve be algebraically smaller than the slope of the LL curve. The slope of the LL curve cannot be negative ($L_Y > 0$, $L_r \geqq 0$). Also general economic considerations suggest that $S_r - I_r > 0$. Hence this condition is necessarily satisfied if $I_Y - S_Y < 0$, i.e., when the IS curve falls from left to right. But this is not necessary. Stability is also possible when the IS curve rises in the neighborhood of the equilibrium point as long as it cuts the LL curve from its concave toward its convex side.[19]

If the stability conditions are satisfied, the variables approach their equilibrium values, which are the same as those obtained by solving the static system of Section 2. In the opposite case they diverge more and more from these values in a process of cumulative contraction or expansion. In the same way, a change in some of the data will lead to a new stable equilibrium if the new functions satisfy the conditions written above.

It is interesting to note that, as long as the money supply is inelastic, the system must always have at least one stable solution since eventually the LL curve becomes perpendicular to the horizontal axis and hence its slope must become larger then the slope of the IS curve.

11. THE DETERMINANTS OF REAL EQUILIBRIUM

It is now time to consider the role of the second part of the system in the determination of equilibrium. Equations (5), (6), and (7) *explain* the forces that determine the real variables of the system: physical output, employment, real wage rate.[20]

[19] It is only as $L_r \to \infty$ (demand for money to hold infinitely elastic, LL curve parallel to the horizontal axis) that the condition $I_Y - S_Y < 0$ becomes necessary for equilibrium. This holds equally if the supply of money is infinitely elastic for this has the same effect as $L_r = \infty$.

[20] The price level is also necessary to determine the real wage rate, given the money wage rate W.

The most important of these equations is (7), which states the conditions of equilibrium in the production of goods whether for consumption or for investment.[21] Production will be extended up to the point at which the given and fixed money wage rate w_0 is equal to the marginal net product of labor, or, if we prefer, up to the point at which price equals marginal labor cost.[22] This assumes that the only variable factor is labor and the quantity of equipment is fixed; a condition that is approximately satisfied in the case we are considering. Eliminating equation (5) by substitution into (7) we can reduce this part of the system to two equations in the two unknowns X and N, where X' is used for dX/dN:

$$W_0 = X'(N)\frac{Y}{X}, \qquad X = X(N).$$

Since the money income is determined exclusively by the *monetary* part of the system, the price level depends only on the amount of output. If, at any given price level, the fixed wage is less than the marginal product of labor, the forces of competition lead to an expansion of employment and output which forces prices down. This lowers the marginal product of labor until it becomes equal to the wage rate. If the wage rate exceeded the marginal product of labor, output and employment would contract, which would force prices up. We see clearly from Figure 3 that the amount of employment thus determined will, in general, not be "full employment"; that is, unless the LL curve intersects the IS curve at (Y_2, r_2) or to the right of it.

12. UNDEREMPLOYMENT EQUILIBRIUM AND LIQUIDITY PREFERENCE

This last result deserves closer consideration. It is usually considered as one of the most important achievements of the Keynesian theory that it explains the consistency of economic equilibrium with the presence of involuntary unemployment. It is, however, not sufficiently recognized that, except in a limiting case to be considered later, this result is due entirely to the assumption of "rigid wages"[23] and not to the Keynesian liquidity preference. Systems with rigid wages share the common property that the equilibrium value of the "real" variables is determined essentially by monetary conditions rather than by "real" factors (e.g., quantity and efficiency of existing equipment, relative

[21] The equilibrium price of each type of physical asset is found by capitalizing a series of expected marginal yields at the current rate of interest. The expected yields of the marginal unit need not be equal in each period.

[22] This is a sufficient condition under assumption of perfect competition; the modifications necessary in the case of monopolies cannot be considered here.

[23] The expression "rigid wages" refers to the infinite elasticity of the supply curve of labor when the level of employment is below "full."

preference for earning and leisure, etc.). The monetary conditions are sufficient to determine money income and, under fixed wages and given technical conditions, to each money income there corresponds a definite equilibrium level of employment. This equilibrium level does not tend to coincide with full employment except by mere chance, since there is no economic mechanism that insures this coincidence. There may be unemployment in the sense that more people would be willing to work at the current real wage rate than are actually employed; but in a free capitalistic economy production is guided by prices and not by desires and since the money wage rate is rigid, this desire fails to be translated into an economic stimulus.

In order to show more clearly that wage rigidities and not liquidity preference explain underemployment equilibrium we may consider the results to be obtained by giving up the liquidity-preference theory and assuming instead the crudest quantity-of-money theory while keeping the assumption of rigid wages. This can be done by merely replacing equation (1) of our system by the equation

$$M = kY. \tag{1a}$$

Since M and k are constant this equation is sufficient to determine money income. Equations (5), (6), and (7) determine directly physical output and employment as we saw in Section 10. Once more there is no reason to expect that the level of employment thus determined will be "full employment"; and yet the system will be in equilibrium since there will be no tendency for income, employment, and output to change.

It is very interesting to see what part is played under these conditions by equations (2) and (3), the saving and investment equations that have been so much stressed by all the Keynesians. Since the income is determined by equation (1a), equation (2) reduces to an "orthodox" supply-of-saving schedule, giving saving as a function of the rate of interest. For the same reason, equation (3) reduces to a demand-for-saving schedule. Both schedules can be represented in a Marshallian supply and demand diagram as is done in Figure 4. The intersection of these curves, i.e., the equilibrium condition, demand = supply, determines the level of the rate of interest.

Finally let us notice that, in this system also, the rate of interest depends on the quantity of money, or more exactly on the ratio M/W. A change in M (W constant) raises real income and shifts both the SS and II curves to the right. The net result will be a fall in the rate of interest, if the increase in income raises the desire to save more than the desire to invest (normal case); a rise, in the opposite case.

In spite of these significant similarities between the present system and the Keynesian system, in which we recognize the existence of liquid-

ity demand for money, there remains one very important difference; this difference is to be found in the role played by the rate of interest in the determination of equilibrium. In both cases the level of employment depends on the quantity of "active" money. But in the Keynesian system this depends on the rate of interest and consequently also on the propensities to save and invest. In the present case the quantity of active money is fixed and independent of the rate of interest. Hence the propensities to save and invest are not a part of the mechanism determining employment; they merely determine the amount of resources devoted to the improvement of the means of production.

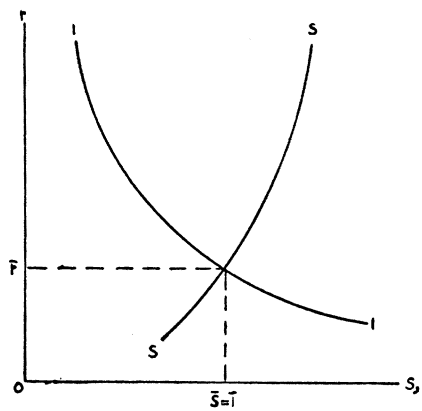

FIGURE 4

We now proceed to consider the determinants of equilibrium in a system in which we do away not only with the liquidity-preference theory but also with the assumption of rigid wages.

13. THE LOGICAL CONSISTENCY OF THE QUANTITY THEORY OF MONEY AND THE DICHOTOMY OF MONETARY AND REAL ECONOMICS

In order to discuss the quantity theory of money we substitute equation (1a) for (1) and replace conditions (10) by the identities (11).

It was shown in Section 8 that a given change in prices will change income, investment, and saving in the same proportion. Consequently, after Y in equations (2) and (3) is replaced by the expression given in (5), the saving and investment equations may be written in the form

$$\frac{I}{W} = I\left(r, \frac{P}{W}X\right), \tag{3.2}$$

$$\frac{S}{W} = S\left(r, \frac{P}{W}X\right). \tag{3.3}$$

Next we divide both members of equations (4) and (5) by W obtaining

$$\frac{S}{W} = \frac{I}{W}, \tag{3.4}$$

$$\frac{Y}{W} \equiv \frac{P}{W} X, \tag{3.5}$$

$$X = X(N), \tag{3.6}$$

$$\frac{W}{P} = X'(N), \tag{3.7}$$

$$N = F\left(\frac{W}{P}\right), \tag{3.9}$$

$$\left[\frac{Y}{W} \equiv \frac{I}{W} + \frac{C}{W}\right]. \tag{3.8}$$

Equations (3.2) to (3.7) and (3.9) form a system of 7 equations in the 7 unknowns I/W, S/W, P/W, Y/W, r, X, N. These unknowns are therefore determined. Next we can write equation (1a) in the form $M = kPX = Wk(P/W)X$. But since P/W and X have already been determined, this equation determines the money wage rate and hence the price level, money income, etc. This is essentially the "classical" procedure, and we can only repeat the classical conclusions to the effect that the real part of the system, namely, employment, *interest rate*, output, or real income, do not depend on the quantity of money. The quantity of money has no other function than to determine the price level.

This result does not, of course, depend on any special feature of our system. It will always follow, provided all the supply and demand functions for commodities[24] and labor are homogeneous of the zero degree; and since we are proceeding under "static" assumptions, all the supply and demand functions must be homogeneous of zero degree, if people behave rationally.[25]

This conclusion, which is very old indeed, has some interest since it has been recently challenged by Oscar Lange. Of all the recent attacks against the traditional dichotomy of monetary and real economics, Lange's criticism is by far the most serious because it maintains that "the traditional procedure of the theory of money involves a

[24] "Commodities" are, in this context, all goods except money.

[25] For a proof of this statement see O. Lange, "Say's Law: A Restatement and Criticism," *op. cit.*, pp. 67 and 68. Professor Lange shows that the homogeneity of first degree of all expectation functions is a sufficient condition for all demand and supply equations for "commodities" to be homogeneous of zero degree.

[logical] contradiction."[26] We propose to show, however, that, while Lange's criticism of Say's law cannot be questioned, it does not invalidate the logical consistency of the procedure of the quantity theory of money.

According to Lange, Say's law implies that the amount of cash people desire to hold is always identically equal to the quantity in existence: denoting by D_n and S_n the demand and supply of money respectively, we can write this as $S_n \equiv D_n$. Lange then states that "a proportional change of all prices does not induce a substitution between different commodities"[27] and concludes that "the demand and supply functions of commodities are, *when Say's law holds*, homogeneous of zero degree."[28] But the homogeneity of the supply and demand functions for commodities does not depend on Say's law: it depends on the assumption of rationality and the homogeneity of the expectation functions. Since a proportional change in all prices does not change the price ratios it also does not change the marginal rate of substitution, and therefore does not induce a substitution between different commodities.

Let us now consider a system in which there are n goods ($n-1$ commodities and money). As is well known, there are only $n-1$ prices to be determined, the price of money being unity, and $n-1$ independent supply and demand equations, for one follows from the rest. Since the supply and demand functions for commodities are homogeneous of zero degree, the quantities demanded of the $n-1$ commodities are functions of the $n-2$ price ratios p_i/p_{n-1} ($i=1, 2, \cdots, n-2$), where p_{n-1} is chosen arbitrarily.[29] At the same time the demand and supply function to be eliminated is also arbitrary; we may, if we choose, eliminate one of the $n-1$ referring to commodities; we are then left with $n-2$ equations for commodities to determine the $n-2$ price ratios. Hence the price ratios are determined. To determine the actual prices we use the demand and supply equation for money as was done above. In Lange's system this is written:

$$k\sum_{i=1}^{n} p_i S_i = M, \quad \text{or also} \quad kp_{n-1}\sum_{i=1}^{n} \frac{p_i}{p_{n-1}} S_i = M,$$

where S_i denotes the equilibrium quantity supplied and demanded of the ith commodity. Since k is a constant this equation determines p_{n-1} and consequently all other prices.

As long as Say's law is not assumed, this procedure is perfectly legitimate; and we cannot escape the classical conclusion that money

[26] *Ibid.*, p. 65.

[27] *Ibid.*, p. 63.

[28] *Ibid.*, p. 63. Italics ours.

[29] In our own system p_{n-1} was arbitrarily chosen as the wage rate.

is "neutral," just a "veil." If, however, Say's law holds, the demand and supply of money are identically equal. The nth equation is therefore not a genuine equation. Thus we have only $n-2$ independent equations to determine $n-1$ prices: the system is not determinate. In Lange's own formulation, the nth equation degenerates into the identity

$$kp_{n-1}\sum_{i=1}^{n}\frac{p_i}{p_{n-1}}S_i \equiv M,$$

which is satisfied by any value of p_{n-1} whatever; the price level is thus indeterminate.[30]

Hence one of Lange's conclusions, namely that "Say's law precludes any monetary theory,"[31] is perfectly justified. But Lange goes on to draw a conclusion which does not follow, namely that "the traditional procedure of the theory of money involves a contradiction. Either Say's law is assumed and money prices are indeterminate, or money prices are made determinate—but then *Say's law and hence the neutrality of money* must be abandoned."[32] But the traditional theory of money is not based on Say's law. The necessary condition for money to be neutral is that the $n-1$ "real" demand and supply equations be homogeneous of order zero and this homogeneity does not "disappear when Say's law is abandoned."[33] Under "static" assumptions money is neutral even without assuming Say's law, if only people are assumed to behave "rationally"; this is all that the classical theory assumes and needs to assume.[34]

The most serious charge against the classical dichotomy can thus be dismissed, as long as we maintain our "static" assumptions.

14. LIQUIDITY PREFERENCE AND THE DETERMINANTS OF THE RATE OF INTEREST UNDER THE ASSUMPTION OF FLEXIBLE WAGES[35]

With this in mind we may now proceed to analyze our third system consisting of equations (1) to (7), (9), and identities (11). In this system we recognize that there are two sources of demand for money,

[30] Then k changes in inverse proportion to p_{n-1} instead of being a constant.

[31] O. Lange, *op. cit.*, p. 66.

[32] *Ibid.*, p. 65. Italics ours.

[33] *Ibid.*, p. 66.

[34] Lange's result seems due to a failure to distinguish between necessary and sufficient conditions. Say's law is a sufficient condition for the neutrality of money but not a necessary one. Lange asks me to inform the reader that he agrees with my conclusion. This conclusion, however, does not invalidate his result that under Say's law the money prices are indeterminate.

[35] The expression "flexible wages" is used here and in the following pages for brevity in place of the more exact expression "homogeneity of zero degree of the supply-of-labor function."

the transaction demand and the liquidity demand. But, as in the case just analyzed, we make no restrictive assumptions as to the supply-of-labor equation. The suppliers of labor as well as the suppliers of all other commodities are supposed to behave "rationally." It follows that the only difference between the present case and the case just considered is in equation (1). As in the previous case, the last 7 equations form a determinate system which is sufficient to determine the 7 unknowns it contains, namely *the "real" variables of the system and the rate of interest.*

By use of equation (5) or (3.5) equation (1) takes the form

$$M = L\left(r, W\frac{P}{W}X\right). \tag{3.1}$$

Since r and P/W are already determined, this equation determines the 8th unknown of the system, the wage rate: and therefore also the price level, money, income, etc.[36]

We thus reach the conclusion that under "static" assumptions and "flexible" wages, *the rate of interest and the level of employment do not depend on the quantity of money.*

Two questions arise at once: (a) what determines the rate of interest and (b) what part do the rate of interest and liquidity demand for money play in the determination of equilibrium.

Strictly speaking, the rate of interest is determined by all the equations of a Walrasian system *except the supply-of-and-demand-for-money equation.* But it is clear that in the first approximation of partial-equilibrium analysis, the determination of the rate of interest must be associated with equations (3.2) and (3.3), the saving and investment schedules. To explain the level of the rate of interest we could use once more Figure 4, changing the variables measured on the horizontal axis from S or I into S/W or I/W. We must add at once, however, that these two schedules should in no way be confused with the schedules of supply of and demand for savings (or supply of and demand for securities) used in the textbook explanation of the determination of the rate of interest.

Equation (3.3) only tells us what part of their real income people wish to devote to increasing their assets rather than to consumption, at different levels of the rate of interest.

In a similar fashion equation (3.2) shows that by devoting output worth I/W to the improvement of the means of production, it is possible to increase real income by an amount $(I/W)(1+r)$ per unit of time. The value of r depends on the given technical conditions, on the

[36] Except in the Keynesian case considered later (Section 16).

quantity I/W and $(P/W)X$ according to the relation expressed by equation (3.2). This shows clearly the fundamental factors that determine the rate of interest. The given technical conditions, expressed by the production function [equation (3.6)], together with *tastes* of people for earning and leisure, expressed by the supply-of-labor function [equation (3.9)], give the level of real income that can be reached.[37] The saving schedule, equation (3.3), tells us what part of this income the community desires to save. The technical conditions (inventions, quantity of capital already in existence, etc.) expressed by the marginal-efficiency-of-investment function (3.2), determine the marginal efficiency of the amount of investment that the giving up of consumption permits undertaking: this is the equilibrium rate of interest.

Let us now examine what part is played by liquidity preference in the present system. On the basis of the given rate of interest determined in the fashion discussed above, people decide what quantity of money they want to hold as an asset. Hence, provided the liquidity demand is finite, the rate of interest, together with the supply of money, determines the quantity of active money and therefore the price level. Thus under "flexible" wages, *the desire to hold assets in liquid form does not determine the rate of interest, but determines the price level.* It follows that any factor that influences the demand for money as an asset, either directly or through the rate of interest, will have a repercussion on the price level, unless it is counteracted by an appropriate change in the quantity of money. This will in particular be the case with changes in the propensities to save and to invest.

15. LIQUIDITY PREFERENCE UNDER RIGID AND FLEXIBLE WAGES—AN EXAMPLE

In order to see clearly the different implications of the liquidity-preference theory under different hypotheses as to the supply of labor we may briefly consider the effects of a shift in the investment schedule [equation (2) or (3.2)].

Suppose that the system is in equilibrium at money income Y_0: the flow of investments is I_0, and its marginal efficiency, r_0, is the equilibrium rate of interest. Now let us assume that for some reason the rate of investment that seems profitable at any level of the rate of interest falls. In particular the marginal efficiency of the rate of investment I_0 falls to the level $r_1 < r_0$. In order for the system to reach a new position of equilibrium, it is necessary that the rate of interest fall to this level. Except under special circumstances, to be considered later, as

[37] Under flexible wages there is, of course, always full employment under the conditions mentioned in Section 16.

the rate of interest falls, the demand for money as an asset rises, and a certain amount of current money savings remains in the *money market* to satisfy the increased demand. If the supply of money is not properly increased, this, in turn, implies a fall in money income.

Under the conditions of our last model (flexible wages) the fall is brought about by an all-around reduction in wages and prices. The price level reaches its new equilibrium position when the supply has been increased sufficiently to satisfy the liquidity demand for money associated with the interest rate r_1.[38] The net effect of the shift is then to depress the interest rate, the money income, and money wages without affecting the real variables of the system, employment, output, real wage rate.[39]

But if money wages are rigid downward, the reduction in money income, made necessary by the fall in the rate of interest, becomes a reduction in real income and employment as well. The effect of the shift in the investment schedule is now to start a typical process of contraction so frequently described in Keynesian literature. As producers of investment goods make losses, they have no other choice than to dismiss workers, even though their physical productivity is unchanged. This, in turn, reduces the demand for consumption goods and causes unemployment to spread to this sector. Real income falls along with money income (the price level is likely to fall to a smaller extent). The fall in money income increases the supply of money to hold; the fall in real income decreases saving and raises its marginal efficiency above the level r_1.[40] This double set of reactions leads finally to a new equilibrium, with a smaller money and real income, less employment, higher real wages (since the price level falls) and a rate of interest somewhere below r_0 and above the new "full employment interest" r_1.[41] In terms of our graphic apparatus, a decreased marginal efficiency of capital (or increased propensity to save), shifts the IS curve to the left, as shown by the curve $I'S'$, and lowers interest rate and income, money as well as real income.

[38] The rate of interest must necessarily fall to the level r_1, for the real income and therefore the amount of real savings will be unchanged, and the marginal efficiency of this amount of real savings is r_1, by hypothesis.

[39] The real wage rate clearly cannot fall. If the real wage rate had fallen, entrepreneurs would try to expand employment while the supply of labor would, if anything, contract. If it had risen, the opposite situation would occur, and neither of these situations is compatible with equilibrium.

[40] Except if the IS curve is not monotonic decreasing, in which case the process of contraction will be more pronounced.

[41] If there was no full employment in the initial situation, then r_1 is simply the rate of interest that would maintain the old level of employment. This conclusion is also subject to the qualification mentioned in footnote 40.

16. TWO LIMITING CASES: (A) THE KEYNESIAN CASE

There is one case in which the Keynesian theory of liquidity preference is sufficient by itself to explain the existence of underemployment equilibrium without starting out with the assumption of rigid wages. We have seen (Section 5) that, since securities are inferior to money as a form of holding assets, there must be some positive level of the rate of interest (previously denoted by r'') at which the demand for money becomes infinitely elastic or practically so. We have the Keynesian case when the "full-employment equilibrium rate of interest" is less than r''. Whenever this situation materializes, the very mechanism that tends to bring about full-employment equilibrium in a system with "flexible" wages breaks down, since there is no possible level of the money wage rate and price level that can establish full-employment equilibrium.

From the analytical point of view the situation is characterized by the fact that we must add to our system a new equation, namely $r = r''$. The system is therefore overdetermined since we have 9 equations to determine only 8 unknowns.

Equations (3.2) and (3.3) are sufficient to determine the value of the real income (since r is already determined). But this value will in general not be consistent with the value of the real income determined by the last four equations. More workers would be willing to work at the ruling real wage rate than are employed, but efforts at reducing real wages and increasing employment are bound to fail. For any fall in wages and prices increases the supply of money to hold but cannot lower the rate of interest below the level r'' since the demand for money as an asset is infinitely elastic. As Keynes would say, labor as a whole will not be able to fix its own real wage rate.

It appears clearly that, in this case, equilibrium is determined by those very factors that are stressed in the typical Keynesian analysis. In particular, real income and employment is determined by the position and shape of the saving and investment function, and changes in the propensity to invest or to save change real income without affecting the interest rate.

The price level on the other hand is in neutral equilibrium (at least for a certain range of values). It will tend to fall indefinitely as long as workers attempt to lower money wages in an effort to increase employment; and it can only find a resting place if and when money wages become rigid.

In this case the Keynesian analysis clearly departs from the classical lines and it leads to conclusions that could scarcely have been reached by following the traditional line of approach.

Whether the situation we have characterized as the "Keynesian case" is typical of some or all modern economic systems is a factual question which we cannot attempt to answer here. It is beyond doubt however that its interest is not purely theoretical.[42]

(B) THE CLASSICAL CASE

We have the classical case when the equilibrium rate of interest is sufficiently high to make the demand for money to hold zero or negligible. Graphically, the IS curve of Figure 3 intersects the LL curve in the range in which LL is perpendicular to the income axis. Under these conditions changes in the rate of interest (except possibly if they are of considerable size) tend to leave the demand for money unchanged or practically so; $L_r=0$ or negligible and $M=L(Y)$. The properties of a system satisfying this condition have already been sufficiently analyzed in Sections 11 and 12.[43]

17. PRELIMINARY CONCLUSIONS

This brings to an end the first part of our analysis which aimed principally at distinguishing, as far as possible, to what extent the results of the Keynesian analysis are due to a more refined theoretical approach (liquidity preference) and to what extent to the assumption of rigid wages. We may summarize the results of our inquiry in the following propositions:

I. The liquidity-preference theory is not necessary to explain under-

[42] In the *General Theory* Keynes explicitly recognizes that the situation described as the "Keynesian case" does not seem, so far, normally to prevail in any economic system. This situation, on the other hand, certainly plays an important part in some phases of the business cycle, when a great feeling of uncertainty and the anticipation of price reductions increase the attractiveness of liquidity and, at the same time, decreases the propensity to invest. Besides, it may also soon become a normal feature of some economies if there should come to prevail a real scarcity of investment outlets that are profitable at rates of interest higher than the institutional minimum. Modifying a well-known statement of Hicks we can say that the Keynesian case is either the Economics of Depression or the Economics of Abundance. (Hicks's original statement: "The General Theory of Employment is the Economics of Depression" is found in "Mr. Keynes and the 'Classics,'" *op. cit.*, p. 155.)

[43] To what extent the "classical case" is met in practice is again a factual question. In our opinion a moderately high rate of interest is sufficient to make it unattractive to hold assets in the form of cash and therefore to induce members of the community to limit their holdings to the amount necessary for transactions (which is determined by the institutional set-up). It is perhaps not unreasonable to expect that under normal conditions a "pure" rate of interest (i.e., net of default risk) in the neighborhood of 5 per cent might be sufficient to reduce the demand for money to hold to negligible proportions.

employment equilibrium; it is sufficient only in a limiting case: the "Keynesian case." In the general case it is neither necessary nor sufficient; it can explain this phenomenon only with the additional assumption of rigid wages.

II. The liquidity-preference theory is neither necessary nor sufficient to explain the dependence of the rate of interest on the quantity of money. This dependence is explained only by the assumption of rigid wages.

III. The result of the liquidity-preference theory is that the quantity of active money depends not only on the total quantity of money but also on the rate of interest and therefore also on the form and position of the propensities to save and to invest. Hence in a system with flexible wages the rate of interest and the propensities to save and to invest are part of the mechanism that determines the price level. And in a system with rigid wages they are part of the mechanism that determines the level of employment and real income.

We proceed now to make use of our results for two purposes: (a) To examine critically some of the theories that have their logical foundation in the Keynesian analysis. (b) To state some general conclusions about the determinants of the rate of interest.

PART II

18. GENERAL REMARKS ABOUT THE ASSUMPTION OF WAGE RIGIDITY IN THE KEYNESIAN THEORIES

In the *General Theory* Keynes does of course recognize the fundamental importance of the relation between money wages and the quantity of money as is shown by his device of the wage units. This very fact, on the other hand, has had the effect of obscuring the part played by wage rigidities in the determination of economic equilibrium. This can be clearly seen in a large body of literature based on the Keynesian analysis, and will be illustrated with a few examples.

(A) Let us first consider the role of investment.

The statement that unemployment is caused by lack of investment, or that a fall in the propensity to invest or an increase in the propensity to save will decrease employment, has become today almost a commonplace.

As we have seen, however, lack of investment is sufficient to explain underemployment equilibrium only in the "Keynesian case," a situation that is the exception and not the rule.

It is true that a reduced level of employment and a reduced level of investment go together, but this is not, in general, the result of causal relationship. It is true instead that the low level of investment and

employment are both the effect of the same cause, namely a basic maladjustment between the quantity of money and the wage rate. It is the fact that money wages are too high relative to the quantity of money that explains why it is unprofitable to expand employment to the "full employment" level. Now to each level of employment and income corresponds a certain distribution of the employment between the production of consumption and investment goods determined by the saving pattern of the community. Hence, when the over-all level of employment is low there will be a reduced level of investment as well as a reduced level of consumption. And the level of investment is low because employment is low and not the other way around.

What is required to improve the situation is an increase in the quantity of money (and not necessarily in the propensity to invest); then employment will increase in every field of production including investment. Again, it is true that, in general, a fall in the propensity to invest (the propensity to save being constant) tends to decrease employment (and that an increase in the same propensity has the opposite effect), but this occurs only because it decreases (or increases) the quantity of money available for transactions relative to the money wage rate and therefore makes it profitable to expand employment. Exactly the same result could be obtained by deflating (or inflating) the quantity of money directly. That a change in the marginal efficiency of investment has no direct influence on aggregate employment can be clearly seen in the "classical case" when the demand for money to hold is zero or negligible. In this case the change mentioned above does not affect employment, but only the rate of interest and therefore, at most, the distribution of the unchanged amount of employment between consumption and investment.

In conclusion, then, the statement that unemployment is caused by lack of investment assumes implicitly that every possible economic system works under the special conditions of the "Keynesian case"; and this is clearly unwarranted. In general the reduced level of employment is not a cause, but just a symptom of unemployment, which in turn is due to essentially monetary disturbances.

This formulation is not only more correct but carries also important implications about the concrete form of economic policies necessary to relieve unemployment.

(B) Another typical result of understressing the assumption of rigid wages is to be found in connection with the concepts of a "natural rate of interest" and of "cumulative inflation" and "deflation" of Wicksellian analysis.[44]

[44] See J. Marschak, "Wicksell's Two Interest Rates," *Social Research*, Vol. 8, November, 1941, pp. 469–478.

This "natural rate" is the equilibrium (and therefore full-employment) interest rate of a system with flexible wages and not of a Keynesian system with rigid wages. Under "flexible" wages, as we know, the equilibrium rate of interest does not depend on the quantity of money. But, because of the time required for a new position of equilibrium to be reached when some of the conditions change, it will depend on the rate of change of M. Thus the money authority will be able to keep r below (or above) its equilibrium value by increasing (or decreasing) the quantity of money without limit; we thus get a process of cumulative inflation or deflation. Under Keynesian assumptions this ceases to be true; but only because wages are assumed rigid and in this condition, as we have seen, it is in general possible to change the rate of interest with a finite change in the quantity of money.[45]

(C) As a last example, we may quote Lange's "optimum propensity to consume."[46] This concept, outside of its theoretical interest, is only of practical importance if for some reason, money wages and money supply are absolutely inelastic. In general all that is required to increase employment is to expand the quantity of money (or at worst reduce wages) without any necessity for interfering with the propensity to consume.[47]

19. LERNER'S THEORY OF THE RATE OF INTEREST

We proceed now to consider the typically "Keynesian" theory of the rate of interest and money due to A. P. Lerner. We choose Lerner's theory, because its extremism and its clear-cut formulation permit of a useful criticism.

[45] The case is more complicated if the relation between Y and r described by the IS curve is not monotonic decreasing in the relevant range. It might then appear that an attempt of the money authority at reducing the interest rate will result in a fall in income and employment. This is the result reached by Marschak. Actually as the money authority expands the quantity of money by open-market policy it finds that the rate of interest eventually rises along with income and employment instead of falling. If the money authority insists on keeping the interest rate at the planned level it will have to go on expanding the quantity of money. This will either push the system to some new equilibrium if the planned rate is equal to or larger than the full-employment rate, or it will cause inflation if the planned rate is below this level. But in no event will an initial attempt at lowering r by open-market policy lead to a contraction of income.

[46] Oscar Lange, "The Rate of Interest and the Optimum Propensity to Consume," *Economica*, Vol. 5 (N. S)., February, 1938, pp. 12–32.

[47] If the demand for money is infinitely elastic the propensity to consume plays an important role in the determination of employment. In this case the optimum level of consumption C' would clearly be $C' = Y' - I(r'', Y')$, where Y' is full-employment income and r'' the critical level of the rate of interest for which $L_r = \infty$.

The substance of Lerner's argument, as far as we can make out, is this: The "classical theory" that saving and investment determine the rate of interest must be rejected: saving and investment, being identically equal, cannot determine interest. This is instead determined by the quantity of money according to a demand-for-money function, say $M=f(r)$.[48]

The first argument is clearly unimportant since it is based on definitions. If one accepts the Keynesian definitions then, of course, actual (or *ex post*) saving and investment are identical; and clearly the *ex post* identity, saving≡investment, cannot determine either the rate of interest or income. This however does not prove that the propensities to save and to invest are irrelevant to the determination of interest.

We know on the contrary, that, under assumption of flexible wages, neither of Lerner's arguments holds. In this case the rate of interest is independent of the quantity of money and, except in limiting cases, is determined only by the propensities to save and to invest [equations (3.2) and (3.3)].

Let us stress, in order to avoid misunderstandings, that we perfectly agree with Lerner and with all the Keynesians that saving and lending are the result of two independent decisions; our equation (3.3) is a saving schedule and not a schedule of supply of loanable funds. However we cannot agree with Lerner that to treat saving as a "demand-for-securities schedule" is, without qualifications, a serious blunder, or that the classical analysis as to the effect of shifts in the desire to invest or to save is right by pure chance. We must remember that saving and lending coincide when the demand for money to hold is zero or constant. The quantity theory of money starts out with the assumption that the demand for money to hold is identically zero: $D_a'(r)\equiv 0$ or $M=L(Y)$. Now this assumption is unsatisfactory for a general theory, but may be fully justified under certain conditions.

We know that, when the equilibrium rate of interest is sufficiently high, the demand for money to hold does become zero, even if it is not assumed to be identically zero. And, under historically realized conditions, the equilibrium rate of interest may be sufficiently high to make the demand for money to hold so negligible and so scarcely affected by observed changes in the interest rate that this demand can, safely, be neglected. Interest becomes a factor of secondary importance and can

[48] See especially, "Alternative Formulations of the Theory of Interest," *Economic Journal*, Vol. 48, June, 1938, pp. 211–230; and "Interest Theory—Supply and Demand for Loans or Supply and Demand for Cash?" This latter paper has been recently made available to me by Mr. Lerner in manuscript form; it is to be published in the *Review of Economic Statistics*. The present criticism is also the result of a long personal discussion and correspondence.

be dropped along with many others which certainly do influence the demand for money but are not sufficiently relevant to warrant separate consideration. Under these conditions, the assumption $M=L(Y)$ will give a satisfactory approximation to economic reality.[49] Under changed historical conditions this assumption is no longer justified and it becomes necessary to take into account new factors to avoid oversimplifications.[50]

When we recognize that the demand for money to hold need not be zero (and as long as it is finite), saving and lending coincide only when the demand for money to hold is constant, that is to say, in equilibrium. The equality of money savings and lending becomes an equilibrium condition which, under flexible wages, *determines the price level, not the rate of interest.* And this in turn may explain the traditional lack of attention to the demand for money to hold in connection with the theory of interest.

Thus Lerner's theory cannot explain the rate of interest in a system with "flexible" wages. Let us then see whether it holds within the limits of his (tacit) assumption of rigid wages. We will agree at once that under this assumption the rate of interest depends on the quantity of money, but this is true only in a very special sense. If we look at our "Keynesian" model we find that we have 7 equations in 7 unknowns and two arbitrary quantities or "parameters," M and W_0. The solution of the system gives each of the 7 variables as functions of these arbitrary parameters: $\bar{r}=r(M, W)$, $\overline{Y}=Y(M, W)$, $\overline{N}=N(M, W)$, etc. On the basis of previous considerations these can be written:

$$(5.1) \qquad \bar{r} = r\left(\frac{M}{W}\right), \qquad (5.2) \qquad \overline{Y} = Y\left(\frac{M}{W}\right), \text{ etc.}$$

If this is the sense in which Lerner states that r is a function of M, his statement is formally correct. But in the first place it is not very helpful for understanding the determinants of the rate of interest. In a system with rigid wages practically every economic variable depends on the quantity of money (and the money wage). The rate of interest depends on M as much as the price of shoes or employment in ice-

[49] The fact that hoarding and unemployment have always developed in certain phases of the business cycle is not an objection to that. For these are features for a theory of business cycles to explain. Here we are only comparing static theories.

[50] Thus for example, the outcome of a certain physical experiment may be influenced, to a slight extent, by changes in humidity. Then, if the experiment is carried out in a place in which the observed variations in humidity are not sufficient to affect the outcome sensibly, it is perfectly justifiable to neglect it. If the same experiment were conducted somewhere else, where humidity is known to be highly unstable, precautions should be taken in interpreting the results.

cream manufacturing. In the second place it has nothing to do with Keynes's liquidity preference: r depends on M even if we neglect the liquidity demand for money (see Section 11). Hence if Lerner's equation, $M=f(r)$, corresponds to our equation (5.1), then it is not a demand-for-money schedule, but an empirical relationship obtained by previous solution of a system of equations of which the demand for money itself is one. And his approach certainly throws no light on the determinants of the rate of interest.

The only alternative is to consider Lerner's equation as a true demand for money corresponding to our equation (1): $M=L(r, Y)$. But why has the second variable been omitted? The answer is clear; by concentrating attention on the liquidity preference and the demand for money to hold, sight has been lost of the demand for money to spend. Thus we go from one extreme to the other; instead of neglecting the influence of the rate of interest as in the "quantity theory," we neglect the part played by income in determining the demand for money. The results of this unjustified omission are serious in many respects. The most serious is that it leads to the conclusion (reached by Lerner) that saving and investment play no part in the determination of the rate of interest.[51] Figure 3 shows on the contrary that equations (2) and (3) play as vital a role as the demand-for-money equation. It is clear also that changes in the propensity to save or to invest or in the wage rate, lead directly to changes in the interest rate.

To defend his point Lerner is forced to say that changes in these propensities affect the rate of interest *because* they change the demand for money, i.e., because they shift the graph of $M=f(r)$.[52] But this is true and by definition only if Lerner identifies $M=f(r)$ with our equation (5.1). Since this equation is obtained by previously solving the whole system, it contains the relevant parameters of the functions which determine the rate of interest. A change in any of these parameters changes or shifts the function $r=r(M/W)$ accordingly. But, as we

[51] In "Alternative Formulations of the Theory of Interest," Lerner writes: "For the first, easy step [from the classical to the modern theory of interest] is the insinuation of Liquidity Preference as a junior partner in the old established one-man firm in the business of interest-determination, and the second . . . step is to put Saving-Investment, the senior partner, to sleep, as a preliminary to kicking him out" (*op. cit.*, p. 221).

[52] That this is Lerner's point of view may be seen for instance in the following passage from a letter written to me in June, 1943. Discussing the effects of an increase in the propensity to invest in the "classical case" (demand for money to hold equal zero) he writes: "Even in that case there must be a fall in income which decreases the need for cash which lowers the rate of interest so that the investors have a signal that they should increase investment, but an infinitesimal decrease in employment is sufficient to bring about any necessary fall in the rate of interest. . . ."

have already seen, equation (5.1) cannot possibly help us in understanding the determinants of the rate of interest.[53]

Another consequence of Lerner's formulation is that it leads to the conclusion that the interest rate can always be lowered by increasing the quantity of money, at least to the point where the demand becomes infinitely elastic; while the truth is that no finite change in the quantity of money can hold the interest rate below the full-employment level.[54]

Let us finally note that Lerner's theory is not fully satisfactory even in the "Keynesian case." It is true that in this case saving and investment do not determine the rate of interest, but it is equally clear that the rate of interest does not depend on the quantity of money.

In conclusion, to say that the rate of interest is determined by the schedule $M = f(r)$ is useless and confusing if this schedule is arrived at by previous solution of the entire system; it is an unwarranted simplification, full of serious consequences, if this function is treated as an ordinary demand function. And the statement that the propensity to save and invest plays no part in determining the rate of interest is true only in a limiting case: the Keynesian case.

20. HICKS'S THEORY—THE RATE OF INTEREST AND THE COST OF INVESTING IN SECURITIES

In *Value and Capital* Hicks has developed what is probably the most daring attempt at reducing the rate of interest to a purely monetary phenomenon.

In Hicks's own words the rate of interest is explained by the "imperfect moneyness" of securities. "The imperfect moneyness of those bills which are not money is due to their lack of general acceptability: it is this lack of general acceptability which causes the trouble of investing in them"[55] and it is this trouble, namely "the trouble of making transactions [i.e., of purchasing securities] which explains the short rate of interest."[56] And these same factors also explain the long rate since the long rate is some average of the short rates plus a premium to cover the risk of (unanticipated) movements in the future short rates.[57]

Thus the rate of interest is explained by the fact that securities are not a medium of exchange and is determined essentially by the cost of

[53] To give another example, we can solve the system to obtain, say, the equilibrium output of shoes (Q) as a function of the quantity of money: $Q = f(M, W)$ or $M = F(Q, W)$. But to say that a change in tastes changes the output *because* it shifts this function is formally correct but perfectly useless as a tool of analysis.

[54] Proper qualifications must be made for the case in which the IS curve is not monotonic decreasing.

[55] *Value and Capital*, p. 166.

[56] *Ibid.*, p. 165.

[57] *Ibid.*, Chapter XI.

making loan transactions. This is certainly an unusual theory of interest and an astonishing one, to say the least; it appears irreconcilable with the theory we have developed throughout this paper.

Hicks's theory finds its origin in an attempt to answer a question posed by the Keynesian analysis. The reason that induces people to hold assets in the form of cash rather than securities is that the value of even the safest type of securities is not certain: it is subject to changes due to movements in the rate of interest. Now, as we have seen, this risk decreases as the duration of the loan transaction becomes shorter: and it disappears entirely on loans that last only one "Hicksian week" (or one income period in our model) since by hypothesis the rate of interest cannot change. There must then be some other reason to stop people from holding all of their assets in the form of securities and thus reducing their demand for "money to hold" to zero; this reason can only be the cost of investing in this riskless type of loans. This is Hicks's starting point: and so far there seems to be no difference from our own approach as developed in Section 5. But from these correct premises Hicks draws the wrong conclusion: namely *that it is the cost of investing that explains the rate of interest.* To say that the cost of investing is necessary to explain *why* the demand for money to hold is not always zero and to say that it *explains* the rate of interest are quite different statements. There is a logical gap between the two. Thus, for example, from the correct premise that the cost of automobiles in New York cannot fall to zero because they have to be transported from Detroit, there does not logically follow the conclusion that the cost of cars in New York is explained or determined by the cost of transporting them.

There is a different way of explaining the rate of interest, which is not less satisfactory for the fact of being obvious: namely that for certain categories of people (entrepreneurs as well as spendthrifts) it is worth while to pay a premium to obtain spot cash against a promise to pay cash in the future. This is the course we have followed: and it is clearly all that is necessary to explain the existence of the rate of interest. The cost of investing continues to play an important part in our theory: (a) it explains why the demand for money to hold is not identically zero; (b) it explains why the rate of interest can never fall below a certain level in a free capitalistic economy; and hence it explains the pecularities of the Keynesian case. But it is clear that it is not necessary to explain the rate of interest.

Our next task is to show that the cost of investing is also not sufficient to explain the nature of interest. To this end we must disprove Hicks's statement that if people were to be "paid in the form of bills . . . there would be no cost of investment and therefore . . . no reason for

the bills to fall to a discount,"[58] i.e., no rate of interest. It is easy to show that, even if "bills" were to be used as medium of exchange, there would be no reason for the rate of interest to fall to zero.

Let us consider first the case of a "stationary state." It is well known that the stationary state is characterized by the fact that the rate of change of the quantity of capital is zero; the marginal efficiency of the existing quantity of capital is equal to the rate of interest, say r_0, that makes net saving equal to zero.[59] Now it is theoretically conceivable that, in this state, securities might replace money as a medium of exchange;[60] their purchasing power would be objectively determined by their discounted value since, by hypothesis, the future rate of interest is known and constant. Their aggregate value would also be constant but, since individual savings need not be zero, there would be a net flow from dissavers to savers. Under these conditions it is clear that securities would continue to yield the rate of interest r_0, even though they would be performing the function of a medium of exchange. Thus, as far as the stationary state goes, Hicks's conclusion does not follow: the interest rate would be zero only in the special case $r_0 = 0$.

Next let us consider an expanding economy, in which the net level of saving and investment is not zero, and let us assume again that it is technically possible for securities to be accepted as a medium of exchange.[61]

In this economy, if there is to be no inflation, it is necessary that the rate of money investment be not larger than the rate of (*ex ante*) saving. Now there are two possibilities:

(a) There exists some mechanism by which the net increase in outstanding securities cannot exceed net savings. Then the competition of borrowers to obtain loans will automatically determine the level of the rate of interest.

(b) There is no limitation as to the issuance of new securities per unit of time. Then, of course, the rate of interest would be zero, since there would be no necessity for borrowers to compete. But the result would clearly be a situation of unending and progressive inflation. In the first case the stability of the quantity of active money and therefore of the price level is assured by the fact that savers would increase their "hoards" of securities-money, at a rate equal to the net increase in the value of outstanding securities. But in the second case there is nothing

[58] *Ibid.*, p. 165.

[59] For a more detailed description of the conditions that give rise to a stationary state see, for instance, M. Timlin, *Keynesian Economics*, Chapter IV.

[60] See, for instance, *ibid.*, p. 53.

[61] This would require that all people agree at all times on the present value of every security.

to stop the price level from rising indefinitely, except if it so happens that the "full employment" rate of interest is zero or negative.[62]

We may therefore safely conclude that the rate of interest is not explained by the fact that securities are not money. Once we recognize this, the complicated and confusing Hicksian theory about the imperfect moneyness of securities becomes unnecessary and should, in our opinion, be abandoned.

To say that different assets share in different degrees the quality of "moneyness" either has no meaning or it is based on a confusion between liquidity and the properties of a medium of exchange. It is true that different assets have different degrees of liquidity, since the liquidity depends on the perfection of the market in which a good is traded. And it is also true that money is probably, under normal conditions, the most liquid of all assets. But the property of money is that it is accepted (freely or by force of law) as a medium of exchange: and liquidity does not make money out of something that is not money. Whatever one's definition of liquidity, to say that a government bond, a speculative share, a house, are money in different degrees, can at best generate unnecessary confusion. It is true that money and securities are close substitutes, but this connection is to be found elsewhere than in degrees of moneyness; it depends on the fact that both money and securities are alternative forms of holding assets in nonphysical form. Securities are thus close substitutes for money, but not for money as a medium of exchange, only for money as an asset.

Having shown that the cost of investment neither explains nor determines the rate of interest, we will agree with Hicks that "the level of that [short] rate of interest measures the trouble involved in investing funds . . . to the marginal lender."[63] One cannot disagree with this statement any more than with the statement that the price of butter measures the marginal utility of butter to each member of the community.[64] Both statements are either tautologies or definitions of rational behavior. They are tautologies if they mean that all those who found it convenient to perform a certain transaction have done so. They are definitions of rational economic behavior if they state the conditions under which economic agents will maximize their satisfac-

[62] We are well aware of the fact that the excess of money investment over (*ex ante*) saving does not lead to inflation, unless there is full employment to begin with, or until full employment is reached. It remains true however that, except in the case mentioned in the text, a zero rate of interest must eventually lead to inflation.

[63] *Op. cit.*, p. 165.

[64] More exactly: the ratio of the price of butter to that of any other commodity measures the ratio of their respective marginal utilities.

tion.[65] But it is clear that whether these statements are tautologies or definitions they are not sufficient to explain either the price of butter or the level of the rate of interest.

To conclude then we agree with Hicks that the rate of interest is at least equal to the cost of investing to the marginal lender, but this statement is not very helpful for understanding the rate of interest. But the Hicksian theory that the rate of interest is determined or simply explained by the imperfect moneyness of securities must be discarded as faulty.

21. SAVING AND INVESTMENT OR SUPPLY OF AND DEMAND FOR CASH?—CONCLUSIONS

It will now be useful, in concluding this paper, to restate in brief form the general theory of interest and money that emerges from our analysis.

We believe that the best way of achieving this aim is to show how, by means of our theory, we can answer the controversial question that has caused so much discussion in recent economic literature.

Is the rate of interest determined by the demand for and supply of cash? Or is it determined by those "real factors," psychological and technological, that can be subsumed under the concepts of propensity to save and marginal efficiency of investment?

We consider it to be a distinct advantage of our theory that we can answer both questions affirmatively. We do not have to choose between these two alternatives any more than between the following two: Is the price of fish determined by the daily demand and the daily supply; or is it determined by the average yearly demand and the cost of fishing?

Since we have maintained throughout this paper that, in general, saving and lending are independent decisions, we must clearly agree that the "daily" rate of interest is determined by the demand for and supply of money to hold (or, for that matter, by demand for and supply of loanable funds).[66] It is this very principle that has formed the base of our analysis of the money market (Section 7). But we cannot stop at this recognition and think that this is sufficient for a general theory of the rate of interest.

To come back to our example, it is certainly true that the daily price

[65] If anything, Hicks's statement is less illuminating, since there is, at least theoretically, the possibility that the rate of interest may exceed the cost of lending idle funds to the marginal lender: it is this very possibility that gives rise to the "classical case."

[66] In this respect we have nothing to add to the arguments developed by Hicks in Chapter XII of *Value and Capital*. There are enough equations to determine all the prices on each Monday and it makes no difference which equation is eliminated.

of fish is entirely explained by the daily catch of fish. But if we want to understand why the daily price fluctuates around a certain level and not around a level ten times as high, we must look for something more fundamental than the good or bad luck of the fishermen on a particular day. We shall then discover that the number of fishermen and the amount of equipment used does not change daily but is determined by the condition that the average returns, through good and bad days, must be sufficiently high to make the occupation of fishing (and investment in fishing equipment) as attractive as alternative ones.

What is obviously true for the price of fish must also hold for the price of loans. The statement that the "daily" rate is determined by the "daily" demand for and supply of money (or, more exactly, of money to hold) does not greatly advance us in the understanding of the true determinants of the rate of interest. This theory by itself is insufficient to explain, for instance, why in countries well-equipped and of great saving capacity, like England or the United States, the system of rates of interest fluctuates around low levels (2 or 3 per cent for the pure long rate and much less for short rates); while it fluctuates around much higher levels (5 or 6 per cent or more for the long rate) in countries poor in savings or rich but scarcely developed. Is that because in the last-mentioned countries the supply of cash is insufficient? Clearly not. The explanation for this difference can only run in terms of those more fundamental factors, technological and psychological, that are included in the propensity to save and the marginal efficiency of investment.

As we have shown in our model the equality of demand and supply of loanable funds is the equilibrium condition for the week (or for our income period) and determines the equilibrium rate of interest (or system of rates) for the week. It corresponds to the short-run equilibrium condition of the Marshallian demand and supply analysis: price equals marginal cost. But the stock of money to hold (the supply) tends itself to change and thus push the "daily" rate toward the level at which the flow of money saving equals the flow of money investment. The condition, (*ex ante*) saving = (*ex ante*) investment, corresponds to the long-run Marshallian condition (under perfect competition): price = average cost including rent.

The first condition is satisfied even in the short period since it is the result of decisions that can be carried out instantaneously (see Section 5). The second is a long-run condition and therefore may actually never be satisfied: but it is necessary to explain the level toward which the weekly rate tends (even though this level may never be reached since the long-run equilibrium rate of interest itself changes).

Thus, to complete our theory, we must be able to explain what de-

termines the level of long-run equilibrium. At this point we find that our answer is not unique since it depends on the assumptions concerning the form of the supply-of-labor schedule.

I. As long as wages are flexible, the long-run equilibrium rate of interest is determined exclusively by real factors, that is to say, essentially by the propensity to save and the marginal efficiency of investment. The condition, money saving = money investment, determines the price level and not the rate of interest.

II. If wages are rigid it is still true that the long-run equilibrium rate of interest is determined by the propensities to save and to invest but the situation is now more complicated; for these propensities depend also on money income and therefore on the quantity of active money which in turn depends itself on the level of the rate of interest. Thus, unless wages are perfectly flexible or the supply of money is always so adjusted as to assure the maintenance of full employment, the long-run equilibrium rate of interest depends also on the quantity of money and it is determined, together with money income, by equations (1), (2), and (3) of our model. We want however to stress again that the dependence of the rate of interest on the quantity of money does not depend on liquidity preference. In a system with rigid wages not only interest but also almost every economic variable depends on the quantity of money.

III. Finally our theory of the rate of interest becomes even less uniform when we take into account the "Keynesian case." In this case clearly the long-run equilibrium rate of interest is the rate which makes the demand for money to hold infinitely elastic. The economic theorist here is forced to recognize that under certain conditions the rate of interest is determined exclusively by institutional factors.

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