Real Exchange Rate and Structural Change in a Kaldorian Balance of Payments Constrained Growth Model

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Resumo: O presente artigo tem por objetivo desenvolver um modelo Kaldoriano de crescimento que (i) incorpore a restrição de balanço de pagamentos, eliminando assim a inconsistência presente nos MCRBP; e (ii) estabeleça um mecanismo pelo qual o nível da taxa real de câmbio possa afetar o crescimento de longo-prazo das economias capitalistas. Uma inovação importante introduzida no modelo que será desenvolvido ao longo desse artigo é a hipótese de que o coeficiente de Kaldor-Verdoorn - que capta a sensibilidade da taxa de crescimento da produtividade do trabalho com respeito a taxa de crescimento da produto real - depende da participação da indústria no PIB. Essa hipótese permitirá introduzir no modelo a possibilidade de ***mudança estrutural***, a qual é entendida como um processo dinâmico mediante o qual a participação da indústria no produto se altera ao longo do tempo. Dessa forma, será possível analisar as propriedades dinâmicas do modelo tanto no caso em que a estrutura produtiva é mantida constante (caso sem mudança estrutural), como no caso em que a mesma se altera em decorrência de algum processo econômico (caso com mudança estrutural).

Palavras-Chave: Crescimento puxado pela demanda, câmbio real, mudança estrutural.

Abstract: The objective of the present article is to develop a Kaldorian Growth model that (i) had a balance of payments constraint, in order to eliminate the inconsistency of balance of payments growth models; and (ii) defines a precise mechanism by which the level of real exchange rate can affect long-term growth. An important innovation introduced in the model is the idea that Kaldor-Verdoorn coefficient – that measures the sensibility of growth rate of labor productivity to output growth – depends on the share of manufacturing output on GDP. This hypothesis allowed us to introduce the possibility of structural change, defined as a dynamic process by which the share of manufacturing industry on real output could change over time. In this case, it will be possible to analyze the dynamic properties of the model either in the case where productive structure is kept constant (case with no structural change), as in the case where it evolves over time as a result of some economic process (case with structural change).

Key-Words: Demand-led Growth, Real Exchange Rate, Structural Change.

Jel-Code: O1, O11; O12

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1. Introduction

The balance of payments constrained growth model, pioneered developed by Anthony Thirwall(1979), holds two fundamental problems. Firstly, they fully disregard the cumulative causation mechanism, so relevant to kaldorian growth models. Indeed, assuming constant terms of trade then productivity gains induced by economic growth have no effect over the dynamics of the system, in a such way they become, strictly, irrelevant. However, in this case, the system no longer has any adjustment mechanism between aggregate supply and demand. This deficiency was observed by Palley (2002) for whom the balance of payments constrained growth model would be inconsistent in the extent that only in a "happy coincidence" would be possible the equality between the growth rate compatible with the balance of payments equilibrium and natural growth rate, i.e., the one that keeps the unemployment rate constant over the time. In this way, the balance of payments constrained growth models are not, in general, compatible with a balanced growth path.

Last but not least, the balance of payments constrained growth models fully neglect the relationship between the real exchange rate and the long-term growth. Indeed, in those models the long-term equilibrium growth rate depends on the ratio of export and import income elasticities multiplied by rest of the world growth rate. Thus, real exchange rate variations are assumed irrelevant to the long-term growth either because empirical evidence shows that finding that export and import price elasticities are low, in a such way that the impact of a real devaluation of exchange rate over the growth path of exports and imports is low; either because the terms of trade do not show a systematic trend to appreciation or depreciation in the long-term.

In recent years, an interesting literature has been developed about the relation between real exchange and economic growth. The Razin and Collins (1997) seminal paper indicated to the existence of important non-linearities in the relationship between exchange rate misalignment - defined as a lasting deviation of the real exchange rate with respect to some reference value, determined by the "fundamentals"- and the real output growth in a sample of 93 developing and developed countries between 1975-1993. Indeed, the empirical results showed that while only very large overvaluations of real exchange rate are associated with a slower economic growth in the long term, even moderated undervaluation of the real exchange rate have a positive effect on economic growth. Rodrik (2008), analyzing the development strategies adopted by a group of countries, noted that an important factor for the ignition of a process of sustained growth of the real output is the maintenance of an undervalued and stable real exchange rate. Similarly, Frenkel (2004) - analyzing the employment and the growth rate performance of Argentina, Brazil, Chile and Mexico – verified that maintaining a competitive and stable real exchange rate is the best contribution the macroeconomic policy can provide to the long-term growth. In the Brazilian case, Oreiro, Punzo e Aráujo (2012) indicated to the existence of a negative and statistically significant effects of exchange rate misalignment over output growth rate in the period 1994-2007. Therefore, the absence of a connection between the level of the real exchange rate and the long-term growth in the context of the balance of payments constrained growth models becomes theoretically unacceptable.

Hence, this article aims to develop a kaldorian growth model that (i) incorporates the balance of payments constraint, eliminating the inconsistency presented on balance of payments constrained growth models; (ii) establishes a mechanism by which the level of the real exchange rate may affect the long-term growth of capitalist economies.

The model to be developed throughout this article incorporates some innovations introduced by Oreiro (2009) into the structure of Kaldorian growth models, such as the conduction of monetary policy in a Inflation Target Regime, nominal interest rate determined by a Taylor rule, a floating exchange rate regime and imperfect capital mobility. In contrast to the Oreiro model, however, we will assume a balance of payments constraint in which the growth rate of international capital inflows is a positve function of the differential between the domestic interest rate and the international interest rate plus the country risk premium. In this context, the differential between the domestic and international interest rates (plus the risk premium) will also determine the rate of depreciation (or appreciation) of the nominal exchange rate.

Another important innovation introduced in the model that will be developed throughout this article is the hypothesis that the Kaldor-Verdoorn coefficient - which captures the sensibility of the rate of growth of labor productivity with respect to the rate of growth of the real output - depends on the manufacturing share on output. This hypothesis will allow to introduce into the model the possibility of structural change, which is understood as a dynamic process by which the manufacturing share of output changes over time. In this way, it will be possible to analyze the dynamic properties of the model both in the case where the productive structure is kept constant (case with no structural change), and in a situation in which it changes due to some economic process (case with structural change).

The structural change, in its turn, will be induced by the exchange rate misalignment, that is, by the difference between the actual value of the real exchange rate and the level of the real exchange rate that would correspond to the "industrial equilibrium", in other words, the exchange rate level in which domestic firms that use state-of-art technologies are competitive in international markets (Bresser-Pereira, Oreiro and Marconi, 2014, 2015).

In the case of an economy with no structural change, the analysis of the short-run equilibrium solution of the model shows that the growth rate compatible with the equilibrium of the balance of payments can be affected by changes in the medium-term inflation target in a such way that money is non-neutral, at least in the short term. In addition, changes in the international economic scenario in the form of variations in the growth rate of the rest of the world´s income and / or in the international inflation rate are transmitted to the domestic economy in the form of changes in the output growth rate and inflation rate.

Analyzing the properties of the balanced growth path in the case of an economy with no structural change, we find two interesting results. The first one is that the output growth rate along this path is independent of the medium-term inflation target, so that money is neutral in the long term. This is another surprising result given that in Kaldorian models output growth is demand-led. The second interesting result is that inflation rate does not converge to the medium-term target defined by the monetary authority.

The result of the monetary policy neutrality in the long run will no longer holds, however, in the case of an economy with structural change. In this context, raising the inflation target pursued by the Monetary Authority has the effect of inducing an increase in the share of the manufacturing industry in GDP, since it induces a devaluation of real exchange with respect to its industrial equilibrium level. Though this devaluation is purely temporary, it is capable of inducing a structural change in the economy, which will eventually increase the Kaldor-Verdoorn coefficient and, thus, the output growth rate along the balanced growth path.

2 – Structure of the Model

Let us consider a small open economy with a free-floating exchange rate regime and imperfect capital mobility, in which growth rate of exports and imports are given by:

$$\hat{x}_t = \mu(\hat{p}_t^* - \hat{p}_t + \hat{e}_t) + \varepsilon\hat{z}_t \quad (1)$$

$$\hat{m}_t = \gamma(\hat{p}_t - \hat{p}_t^* - \hat{e}_t) + \pi\hat{y}_t \quad (2)$$

In which $\hat{x}_t$ is the growth rate of exports (quantum) the period t, $\hat{m}_t$ is the growth rate of imports (quantum) in the period t, $\hat{p}_t$ is the domestic rate of inflation in the period t, $\hat{p}_t^*$ is the rest of the world inflation in the period t, $\hat{e}_t$ is the rate of depreciation of nominal exchange in period t, $\hat{y}_t$ is the domestic output/income growth rate in the period t, $\hat{z}_t$ is the rest of the world output/income growth rate in the period t, μ is price elasticity of exports, γ is the price elasticity of imposrts, ε is the income elasticity of exports, π is the income elasticity of imports.

We will assume the validity of Marshall-Lerner's condition, so that:

$$\mu + \gamma > 1 \quad (3)$$

Such as in Moreno-Brid's (2003)[1] model we will assume that the Balance of Payments restriction in period t is given by:

$$\hat{e}_t + \hat{p}_t^* + \hat{m}_t = \theta_1(\hat{p}_t + \hat{x}_t) - \theta_2(\hat{p}_t + \hat{r}_t) + (1 - \theta_1 + \theta_2)(\hat{p}_t + \hat{f}_t) \quad (4)$$

In which: $\theta_1 = \frac{px}{ep^*m}$ is the ratio between the initial value of exports and the initial value of imports; $\theta_2 = \frac{pr}{ep^*m}$ is the ratio between the initial value of external liability services and the initial value of imports; $\hat{r}_t$ is the growth rate of services (interest and dividends) related to the external liabilities in the period t, e $\hat{f}$ is the real growth rate of external capital flows in period t.

Two important points can be observed in this equation. The first one is that the constraint imposed here is "deflated" in terms of value paid by imports. The second one is that we are considering an economy with a net debt to the rest of the world, since θ_2 is a positive parameter and there is a negative signal before it.

Assuming capital mobility to be imperfect in Mundell's sense, the real rate of growth of external capital flows will be a function of the difference between the domestic interest rate and the international interest rate adjusted by the country risk premium. We have :

$$\hat{f}_t = h(i_t - i_t^* - \rho) \quad (5)$$

[1] This approach advances Thirlwall and Hussain (1982), since they did not take into account the role of interest payments.

In which h is the sensibility of external capital flow growth rate to the interest differential[2], i_t is the domestic interest rate in the period t, i_t^* international interest rate and ρ country risk premium[3].

In an economy with an open capital account, the dynamics of the nominal exchange rate, assuming a free-floating exchange rate regime, depends fundamentally on inflows and outflows of foreign capital. Thus, we will assume that the rate of change of the nominal exchange rate will be a (negative) function of the growth rate of the external capital flows as in equation (6) below:

$$\hat{e}_t = -k\hat{f}_t \quad (6)$$

Where k is the coefficient of sensibility of the variation of nominal exchange rate in relation to the growth rate of external capital flows[4].

Regarding the determination of the domestic interest rate, we will assume that the economy under consideration operates with an inflation targeting regime, so that the monetary authority should deliver to society, in the medium term, an inflation rate equal to the target $\hat{p}^T$. To achieve this goal, the monetary authority sets the interest rate based on a modified version of the Taylor[5] rule such as the one assumed below:

$$i_t = (i_t^* + \rho) + \beta(\hat{p}_t - \hat{p}^T) \quad (7)$$

In which: β represents the degree of aversion of the monetary authority to the deviations of the inflation rate from the medium-term inflation target.

With regard to domestic inflation rate, we will assume that it is equal to the difference between wage inflation and the rate of growth of labor productivity[6], according to equation (8) below.

$$\hat{p}_t = \hat{w}_t - \hat{q}_t \quad (8)$$

Regarding the determination of the growth rate of labor productivity, we will assume the existence of static and dynamics economies of scale so that the so-called Kaldor-Verdoorn law is valid. Then, we have[7]:

$$\hat{q}_t = c + \alpha\lambda_{t-1}\hat{y}_{t-1} \quad (9)$$

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[2] This parameter h reflects, among other things, the level of capital controls in the economy. Indeed, if the inflow of foreign capital is prohibited by law, as occurred during the period of the Bretton Woods agreement, then h = 0, so that the differential between domestic and external interest will have no consequence in terms of attraction or repulsion of foreign capital from the country. On the other hand, the higher the value of h, the greater the sensibility of external capital flows to the differential between internal and external interest rates and, therefore, lower will be the level of capital controls. On regard to the economics of capital controls, see Oreiro (2004).

[3] Without loss of generality we will assume that the country risk premium is constant over time.

[4] This parameter fundamentally reflects the density of the foreign exchange market, that is, the volume of operations that take place daily in that market. As higher the exchange market density is, the sensitivity of the nominal exchange rate to inflows and outflows of foreign capital will be the lower.

[5] This is a modified version because the output (or growth) gap is absent from the equation, meaning that the monetary authority is only concerned with the deviations of inflation from the medium-term target. A specification similar to this can be found in Carlin and Soskice (2006, p.152).

[6] This equation can easily be deduced from a pricing rule based on mark-up of the type: $p = (1 + \tau)\, w / q$, where p is the price of the domestic product, τ is the mark-up rate -up, w is the nominal wage rate and q is the labor productivity. To arrive at equation (8) it is enough to consider that the mark-up rate is constant and that the work is the only input used in the production.

[7] This approach of Kaldor-Verdoorn law is based on Botta (2009) and Gabriel, Gonzaga and Oreiro (2016)

In which α is the the so-called Kaldor-Verdoorn coefficient, which reflects the degree of productivity dynamism of the economy, that is, the extent in which output growth (from the previous period) induces productivity growth (in the current period); and λ_{t-1} is the manufacturing share on output in the period t-1. This Kaldor-Verdoorn law approach gives relevance to the manufacturing industrial sector in the productivity dynamics of the economy, as Kaldor believed this sector to be the "engine of growth" of outout and productivity.

Wage inflation, in turn, depends on the rate of domestic inflation in the previous period and the behavior of the labor market. The idea here is that nominal wages are determined by a process of collective bargaining, in which unions seeks, in first place, to defend the wages' purchasing power from losses due to inflation. In this way, unions will demand nominal wages changes to be at least equal to the inflation observed in the previous period. However, depending on the actual situation in the labor market, unions may demand real wage gains, that is, they may require changes of nominal wages that surpasses, for a certain margin, the inflation observed in the previous period. This should happen in those periods when labor demand is growing ahead of the labor supply so that unemployment rate is decreasing consistently over time. Otherwise, unions may be forced to accept a nominal wage changes that are lower than inflation in the previous period. In this case, there will be a real wage loss.

So, the wage inflation determination equation is given by:

$$\hat{w}_t = \hat{p}_{t-1} + \hat{l}_{d,t} - \hat{l}_{s,t} \quad (10)$$ [8]

In which $\hat{l}_{d,t}$ is the rate of growth of the labor demand in period t, $\hat{l}_{s,t}$ is the rate of growth of the labor supply in period t.

The labor demand growth rate is equal to the difference between the output growth rate and the labor productivity growth rate, as we can see in equation (11) below.

$$\hat{l}_{d,t} = \hat{y}_t - \hat{q}_t \quad (11)$$

Finally, without loss of generality, we will assume that the rate of growth of labor supply is constant and equal to η.

$$\hat{l}_{s,t} = \eta \quad (12)$$

[8] This equation is derived from a Phillips Curve with adaptive expectations, in which the increase in wages (wage inflation) will be a function of the change of unemployment in the economy and the rate of inflation of the previous period.

2.1 – Short-term Equilibrium

The kaldorian growth model presented in the previous section is compounded by the following equations:

$$\hat{x}_t = \mu(\hat{p}_t^* - \hat{p}_t + \hat{e}_t) + \varepsilon\hat{z}_t \quad (1)$$

$$\hat{m}_t = \gamma(\hat{p}_t - \hat{p}_t^* - \hat{e}_t) + \pi\hat{y}_t \quad (2)$$

$$\hat{e}_t + \hat{p}_t^* + \hat{m}_t = \theta_1(\hat{p}_t + \hat{x}_t) + \theta_2(\hat{p}_t + \hat{r}_t) + (1 - \theta_1 + \theta_2)(\hat{p}_t + \hat{f}_t) \quad (4)$$

$$\hat{f}_t = h(i_t - i_t^* - \rho) \quad (5)$$

$$\hat{e}_t = -k\hat{f}_t \quad (6)$$

$$i_t = (i_t^* + \rho) + \beta(\hat{p}_t - \hat{p}^T) \quad (7)$$

$$\hat{p}_t = \hat{w}_t - \hat{q}_t \quad (8)$$

$$\hat{q}_t = c + \alpha\lambda_{t-1}\hat{y}_{t-1} \quad (9)$$

$$\hat{w}_t = \hat{p}_{t-1} + \hat{l}_{d,t} - \hat{l}_{s,t} \quad (10)$$

$$\hat{l}_{d,t} = \hat{y}_t - \hat{q}_t \quad (11)$$

$$\hat{l}_{s,t} = \eta \quad (12)$$

The dependent variables of the model are: $\hat{x}_t$, $\hat{m}_t$, $\hat{y}_t$, $\hat{q}_t$, $\hat{l}_{d,t}$, $\hat{l}_{s,t}$, $\hat{f}_t$, $\hat{e}_t$, $\hat{p}_t$, i_t e $\hat{w}_t$. There are 11 dependent variables to be determined by a system with 11 linearly independent equations. It follows that this is a *determined* system.

The exogenous variables and model parameters are: $\hat{z}_t$, ρ, $\hat{p}_t^*$, $\hat{p}^T$, $\hat{r}_t$, η, i_t^*, μ, γ, ε, π, h, k, β, α, c, λ_{t-1}, θ_1 e θ_2. In addition to these variables, the system also has pre-determined variables, that is, endogenous variables whose value was determined in the previous period and which, therefore, are constant from the point of view of the current period. The pre-determined variables are: $\hat{p}_{t-1}$ and $\hat{y}_{t-1}$.

First, we will determine the short-period equilibrium of the model, that is, the values for the endogenous variables that satisfy the equations of the system formed by (1), (2), (4) - (11). The solution thus obtained will not necessarily be compatible with a balanced growth path, that is, with a path in which endogenous variables are growing at a constant rate. This solution will be derived in the next session.

To obtain the short-period equilibrium solution we will initially substitute equation (5) in (6), obtaining

$$\hat{e}_t = -kh\,(i_t - i_t^* - \rho) \quad (6a)$$

From (7), we have:

$$(i_t - i_t^* - \rho) = \beta(\hat{p}_t - \hat{p}^T) \quad (7a)$$

Substituting (7a) in (6a) we obtain:

$$\hat{e}_t = -kh\beta(\hat{p}_t - \hat{p}^T) \quad (6b)$$

Equation (6b) shows that the rate of change of nominal exchange rate is a function of the difference between the domestic inflation rate and the medium-term inflation target. Thus, if domestic inflation is higher than the target, there will be an appreciation of the nominal exchange rate, since the monetary authority will raise the nominal interest rate above its equilibrium level given by the sum between the international interest rate and the Country risk premium. On the other hand, if domestic inflation is lower than the medium-term target there will be a depreciation of the nominal exchange rate as the monetary authority reduces the nominal interest rate below its equilibrium level.

Substituting (6b) into (1) and (2), we obtain after the necessary algebraic manipulations that:

$$\hat{x}_t = \mu(\hat{p}_t^* + \alpha_1\hat{p}^T - (1+\alpha_1)\hat{p}_t) + \varepsilon\hat{z}_t \quad (1a)$$

$$\hat{m}_t = \gamma((1+\alpha_1)\,\hat{p}_t - \hat{p}_t^* - \alpha_1\hat{p}^T) + \pi\hat{y}_t \quad (2a)$$

Where: $\alpha_1 = kh\beta$.

Substituting (1a), (2a) and (6b) in (4), we obtain the following:

$$\hat{y}_t = \left(\frac{\theta_1\varepsilon}{\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi}\right)\hat{r}_t + \left[\frac{h\beta(1-\theta_1+\theta_2)+(1+\alpha_1)(1-\gamma-\theta_1\mu)}{\pi}\right]\hat{p}_t - \left(\frac{1-\gamma-\theta_1\mu}{\pi}\right)\hat{p}_t^* - \left(\frac{\alpha_1(1-\gamma-\theta_1\mu)+h\beta(1-\theta_1+\theta_2)}{\pi}\right)\hat{p}^T \quad (13)$$

The growth rate of external debt payments can be expressed by:

$$\hat{r}_t = \frac{\left(dD_t/dt\right)}{D_t} = \frac{f_t}{D_t} = \frac{f_t}{y_t}\frac{y_t}{D_t} \quad (14)$$

Where: D_t is the external debt of the economy, e dD_t/dt is, by definition, the current account deficit.

Equation (14) shows that the growth rate of payments elated to external liabilities is equal to the ratio of current account deficit as a proportion of GDP and external liabilities as a proportion of GDP. As in Moreno-Brid (2003) we will assume that external liabilities grow in the same proportion of the domestic product. Thus, both the current account deficit and the ratio of GDP to external debt as a proportion of GDP are constant over time[9]. Therefore, we must:

$$\hat{r}_t = \sigma \quad (15)$$

Substituting (15) in (13) and defining $\beta_1 = \left(\frac{h\beta(1-\theta_1+\theta_2)}{\pi}\right)$, $\beta_2 = -\left(\frac{1-\gamma-\theta_1\mu}{\pi}\right)$. We have:

$$\hat{y}_t = \left(\frac{\theta_1\varepsilon}{\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi}\right)\sigma + [\beta_1 - (1+\alpha_1)\beta_2]\hat{p}_t + \beta_2\hat{p}_t^* + (\beta_2\alpha_1 - \beta_1)\,\hat{p}^T \quad (16)$$

In what follows, we will assume that $\beta_1 > 0,\ \beta_2 > 0$ and $\beta_1 < \alpha_1\beta_2$.

Equation (16) presents the combinations locus between entre $\hat{y}_t$e $\hat{p}_t$ for which the combinations of the balance of payments is in equilibrium. Basedo on (16) we know that:

[9] We can assume that debt service is composed of interest plus amortizations, these two components are considered constant as a ratio to the level of external debt itself. This, in turn, is assumed to grow at a constant rate, as specified in equation (15)

$$\left|\frac{\partial \hat{y}_t}{\partial \hat{p}_t}\right|_{BOP} = (\beta_1 - (1+\alpha_1)\beta_2) < 0 \;\; (16a) \;\; \frac{\partial \hat{y}_t}{\partial \hat{z}_t} = \left(\frac{\theta_1 \varepsilon}{\pi}\right) > 0 \;\; (16b)$$

$$\frac{\partial \hat{y}_t}{\partial \sigma} = -\left(\frac{\theta_2}{\pi}\right)\sigma < 0 \;\; (16c) \quad \frac{\partial \hat{y}_t}{\partial \hat{p}_t^*} = \beta_2 > 0 \;\; (16d) \qquad \frac{\partial \hat{y}_t}{\partial \hat{p}^T} = (\beta_2\alpha_1 - \beta_1) > 0 \; (16e)$$

As expected, in equation (16a) the rate of change of domestic output is a negative function of domestic inflation rate, since equation (16) refers to the demand side of the economy, even if it is restricted by the Balance of Payments equilibrium condition; (16b), in a way, sums up this restriction since it shows that an increase in the income of the rest of the world stimulates output growth, precisely by relaxing the restriction of the Balance of Payments and increasing exports; in equation (16c) we observe an interesting result albeit analogous to the previous one, since an increase in commitments with the rest of the world in terms of debt service further tightens the restriction of the Payments balance and generates a reduction of growth; The equation (16d) sums up the price effect of foreign trade, since raising the inflation of the rest of the world makes domestic goods more competitive in international markets, inducing a faster growth rate of domestic output; Finally, equation (16e) indicates that a higher target for domestic inflation is associates with a induces a faster economic growth.

Let's turn now to the supply side of the economy. Substituting (9), (10), (11) and (12) into (8), we have

$$\hat{p}_t = \hat{p}_{t-1} + \hat{y}_t - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1}) \qquad (17)$$

The equation (8a) is the supply curve of the economy. We know that:

$$\left|\frac{\partial \hat{p}_t}{\partial \hat{y}_t}\right|_{OA} = 1 \;\; (17a) \quad \frac{\partial \hat{p}_t}{\partial \hat{p}_{t-1}} = 1 \;\; (17b) \;\; \frac{\partial \hat{p}_t}{\partial \eta} = -1 \;\; (17c)$$

$$\frac{\partial \hat{p}_t}{\partial \hat{y}_{t-1}} = -2\lambda_{t-1}\alpha < 0 \; (17d) \; \frac{\partial \hat{p}_t}{\partial \lambda_{t-1}} = -2\alpha\hat{y}_{t-1} < 0 \;\; (17e)$$

The equations (17a) to (17e) present the analysis of the partial derivatives with respect to the supply curve of the economy, so contrary to what occurs in the demand equation presented in (16), inflation and output growth have a positive relation between themselves; in equation (17b) the inflation inertia in this model is made explicit; (17c) shows that an increase in the supply of labor is associteed with a reduction in domestic rate of inflation, equation (17d) follows the same logic as (17a); and (17e) presents an essential result for the dynamics presented here: an increase of the manufacturing share generates gains of competitiveness that will engender a reduction of domestic inflation.

The dynamic system is, thus, composed of two equations:

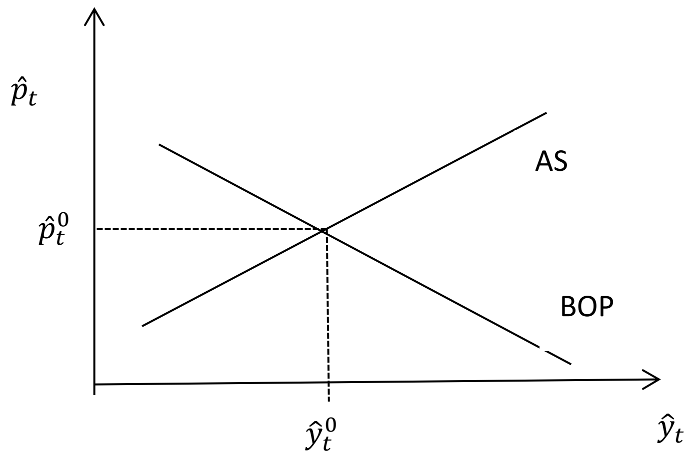
$$\hat{y}_t = \left(\frac{\theta_1 \varepsilon}{\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi}\right)\sigma + [\beta_1 - (1+\alpha_1)\beta_2]\hat{p}_t + \beta_2\hat{p}_t^* + (\beta_2\alpha_1 - \beta_1)\,\hat{p}^T \quad (16)$$

$$\hat{p}_t = \hat{p}_{t-1} + \hat{y}_t - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1}) \qquad (17)$$

We will solve the system for $\hat{y}_t$ and $\hat{p}_t$ taking the values of the parameters and the predetermined variables as given.

The short-term equilibrium visual feature of $\hat{y}_t$ and $\hat{p}_t$ can be done through Figure 1 below:

Figure 1: Short term equilibrium with no structural change



Substituting (17) into (16) we have:

$$\hat{y}_t = \left(\frac{\theta_1\varepsilon}{(1-[\beta_1-(1+\alpha_1)\beta_2]\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi(1-[\beta_1-(1+\alpha_1)\beta_2])}\right)\sigma - \frac{[\beta_1-(1+\alpha_1)\beta_2](\eta+2c-\hat{p}_{t-1})}{(1-[\beta_1-(1+\alpha_1)\beta_2])} - \frac{2\alpha[\beta_1-(1+\alpha_1)\beta_2]}{(1-[\beta_1-(1+\alpha_1)\beta_2])}\lambda_{t-1}\hat{y}_{t-1} + \frac{\beta_2}{(1-[\beta_1-(1+\alpha_1)\beta_2])}\hat{p}_t^* + \frac{(\beta_2\alpha_1-\beta_1)}{(1-[\beta_1-(1+\alpha_1)\beta_2])}\hat{p}_t^T \quad (18)$$

The equation (18) presents the formal expression for domestic output short-term equilibrium growth rate. Based on (18) we know that:

$$\frac{\partial\hat{y}_t}{\partial\hat{z}_t} = \left(\frac{\theta_1\varepsilon}{(1-[\beta_1-(1+\alpha_1)\beta_2]\pi}\right)\hat{z}_t > 0 \quad (18a) \quad \frac{\partial\hat{y}_t}{\partial\sigma} = -\left(\frac{\theta_2}{\pi(1-[\beta_1-(1+\alpha_1)\beta_2])}\right) < 0 \quad (18b)$$

$$\frac{\partial\hat{y}_t}{\partial\eta} = \left(\frac{[\beta_1-(1+\alpha_1)\beta_2]}{(1-[\beta_1-(1+\alpha_1)\beta_2])}\right) > 0 \quad (18c) \quad \frac{\partial\hat{y}_t}{\partial\hat{y}_{t-1}} = -\lambda_{t-1}\frac{2\alpha[\beta_1-(1+\alpha_1)\beta_2]}{(1-(\beta_1-(1+\alpha_1)\beta_2)} > 0 \quad (18d)$$

$$\frac{\partial\hat{y}_t}{\partial\hat{p}_t^*} = \frac{\beta_2}{(1-[\beta_1-(1+\alpha_1)\beta_2])} > 0 \quad (18e) \quad \frac{\partial\hat{y}_t}{\partial\hat{p}^T} = \frac{(\beta_2\alpha_1-\beta_1)}{(1-[\beta_1-(1+\alpha_1)\beta_2])} > 0 \quad (18f)$$

$$\frac{\partial\hat{y}_t}{\partial\lambda_{t-1}} = -\hat{y}_{t-1}\frac{2\alpha[\beta_1-(1+\alpha_1)\beta_2]}{(1-(\beta_1-(1+\alpha_1)\beta_2)} > 0 \quad (18g)$$

Equations (18a) - (18f) show some interesting properties of the short-period equilibrium of the model presented here. First, as in the models inspired on Thirwall, an increase in the income growth rate of the rest of the world is associated with an increase in the rate of growth of domestic income that is compatible with the equilibrium of the Balance of Payments. However, an increase in the current account deficit is associated with a reduction in the growth rate that allows the equilibrium of Balance of Payments over time. This is due to an increase in the current account deficit, which generates an increase in the growth rate of services related to external debt, increasing, thus, the external constraint on growth. Here follows, therefore, that in the model under consideration there is an inverse relationship between external saving and growth.[10]

[10] Bresser-Pereira and Nakano (2003) emphasized this result which is a by-prodduct of the fact that resources derived from foreign indebtedness are, in general, not directed to investment but to consumption, so that the economy does not increase its productive capacity and, consequently, does not

Another interesting result of the model refers to the impact of increase in rate of growth of labor force on the outout growth rate compatible with the equilibrium in the balance of payments. According to equation (18c) the impact is positive. This is due to an increase in the rate of growth of the labor force which, ceteris paribus, generates a reduction on wage inflation, thus leading to a reduction on the domestic inflation rate. Reducing the pace of the domestic price inflation results in a depreciation of the real exchange rate, this increases the pace of export growth and slows down the growth of imports, thus increasing the rate of output growth that is compatible with the balance of payments equilibrium.

In equation (18d) we find that an increase in the growth rate of the output in the previous period generates an increase of the growth rate of the outout in current period. This result is a simple consequence of the existence of static and dynamic economies of scale. In fact, the increase in ouput in the previous period generates an increase in productivity in the current period, which results in a reduction of the domestic inflation rate and, ceteris paribus, in a depreciation of real exchange rate. In this context, there will be an increase in the rate of growth of exports and a reduction in the rate of growth of imports, thus leading to an increase in the output growth rate which is compatible with the balance of payments equilibrium.

Equation (18e) shows that an increase in international inflation is associated with an increase in the output growth rate that is compatible with the balance of payments equilibrium. The interpretation of this result is trivial.

Equation (18f) shows the most interesting result of the short-run equilibrium of the model. We note that raising the medium-term inflation target is associated with an increase in the output growth rate that is compatible with the balance of payments equilibrium. In this way, monetary policy is not neutral in the short period. This is due to the following: if the monetary authority raises the medium-term inflation target, given the domestic inflation rate - or, equivalently, considering a reduction of the interest rate, ceteris paribus – there will be an increase in the depreciation rate of the nominal exchange rate leading the real exchange rate to depreciate. Since Marshall-Lerner's condition is valid, it follows that there will be an increase in the rate of growth of exports and a reduction in the rate of growth of imports, making the output growth rate compatible with the balance of payments equilibrium to increase. Hence, changes in the level of domestic interest rate have impact on output growth, making monetary policy non-neutral in the short period.

Finally, equation (18g) shows the impact of changes in the share of manufacturing industry in the previous period on output growth; In this equation we can see that the greater the share of manufacturing industry in the economy, the greater will the economic growth and the lower will the rate of inflation, due to the productivity gains that this sector generates and spreads over the whole economy.

Substituting (18) in (17), we have:

$$\hat{p}_t = \left(\frac{\theta_1\varepsilon}{(1-[\beta_1-(1+\alpha_1)\beta_2]\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi(1-[\beta_1-(1+\alpha_1)\beta_2])}\right)\sigma - \frac{(\eta+2c-\hat{p}_{t-1})}{(1-[\beta_1-(1+\alpha_1)\beta_2])} - \frac{2\alpha}{(1-(\beta_1-(1+\alpha_1)\beta_2)}\lambda_{t-1}\hat{y}_{t-1} + \frac{\beta_2}{(1-[\beta_1-(1+\alpha_1)\beta_2])}\hat{p}_t^* + \frac{(\beta_2\alpha_1-\beta_1)}{(1-[\beta_1-(1+\alpha_1)\beta_2])}\hat{p}_t^T \quad (19)$$

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increase the ability to meet external debt commitments. In this way, debt service increases its ratio in proportion of domestic income and limits it to the extent that it diverts resources that could be channeled to other purposes.

Based on (19), we may conclude that:

$$\frac{\partial \hat{p}_t}{\partial \hat{p}_{t-1}} = \frac{1}{(1-[\beta_1-(1+\alpha_1)\beta_2])} > 0 (19a) \quad \frac{\partial \hat{p}_t}{\partial \hat{z}_t} = \left(\frac{\theta_1 \varepsilon}{(1-[\beta_1-(1+\alpha_1)\beta_2]\pi}\right) > 0 \ (19b)$$

$$\frac{\partial \hat{p}_t}{\partial \sigma} = -\left(\frac{\theta_2}{\pi(1-[\beta_1-(1+\alpha_1)\beta_2])}\right) < 0 \ (19c) \quad \frac{\partial \hat{p}_t}{\partial \eta} = -\frac{1}{(1-[\beta_1-(1+\alpha_1)\beta_2])} < 0 \ (19d)$$

$$\frac{\partial \hat{p}_t}{\partial \hat{y}_{t-1}} = -\lambda_{t-1} \frac{2\alpha}{(1-(\beta_1-(1+\alpha_1)\beta_2)} < 0 (19e) \quad \frac{\partial \hat{p}_t}{\partial \hat{p}_t^*} = \frac{\beta_2}{(1-[\beta_1-(1+\alpha_1)\beta_2])} > 0 \ (19f)$$

$$\frac{\partial \hat{p}_t}{\partial \hat{p}^T} = \frac{(\beta_2\alpha_1-\beta_1)}{(1-[\beta_1-(1+\alpha_1)\beta_2])} > 0 \ (19g) \frac{\partial \hat{p}_t}{\partial \lambda_{t-1}} = -\frac{2\alpha \hat{y}_{t-1}}{(1-(\beta_1-(1+\alpha_1)\beta_2)} < 0 (19h)$$

From the expressions (19a) - (19g) we can conclude that equilibrium short-run domestic inflation rate is a positive function of the inflation rate of the previous period, of the income growth rate of the rest of the world, of the inflation rate of the rest of the world and the medium-term inflation target; and an inverse function of the current account deficit as a proportion of GDP, labor force growth rate, domestic output growth rate, and manufacturing share of the previous period.

2.2 – Balanced Growth: Existence and Stability

Along to the balanced growth path, we have:

$$\hat{p}_t = \hat{p}_{t-1} = \hat{p} \quad (20) \ \hat{y}_t = \hat{y}_{t-1} = \hat{y} \quad (21) \ \lambda_t = \lambda_{t-1} = \lambda \quad (22)$$

Substituting (21) and (22) in (17), we have:

$$\hat{y} = \frac{\eta + 2c}{1 - 2\lambda\alpha} \quad (23)$$

Equation (23) shows the growth rate of the output along the balanced growth path, which is called the *natural growth rate*. For $\hat{y} > 0$ it is necessary and sufficient that $\alpha\lambda < 0.5$, since the numerator is positive and therefore the denominator will determine the signal of the equation. Based on this result, we can verify that the Kaldor-Verdoorn Coefficient is of fundamental importance to ensure the existence of a positive output growth rate.

Moreover, the existence of a balanced growth path requires a limited value for the Kaldor-Verdoorn Coefficient, that is, the extent of static and dynamic economies of scale cannot be very large, since otherwise they would cause instability and would not sustain a steady-state outcome. It is important to emphasize that this coefficient is the source of dynamic instability in the economy, since it reinforces possible deviations from the equilibrium path.

We also verified that the natural growth rate depends only on the parameters of the labor productivity growth function, the manufacturing share in the economy and the labor force growth rate, and is therefore independent of monetary policy. It follows that in this version of the Kaldorian growth model money is neutral in the long run.

Substituting (20) and (21) on (16) we have:

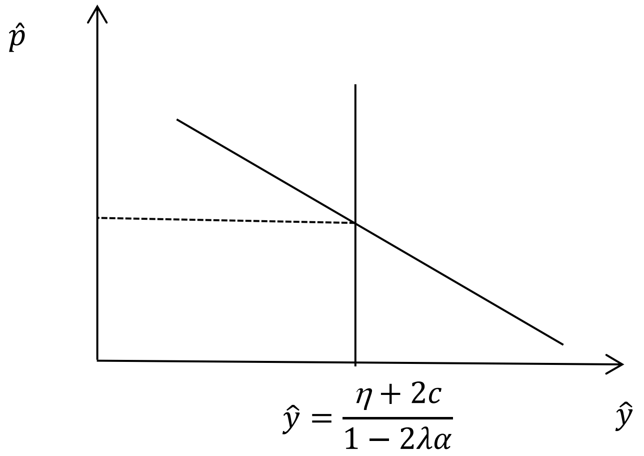
$$\hat{y} = \left(\frac{\theta_1 \varepsilon}{\pi}\right) \hat{z} - \left(\frac{\theta_2}{\pi}\right) \sigma + [\beta_1 - (1 + \alpha_1)\beta_2]\hat{p} + \beta_2 \hat{p}^* + (\beta_2\alpha_1 - \beta_1)\, \hat{p}^T (24)$$

Equation (24) presents the locus of the combinations between $\hat{y}$ and $\hat{p}$ for which the balance of payments is in equilibrium along the balanced growth path. Substituting (23) into (24), we obtain:

$$\hat{p} = \frac{\eta + 2c}{[\beta_1 - (1+\alpha_1)\beta_2](1-2\lambda\alpha)} - \frac{\theta_1}{[\beta_1 - (1+\alpha_1)\beta_2]}\frac{\varepsilon}{\pi} z + \frac{\theta_2}{[\beta_1 - (1+\alpha_1)\beta_2]\pi}\sigma - \frac{\beta_2}{[\beta_1 - (1+\alpha_1)\beta_2]}\hat{p}^* - \frac{(\beta_2\alpha_1 - \beta_1)}{[\beta_1 - (1+\alpha_1)\beta_2]}\hat{p}^T \quad (25)$$

The visualization of the determination of the long-term equilibrium values of $\hat{p}$ and $\hat{y}$ can be made by means of figure 2 below:

Figure 2: Long-term equilibrium without structural change.



Once the conditions of existence of the balanced growth trajectory have been defined, we must do the stability analysis.

The system formed by equations (16) and (17) has an intrinsic dynamic, which can be presented by the following system of finite difference equations:

$$\Delta\hat{y}_t = \hat{y}_t - \hat{y}_{t-1} = \left(\frac{\theta_1\varepsilon}{\pi}\right)\hat{z}_t - \hat{y}_{t-1} - \left(\frac{\theta_2}{\pi}\right)\sigma + [\beta_1 - (1+\alpha_1)\beta_2]\hat{p}_t + \beta_2\hat{p}_t^* + (\beta_2\alpha_1 - \beta_1)\,\hat{p}^T \quad (16a)$$

$$\Delta\hat{p}_t = \hat{p}_t - \hat{p}_{t-1} = \hat{y}_t - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1}) \quad (17a)$$

According to Shone (1997), in order to make this system stable, converging to equilibrium, two conditions are necessary, namely[11]:

i. $[\beta_1 - (1+\alpha_1)\beta_2](1-2\alpha\lambda)<0$
ii. $[\beta_1 - (1+\alpha_1)\beta_2](1+2\alpha\lambda)<2$

As it has already been defined in (16) that $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_1 < \alpha_1\beta_2$, consequently, it necessarily remains that $0.5> \alpha\lambda$ - which has also been defined in (23) - to satisfy the first constraint. Already to meet the second constraint it is sufficient that $\alpha\lambda> 0$; which is perfectly reasonable assumption. It follows that, once respecting all the restrictions imposed so far, the system is stable and therefore converges to the equilibrium positions defined at (23) and (25). Annex I details how this result is achieved.

[11] See Annex I

3 – Growth Model with Structural Change

3.1 –Dynamics of Structural Change

In the previous section, it was considered through equation (9) that the growth rate of productivity is determined by the output growth rate, given the Kaldor-Verdoorn coefficient and the share of manufatuing in domestic output.

In this section, we will take consider manufacturing share in output to be an endogenous variable and then analyze its effects over the system's long-term equilibrium structure.

The Recent literature related to the Structuralist Delopment Macroeconomics[12] insists on the central role of the real exchange rate as an explanatory variable for the growth or reduction of the share of manufacturing industry in domestic output, especially in developing economies. According to this literature, an overvalued exchange rate, that is, an exchange rate that is below to the level that makes the domestic industries operating with the state of the art techonology to be competitive in the international market, leads to a progressive reduction of the manufacturing share in output, since this situation induces a growing overseas transfer of productive activities (see Bresser-Pereira, Oreiro and Marconi, 2014, 2015). This level of the exchange rate is called the industrial equilibrium. Thus, a situation of exchange rate overvaluation is associated with a negative structural change on the economy, which may be called premature de-industrialization (Palma, 2005). An undervalued exchange rate, that is, above the level of industrial equilibrium, would have the opposite effect, the one of inducing a transfer of productive activities into domestic economy, thus increasing the manufacturing share in domestic output.

The equation that defines the rate of change of manufacturing share in the GDP can be written as:

$$\hat{\lambda}_t = \cap \left[\psi_t - \psi^i\right] \quad (26)$$

In which $\hat{\lambda}_t$ is the change in industry share in the product, ψ^i is the "industrial equilibrium" real exchange rate of the economy indicated by the superscript i, ψ_t Is the real exchange rate of the previous period, $\cap$ is a parameter that reflects the sensitivity of the impact of the real exchange differential in relation to its "industrial equilibrium" on the variation of the industry share.

On the other hand, we know that the real exchange rate variation over time can be written as:

$$\widehat{\psi}_t = \hat{e}_t + \hat{p}_t^* - \hat{p}_t (27)$$

By inserting equations (6b) and (25) into (27), we will have:

$$\widehat{\psi}_t = -\alpha_1[\hat{p}_t(\alpha, \lambda, \eta, \varepsilon, \pi, \sigma, \hat{p}_t^*, \hat{p}^T) - \hat{p}^T] + [\hat{p}_t^* - \hat{p}_t(\alpha, \lambda, \eta, \varepsilon, \pi, \sigma, \hat{p}_t^*, \hat{p}^T)] (27a)$$

Equation (27a) defines the variation of the real exchange rate as a function of the gap between domestic inflation and the inflation target and the gap between international inflation and domestic inflation.

[12] See Bresser-Pereira, Oreiro e Marconi (2014, 2015)

3.2 Long-Term Equilibrium and Stability Analysis

Equations (26) and (27a) describe the dynamics of the economy with structural change, that is, in the case where the share of industry in the product varies over time, depending on the relation between the current value of the exchange rate and the Industrial equilibrium level.

The long-run equilibrium of the system corresponds to a situation in which both the real exchange rate and the share of the industry in the product are kept constant over time.

Under these conditions, we must:

$\hat{\lambda}_{\mathrm{t}} = 0$ (28a) ; $\widehat{\psi}_t = 0\ (28b)$

Substituting (28a) in (26), we have:

$\psi = \psi^i \quad (29)$

That is, in the long-run equilibrium position of the system, the real exchange rate is constant and equal to the level corresponding to the industrial equilibrium.

Substituting (28b) into (27a), we have:

$$\alpha_1[\hat{p}_t(\alpha, \lambda, \eta, \varepsilon, \pi, \sigma, \hat{p}_t^*, \hat{p}^T) - \hat{p}^T] = \hat{p}_t^* - \hat{p}_t(\alpha, \lambda, \eta, \varepsilon, \pi, \sigma, \hat{p}_t^*, \hat{p}^T) (30)$$

Reorganizing equation (30), we will find the domestic inflation rate that is compatible with the maintenance of the real exchange rate at a constant level over time.

$$\hat{p}_t = \frac{1}{(1+\alpha_1)}\,\hat{p}_t^* + \frac{\alpha_1}{(1+\alpha_1)}\hat{p}^T (30a)$$

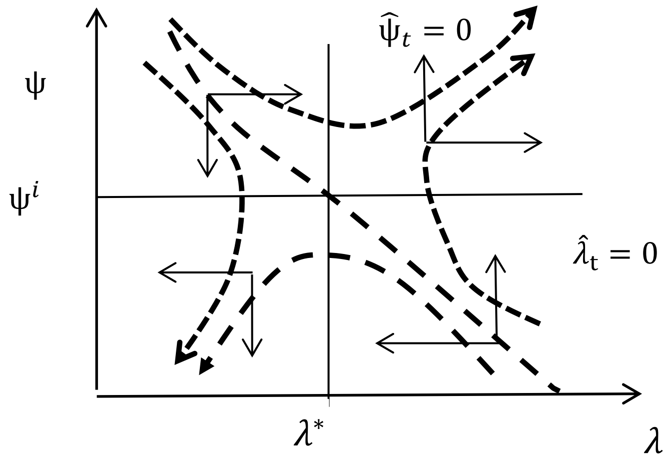
In equation (25) presented in the previous section we obtained the value of the domestic inflation rate for which the economy is in its balanced growth path, given the industry's share of the product. In order to not have an over-determined model, that is, with more equations than unknowns variables, it is necessary to add some other endogenous variable to the system. It is clear that the variable that has to be endogenous is the share of the industry in the product, which will be determined by the making the equations (25) and (30a) equals. In this way, we have:

$$\lambda^* = \left(\frac{1}{2\alpha}\right)\left(\frac{\eta+2c}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\left\{\left(\frac{\theta_2}{[\beta_1-(1+\alpha_1)\beta_2]\pi}\right)\sigma - \left[\left(\frac{\theta_1}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\left(\frac{\varepsilon}{\pi}\right)z^* + \left(\frac{1}{(1+\alpha_1)} + \frac{\beta_2}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\hat{p}^* + \left(\frac{\alpha_1}{(1+\alpha_1)} + \frac{(\beta_2\alpha_1-\beta_1)}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\hat{p}^T\right]\right\}^{-1} \qquad (31)$$

Equation (31) presents the long-run equilibrium value for the industry's participation in the product. The share of industry in the product is adjusted so that the inflation rate for which the real exchange rate is constant over time is equal to inflation rate that makes the balance of payments restriction compatible with aggregate supply.

The long-term equilibrium outlook of the economy under consideration can be made by figure 3 below:

Figure 3 – Long Term Equilibrium with Structural Change.



To analyze the stability of the model, we will linearize the model around its position of long-term equilibrium, using the first term of Taylor's expansion (Sargent, 1987, pp. 29-30)[13]. In this way, we have to:

$$\begin{bmatrix} \hat{\lambda}_t \\ \hat{\psi}_t \end{bmatrix} = \begin{bmatrix} 0 & \cap \\ -(1+\alpha_1)\dfrac{\partial \hat{p}_t}{\partial \lambda_t} & 0 \end{bmatrix} \begin{bmatrix} \lambda_t - \lambda^* \\ \psi_t - \psi^i \end{bmatrix} \qquad (32)$$

The Jacobian matrix $\begin{bmatrix} 0 & \cap \\ -(1+\alpha_1)\frac{\partial \hat{p}_t}{\partial \lambda_t} & 0 \end{bmatrix}$ has trace equal to zero and determinant equal to $\cap\,(1+\alpha_1)\frac{\partial \hat{p}_t}{\partial \lambda_t} < 0$. It follows that the dynamics of the system around the long-term equilibrium position is characterized by a saddle path (Takayama, 1993, p.408). This means that there is only one convergent path, all the others are divergent. In this context, we will adopt the Sargent and Wallace (1973) methodology of considering only the convergent trajectory as the only one possible for the economy under consideration.To do so, we will assume that the level of the real exchange rate adjusts instantly to put the economy exactly on the convergent path. This hypothesis will have strong implications for the effects of variations in the inflation target over the real exchange rate path, as we will see in the next section.

3.3 Non Neutrality of Monetary Policy

In the Kaldorian model with no structural change presented in section 2, we saw that the monetary policy was neutral in the long run, since changes in the inflation target or in the inflation aversion coefficient of the monetary policy rule had no impact on the rate GDP growth along the balanced growth path.

We will now assess whether the result of monetary policy neutrality remains valid in a model with structural change, that is, if in a context where the share of industry in the product is an endogenous variable that adjusts to the gap between the current value of the

[13] This procedure, in effect, has the effect of transforming the system of equations of finite differences into a system of differential equations.

real rate of Exchange rate and its industrial equilibrium value, it remains true that changes in the inflation target do not affect the real variables of the economy.

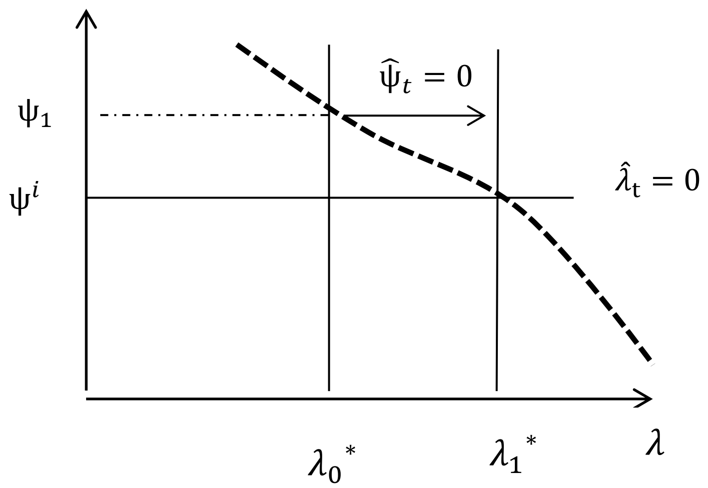
We know from equation (6b) that raising the inflation target is associated with a depreciation of the exchange rate. This is due to the fact that raising the inflation target allows the monetary authority to reduce the domestic interest rate, a reduction that generates an outflow of capital from the country and, consequently, a depreciation of the nominal exchange rate. The devaluation of the nominal exchange rate, given the domestic rate of inflation, should result in a devaluation of the real exchange rate, which, in turn, will induce an increase in the share of the industry in the economy. This increase in the industry share, in turn, will increase the Kaldor-Verdoorn coefficient in equation (23), which will result in an increase in the growth rate of the product along the balanced growth path.

The validity of this reasoning can be attested by the differentiation of (31a) with respect to λ and $\hat{p}^T$. So we have:

$$\frac{\partial\lambda}{\partial\hat{p}^T} = \left(\frac{1}{2\alpha}\right)\left(\frac{\eta+2c}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\left\{\left(\frac{\theta_2}{[\beta_1-(1+\alpha_1)\beta_2]\pi}\right)\sigma - \left[\left(\frac{\theta_1}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\left(\frac{\varepsilon}{\pi}\right)z^* + \left(\frac{1}{(1+\alpha_1)} + \frac{\beta_2}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\hat{p}^* + \left(\frac{\alpha_1}{(1+\alpha_1)} + \frac{(\beta_2\alpha_1-\beta_1)}{[\beta_1-(1+\alpha_1)\beta_2]}\right)\hat{p}^T\right]\right\}^{-2}\left[\frac{\alpha_1}{(1+\alpha_1)} + \frac{(\beta_2\alpha_1-\beta_1)}{[\beta_1-(1+\alpha_1)\beta_2]}\right] > 0 \quad (33)$$

The effect of raising the inflation target on the share of industry can be visualized by means of figure 4 below

Figure 4 – Effects of an Increase of Inflation Target over the Long Term Equilibrium with Structural Change



The economy is initially in the long-run equilibrium with a real exchange rate equal to the one that is of industrial equilibrium and an industry share in the product equal to $\lambda_0{}^*$. When the Central Bank raises the inflation target, the locus of $\widehat{\psi}_t = 0$ moves to the right, defining a new long-run equilibrium point, in which the industry share is higher than initial one. Since equilibrium is unstable of the saddle path type, convergence to it requires that the real exchange rate devalue to ψ_1, exactly at the same time as the Central Bank raises the inflation target. Thus, the announcement of the raising of inflation target will be followed by a sharp and sudden devaluation of the real exchange rate, which will be above to the industrial equilibrium level. In this context, the share of industry in the product will gradually increase until it reaches its new long-term equilibrium point, ${\lambda_1}^*$. Throughout the adjustment path towards the new equilibrium point, the real exchange rate will be appreciated, albeit it remains above to level of equilibrium point. Therefore, raising the inflation target results in (i) a permanent increase in the share of industry in the GDP – and consequently an increase in long-term growth rate – and a (ii) temporary devaluation of the real exchange rate.

4 – Conclusions

Throughout this paper we presented a kaldorian model that incorporates a balance of payments constraint similar to the one developed by Moreno-Brid (2003), as well as incorporating into the dynamic equation of productivity growth the idea that the Kaldor-Verdoorn coefficient depends on the industry share of the product. These innovations represent a step forward not only to eliminate the inconsistency present in growth models with balance of payments constraint, which are unable to reconcile the balance of payments constraint with the supply side of the economy; as well as in the sense of permitting the occurrence of endogenous structural change associated with the misalignment of the real exchange rate, defined as the difference between the current level of the real exchange rate and the value corresponding to the "industrial equilibrium". Thus, the model presented here allows integration between Kaldorian growth models led by aggregate demand and the Structuralist Macroeconomics of Development.

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Annex I – Stability Analysis of the Dynamic Model in the Case with No Structural Change

Equations (16) and (17) make up the system of the model of Section 2.2:

$$\hat{y}_t = \left(\frac{\theta_1 \varepsilon}{\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi}\right)\sigma + [\beta_1 - (1+\alpha_1)\beta_2]\hat{p}_t + \beta_2\hat{p}_t^* + (\beta_2\alpha_1 - \beta_1)\,\hat{p}^T \quad (16)$$

$$\hat{p}_t = \hat{p}_{t-1} + \hat{y}_t - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1}) \qquad (17)$$

By substituting (17) in (16), we will have:

$$\hat{y}_t = \left\{\left(\frac{\theta_1 \varepsilon}{\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi}\right)\sigma + [\beta_1 - (1+\alpha_1)\beta_2][\hat{p}_{t-1} - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1})] + \beta_2\hat{p}_t^* + (\beta_2\alpha_1 - \beta_1)\,\hat{p}^T\right\}\{1 - [\beta_1 - (1+\alpha_1)\beta_2]\}^{-1} (16')$$

Replacing (16') in (17) we will have:

$$\hat{p}_t = \hat{p}_{t-1} + \left\{\left(\frac{\theta_1 \varepsilon}{\pi}\right)\hat{z}_t - \left(\frac{\theta_2}{\pi}\right)\sigma + [\beta_1 - (1+\alpha_1)\beta_2][\hat{p}_{t-1} - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1})] + \beta_2\hat{p}_t^* + (\beta_2\alpha_1 - \beta_1)\,\hat{p}^T\right\}\{1 - [\beta_1 - (1+\alpha_1)\beta_2]\}^{-1} - \eta - 2(c + \alpha\lambda_{t-1}\hat{y}_{t-1}) \ \ (17')$$

Equations (16') and (17') are under the following arrangement: $u_t = Au_{t-1} + b$ which is a nonhomogeneous equation. It can be transformed into a homogeneous one by performing the following operation $(u_t - u^*) = A(u_{t-1} - u^*)$ in which $u^* = Au^* + b$ is the equilibrium vector. Thus, we have a homogenous equation $z_t = Az_{t-1}$.

So, the matrix form of the system (16') and (17') has the following arrangement[14]:

$$\begin{bmatrix}\hat{y}_t \\ \hat{p}_t\end{bmatrix} = \begin{bmatrix} \frac{-2\alpha\lambda £}{1-£} & \frac{£}{1-£} \\ -2\alpha\lambda\{\frac{£}{1-£} + 1\} & \frac{£}{1-£} + 1 \end{bmatrix}\begin{bmatrix}\hat{y}_{t-1} \\ \hat{p}_{t-1}\end{bmatrix}$$

To have a stable system, let R1 and R2 be the eigenvalues of the matrix above, it is necessary that | R1 | <1 and | R2 | <1. Hence, we will necessarily have that if R1 = TrA, then R2 = 0; otherwise R1 = 0, then R2 = TrA, since DetA = 0. Thus, aiming to have a stable system, it is necessary that | TrA | <1.

$$-1 < \frac{-2\alpha\lambda £}{1 - £}\left[\frac{£}{1 - £} + 1\right] < 1$$

Rearranging, we will have:

i. $[\beta_1 - (1+\alpha_1)\beta_2](1 - 2\alpha\lambda) < 0$

ii. $[\beta_1 - (1+\alpha_1)\beta_2](1 + 2\alpha\lambda) < 2$

In accordance with the conditions imposed at the end of section 2.3, *quod erat demonstrandum*.

[14] We are going to call $£ = [\beta_1 - (1+\alpha_1)\beta_2]$: