Cash flow, investment, and Keynes–Minsky cycles

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Abstract

This paper analyzes a dynamic model with (1) an investment function that emphasizes cash flow, (2) a Keynesian macroeconomic framework that determines cash flow endogenously, (3) a dynamic labor market model that drives wage and price adjustments, and (4) boundedly rational expectations. Simulations from the calibrated model generate endogenous cycles with characteristics described in Hyman Minsky’s research. The cycle arises from the link between investment, interest rates, debt service, and cash flow. The amplitude and frequency of the cycle are related to the importance of cash flow for investment, the dynamics of inflation, and the distribution of income.

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1. Introduction

The defining feature of Keynesian macroeconomics is that changes in aggregate demand cause fluctuations in output and employment. These fluctuations are often thought to take the form of business cycles; that is, the path of output through time displays well-defined stages of recession and growth separated by turning points. Much research in the Keynesian tradition, however, pays little attention to what generates a cyclical process, focusing instead on the narrower issue of how aggregate demand shocks cause macro fluctuations. An exception is the research of Hyman

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Minsky, whose macroeconomic theory is inherently cyclical. The Minsky cycle arises from the interaction between finance and investment, and the resulting cycle in investment induces a cycle in aggregate demand and output.1

This paper presents a model in which cycles arise from the financing of investment. We pay particular attention to the need for consistency between the model and empirical evidence. The investment specification and parameter values chosen in this paper follow from extensive recent empirical research on finance and investment. In this respect, this paper addresses the criticism of Crotty (1996) that much recent microeconomic empirical work on the effect of financial variables on investment does not explore the macroeconomic implications of the findings. This paper demonstrates the systemic effects of an empirically based specification that ties investment and finance together along the lines proposed by Minsky.

In Minsky’s theory, investment is financed through internally generated cash flows and external debt. Minsky emphasizes how the accumulation of debt varies systematically under different business conditions and how the impact of debt on investment causes endogenous business cycles. Our Keynesian growth model determines cash flow endogenously from macroeconomic conditions. Aggregate demand determines sales and a markup model determines cash flow, as in Fazzari et al. (1998).2 If cash flow is insufficient to finance investment, firms take on debt.

This model formalizes a part of Minsky’s theory, in contrast to the almost exclusively descriptive approach Minsky adopted. We appreciate the descriptive accounts and recognize that important aspects of these accounts may be lost in a mathematical model.3 A formal model, however, can illuminate the dynamic implications of interactions between variables more rigorously than is possible in purely descriptive models. For example, Minsky asserts that boom conditions lead to an increase in the ratio of debt to income. But because the boom causes both the numerator and the denominator of this ratio to rise, it is not obvious from a descriptive account alone whether the debt–income ratio rises or falls. A formal model is a natural vehicle for addressing this kind of issue.

Because the model is nonlinear we explore its dynamic properties with simulations. We also provide some analytic results from a linearized version of the model. We demonstrate that an investment function calibrated to recent empirical results and embedded in a Keynesian macroeconomic model generates well-defined cyclical output fluctuations. The amplitude and frequency of the cycles depend critically on how nominal interest rates respond to stages of the business cycle. In boom times, nominal interest rates rise. Higher nominal interest rates increase firms’ debt service, reduce internal cash flow, and, other things equal, lower investment. This financial process eventually brings the boom to an end. Symmetrically, lower nominal interest rates in the downturn reduce debt service, restore internal cash flow, and establish the conditions necessary for a recovery in investment and growth. The greater the response of interest rates to macroeconomic conditions, the faster this process takes place, creating more volatile business cycles. This dynamic process identifies a fundamental nonneutrality of money and monetary policy operating through the financing of investment. If investment depends on cash flow as emphasized in numerous recent empirical studies, nominal interest rates drive real investment.

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2 Fazzari et al. (2001) set the micro analysis in a macro perspective. The connection between aggregate economic conditions and profits has been recognized since the work of Michal Kalecki, which was explicitly adopted in the later writing of Minsky.

3 Also see Taylor and O’Connell (1985, p. 871), who write that much of the microeconomic and institutional detail in Minsky’s work “...is rich and illuminating but beyond the reach of mere algebra.”
The structure of the paper is the following. Section 2 reviews previous literature that formalizes aspects of Minsky’s financial theory of investment. Section 3 presents the model and Section 4 describes the calibration of the model along with the steady state values of the key variables. Section 5 discusses the simulation results prior to the conclusion in Section 6.

2. The model in perspective

Our model differs from both of the major, mainstream macroeconomic paradigms—new classical equilibrium models and new Keynesian models. Endogenous aggregate cycles here are driven by demand side rather than the supply-side factors that are emphasized in real business cycle models. Furthermore, macroeconomic fluctuations are endogenous and display a true cyclical pattern with identifiable dynamic processes that cause turning points. The cycles do not rely on stochastic shocks. New classical macroeconomic models, in which fluctuations arise from stochastic shocks and propagation mechanisms, usually do not demonstrate such endogenous cycles. Our model also differs from the new Keynesian approach, which, although sharing with new classical theory an emphasis on microfoundations, differs in its welfare and policy implications; see Delli Gatti et al. (1993) and Delli Gatti and Gallegati (2001). In new Keynesian models, output deviates from its “natural rate” as the result of nominal rigidity, and faster adjustments of wages and prices stabilize the system (see Woodford, 2003). In our model, in contrast, faster adjustment of nominal wages and prices can increase the cyclical volatility of the economy. Another strand of research, often considered part of the new Keynesian literature, consists of the “financial accelerator” models. While the foundation of these models are varied (see the overview by Bernanke and Gertler, 1995) most of the detailed formal models of this type operate through the supply side of the economy, as in Bernanke and Gertler (1989) and Bernanke et al. (1999). In our model the investment–finance link has Keynesian effects on the demand side.

Previous papers have proposed a variety of formal models that capture aspects of Minsky’s macroeconomic theory. The remainder of this section briefly surveys the results of related papers and contrasts them with the approach presented here. We focus on broad issues in this section; detailed structural differences between our model and those found in related literature are discussed in subsequent sections as the model is described.

The model in Taylor and O’Connell links investment to finance through asset prices. This contribution develops a theme central to Minsky’s analysis, but it differs from our objectives here. Rather than asset prices, we focus on the effect of cash flow (internal finance) on investment, also an important element of Minsky’s theory. This approach complements the work of Taylor and O’Connell, and it allows us to calibrate our model to recent empirical evidence because the effect of cash flow on investment has been studied widely in the empirical literature cited below. Furthermore, our simulation analysis explores short-term and medium-term cyclical dynamics rather than asymptotic instability, as in Taylor and O’Connell.

In our model, the cash flow to income ratio is mainly governed by debt service, income distribution remaining constant. This differs from models that, starting from the supply-driven Goodwin model, have studied the evolution of debt in its interaction with income distribution changes. Keen (1995, 1999), for instance, presents models in which endogenous cycles in debt and income shares can lead to an explosion in debt relative to output. Asada (1989) also begins with the Goodwin model, but adds Keynesian aggregate demand effects that are also crucial in our approach.

Jarsulic (1989) and Asada emphasize money and credit markets as the key connection between investment and the financial system. This research captures aspects of Minsky’s theory and com-
emulates our focus on the investment–cash flow link. As mentioned above, Minsky’s theory is rich and deep. Thus, it is not surprising that formal models explore different parts of his theory.

Our approach is more in line with the work of Andresen (1996), who applies a systems dynamics approach and simulations to demonstrate the possibility of debt deflation. Our model also has features in common with Chiarella et al. (2001), who study the endogenous fluctuations of an economy with debt. This paper embodies a wide range of structural features beyond the scope of this study (inventory dynamics and international trade, among other things), and the basic investment structure is different from the model presented in the next section. Chiarella et al. do not include a direct effect of cash flow on investment, and their specification is not calibrated to empirical research on financial constraints and investment. Like the work of Taylor and O’Connell, Chiarella et al. focus their dynamic analysis primarily on asymptotic stability while we emphasize cyclical dynamics at business-cycle frequencies.

Perhaps the paper most closely related to our work is Delli Gatti et al. (1993). The investment function in that paper incorporates cash flow and accelerator effects much like ours (see Eq. (1), p. 165). The model generates cyclical, even chaotic, dynamic paths for the key variables. The dynamic analysis in Delli Gatti et al. (1993), however, has a different emphasis from what we present below. In particular, the authors study the stability of general cyclical patterns predicted by the model, while we focus on the detailed evolution of individual cycles. Furthermore, similar to literature comparisons summarized above, our work puts more emphasis on calibrating the model to recent empirical evidence.

3. The macroeconomic model

We now specify the simulation model. Variables are determined sequentially as the simulation solves for period \( t \) values based on period \( t - 1 \) information. Greek letters represent parameters in all our equations. Variables with “hats” (such as \( \hat{X}_t \)) denote expected values of variables in simulation period \( t \) based on period \( t - 1 \) information. We discuss the dynamic formation of expectations below.

3.1. Investment and finance

The core behavioral relationship in the model is the function that links investment to the expected growth in output and the expected flow of internal funds. A relationship between investment and the change in output follows from the accelerator model, one of the most empirically successful investment models.\(^4\) The accelerator relates the real level of investment (\( I_t \)) to the change in real output (\( Y_t \))

\[
I_t = \eta_0 Y_{t-1} + \eta_1 (\hat{Y}_t - Y_{t-1}) = \eta_0 Y_{t-1} + \eta_1 \hat{g}_t Y_{t-1},
\]

where \( \hat{g}_t \) is the expected growth rate of output between period \( t - 1 \) and \( t \). The \( \eta_0 Y_{t-1} \) term can be interpreted as replacement investment, assuming geometric depreciation and a constant capital–output ratio.\(^5\)

\(^4\) See the survey by Chirinko (1993). The accelerator principle dates back to a classic paper by Aftalion (1913); we thank the editor for this reference. Samuelson (1939) introduced the accelerator into a formal dynamic macroeconomic model.

\(^5\) With a depreciation rate of \( \delta \) and a capital–output ratio of \( \varphi \), \( \eta_0 = \delta \varphi \).
In addition to the accelerator, investment depends on the availability of internal cash flow as in Minsky (1975, pp. 134–135). Greater cash flow raises the amount of investment that firms can undertake without incurring the risks and costs associated with debt or new share issues. As Jarsulic (p. 39) writes “... the greater the flow of current profits, the more easily a firm can pay for already existing investments or begin new ones without seeking financing.” In addition, much recent literature, as surveyed by Hubbard (1998), argues that internal finance helps firms overcome asymmetric information problems that limit firms’ access to external finance. Following Fazzari et al. (1988) and many other empirical studies of investment, we add a cash flow term to the investment function

\[ I_t = \eta_0 Y_{t-1} + \eta_1 \hat{g}_t Y_{t-1} + \eta_2 \left( \frac{1}{p_t} \right) \hat{C}F_t. \]

Expected nominal cash flow (\( \hat{C}F_t \)) is deflated by the price level \( p_t \) to correspond to real investment. Prices in period \( t \) are pre-determined (see Section 2.4 below) and therefore known when firms choose period \( t \) investment. Cash flow, however, depends on period \( t \) output, which depends in turn on period \( t \) investment. Nominal cash flow therefore appears as an expectation in the investment function.

Expected nominal cash flow is expected nominal revenue less the expected wage bill and interest costs

\[ \hat{C}F_t = p_t \hat{Y}_t - \hat{W}_t - R_t D_t, \]

where \( \hat{W}_t \) represents the expected nominal wage bill, \( R_t \) is the pre-determined nominal interest rate, and \( D_t \) is the pre-determined nominal stock of debt outstanding at the beginning of period \( t \). To assure a nonnegative nominal interest rate, \( R_t \) is determined by

\[ R_t = \max[(1 + r)(1 + \hat{\pi}_t) - 1, 0], \]

where \( \hat{\pi}_t \) is the expected inflation rate and \( r \) is the constant real interest rate. We assume a constant wage share in expected nominal aggregate income denoted by \( \omega \). This assumption is consistent with a fixed markup model of pricing that arises from monopolistic competition as in Fazzari et al. (1998). With this assumption, the expected wage bill is \( \omega p_t \hat{Y}_t \), and cash flow can be written as

\[ \hat{C}F_t = (1 - \omega) p_t \hat{Y}_t - R_t D_t. \]

After substitution, the investment function becomes

\[ I_t = \eta_0 Y_{t-1} + \eta_1 \hat{g}_t Y_{t-1} + \eta_2 \left( \frac{1}{p_t} \right) [(1 - \omega) p_t \hat{Y}_t - R_t D_t] = \eta_0 Y_{t-1} + \eta_1 \hat{g}_t Y_{t-1} + \eta_2 (1 - \omega) \hat{Y}_t - \eta_2 R_t \left( \frac{D_t}{p_t} \right). \]

This investment equation demonstrates a real-nominal linkage of fundamental importance for the macroeconomic analysis that follows: real investment is a function of nominal interest rates. This

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6 Delli Gatti et al. (1993) use a similar specification although they make the effect of internal finance on investment a procyclical function rather than a constant.

7 Empirical evidence of the importance of cash flow for investment is extensive. See the survey by Hubbard.

8 Jarsulic (p. 39) makes the same assumption and argues that “... it is an acceptable simplification in a model that seeks to isolate the contribution of financial factors to the generation of cycles.” Below, we consider the effect on our simulations of changing the distributional parameter.
“nonneutrality” arises from the appearance of nominal interest expenses in the definition of cash flow. It appears in any formulation that links investment to the flow of internal financing, as in much of the recent empirical work on investment.\(^9\)

To analyze the growth path of the model, it is convenient to divide investment by \(Y_{t-1}\) to obtain the intensive form

\[
i_t = \frac{I_t}{Y_{t-1}} = \eta_0 + \eta_1 \hat{g}_t + \eta_2 \left( \frac{(1 - \omega)p_t \hat{Y}_t - R_tD_t}{p_t Y_{t-1}} \right)
\]

\[
= \eta_0 + \eta_1 \hat{g}_t + \eta_2 (1 - \omega)(1 + \hat{g}_t) - \eta_2 \frac{R_tD_t}{p_t Y_{t-1}}.
\]

Define the ratio of beginning-of-period nominal debt to lagged nominal income as \(d_t = D_t/(p_{t-1}Y_{t-1})\) and re-write the intensive-form investment equation as

\[
i_t = \eta_0 + \eta_1 \hat{g}_t + \eta_2 (1 - \omega)(1 + \hat{g}_t) - \eta_2 \frac{R_tD_t}{(1 + \pi_t)}.
\]  

Eq. (1) captures several key features of Minsky’s investment theory. First, it incorporates accelerator effects through the growth term. Second, income distribution affects investment through the impact of the wage share on cash flow. Third, Minsky argued that past debt incurred to finance investment constrains current spending because of contractual debt service. Minsky described this effect as the way in which the “financial trails” of past investment, that is, the stock of accumulated debt from past financing activities, affect current investment. The debt–investment link thus arises naturally in a specification that is widely employed in empirical studies of investment undertaken from a variety of theoretical perspectives.

3.2. Debt dynamics

To embed Eq. (1) in a dynamic model, we specify the dynamic accounting relationship for debt accumulation

\[
D_t = D_{t-1} + W_{t-1} + p_{t-1}I_{t-1} + R_{t-1}D_{t-1} - p_{t-1}Y_{t-1}.
\]

Debt at the beginning of period \(t\) equals debt at the beginning of period \(t - 1\) plus cash expenses in \(t - 1\) (including interest) less cash revenue in \(t - 1\).\(^{10}\) This specification embodies two assumptions. First, new borrowing takes place at the end of the period so that interest does not accrue on new loans. This assumption can be modified with only negligible effects on the results. Second, and more important, is the assumption that debt rolls over each period with a new interest rate (as in Chiarella et al.). We discuss the impact of this assumption on the results below. Dividing the

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\(^9\) Because we assume a constant real interest rate, the conventional link between investment and the real interest rate could be incorporated into the constant term of the investment equation (\(\eta_0\)) without a change of specification.

\(^{10}\) In Chiarella et al. and Keen (1999) new debt finances the difference between net investment and profits. By adding depreciation to both net investment and profits, one obtains our specification that new debt finances the difference between gross investment and cash flow.
debt accumulation equation by lagged nominal income and substituting yields the intensive-form debt equation

\[
d_t = \left[ \frac{1 + R_{t-1}}{(1 + g_{t-1})(1 + \pi_{t-1})} \right] d_{t-1} + \frac{i_{t-1}}{(1 + g_{t-1})} - (1 - \omega). \tag{2}
\]

Once again, the distributional wage share variable \((\omega)\) enters the model. In (2) a lower wage share implies a higher cash flow for a given level of output and investment, which implies that a lower value of \(\omega\), other things equal, reduces the accumulation of debt.

### 3.3. Consumption, aggregate demand and output

In this Keynesian model, output is determined by aggregate demand. We specify consumption with a combination of forward-looking and “rule-of-thumb” behavior

\[
C_t = \lambda_1(1 + \hat{g}_t)Y_{t-1} + \lambda_2Y_{t-1}.
\]

Some consumers forecast period \(t\) income with an expected growth rate. The coefficient \(\lambda_1\) combines the share of forward-looking consumers with their marginal propensity to consume. Other consumers base their period \(t\) consumption on period \(t - 1\) income as reflected by the second term in the consumption function. The share of these consumers times their marginal propensity to consume is \(\lambda_2\).\(^{11}\)

Output (aggregate supply) is determined by demand, \(Y_t = I_t + C_t\). Substituting for consumption and dividing by lagged output gives aggregate supply in intensive form and determines the actual growth rate of the economy \((g_t)\)

\[
1 + g_t = \frac{Y_t}{Y_{t-1}} = i_t + \lambda_1(1 + \hat{g}_t) + \lambda_2. \tag{3}
\]

### 3.4. The labor market, wages, and prices

Nominal variables in our model are driven by a Phillips curve and productivity growth. Wage inflation \((\pi^w_t)\) results from multiplying labor productivity growth \((\tau)\) by a term that depends on labor market conditions

\[
1 + \pi^w_t = (1 + \tau)[1 + \hat{\pi}_t - \sigma_1(u_{t-1} - u^*) - \sigma_2(u_{t-1} - u_{t-2})]. \tag{4}
\]

The labor market term in brackets includes expected price inflation \((\hat{\pi}_t)\), so there is no money illusion in the model. Wage inflation varies with the lagged unemployment gap, \(u_{t-1} - u^*\), where the parameter \(\sigma_1\) is the slope of the conventional Phillips curve and \(u^*\) is the unemployment rate at which wage inflation is unchanged.\(^{12}\) The final term captures “hysteresis,” which is the effect on wage inflation of the change in the unemployment rate, with an impact measured by the parameter \(\sigma_2\). Theoretical and empirical work support the importance of this effect in wage inflation equations for developed countries.\(^{13}\) Let \(l_t\) be the ratio of employment to the (constant)

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\(^{11}\) For a discussion of this consumption function as a rule-of-thumb approximation to an optimal choice in an intertemporal environment, see Allen and Carroll (2001).

\(^{12}\) Keen (1995, p. 614) links the Phillips curve to the Goodwin trade cycle model.

\(^{13}\) See Layard et al. (1991) and McMorrow (1996).
labor force. Assuming linear production, the evolution of $l_t$ and $u_t$ will depend on output growth and productivity growth as follows:

\[ l_t = l_{t-1} \left( \frac{1 + g_t}{1 + \tau} \right), \]  

\[ u_t = 1 - l_t. \]  

Nominal wages determine prices via a constant markup, and therefore actual price inflation ($\pi_t$) equals wage inflation adjusted for the rate of productivity growth ($\tau$)

\[ \pi_t = \frac{1 + \pi^w_t}{1 + \tau} - 1. \]  

A constant markup is widely used in post Keynesian models and is consistent with models of monopolistic competition in which firms face demand with constant price elasticity, as discussed in Fazzari et al. (1998).

### 3.5. Expectations, forecasts, and bounded rationality

To close the model we must specify how expectations of growth and inflation are formed. We take a boundedly rational, rather than a rational expectations, approach to forecasting. The possibility that the rational expectation hypothesis is too demanding has been noted not only by authors opposing new classical macroeconomics, but increasingly from its supporters (see Sargent, 1993, 1999). A concept weaker than rational expectations is an expectations function that requires consistency between the forecasts of the agents and the outcomes of the economic model, even though the true model of the economy is not known.

Two aspects of this formulation should be stressed. First, in our “bounded rationality” perspective, agents do not know the model but forecast according to an empirical expectations function. This approach differs from “rational learning” models, in which agents start out with knowledge of the underlying model but are incompletely informed about parameter values. Second, expectations must be consistent in the sense that they reproduce the behavior of the macro variables of the system. As explained by Grandmont (1998, p. 776),

\ldots global nonlinearities, originating in the agents’ expectations formation process themselves, may keep the motion bounded, and lead to convergence to complex nonlinear “learning equilibria,” along which forecasting errors would never vanish . . . .

Such complex “learning equilibria” may be at first sight good candidates to explain why agents keep making significant and recurrent mistakes when trying to predict the fate of the socioeconomic systems in which they participate. To be acceptable, however, the observed pattern along such “learning equilibria” should display some reasonable degree of consistency with the agents’ beliefs.

In our model consistency is treated as a statistical concept. It is interpreted as a high correlation between expected output and actual output. Our initial simulations employ the simplest possible static expectations rule, $\hat{x}_t = x_{t-1}$. Despite its simple form this rule tracks the evolution of both growth and inflation in our model remarkably well. We also explore the implications of an autoregressive forecasting rule with learning

\[ \hat{x}_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2}. \]
The parameters $\alpha_1$ and $\alpha_2$ are estimated by “rolling regressions” of 50 observations. In the terminology of Evans and Honkapohja (2001) this is a deterministic, “bounded-memory” approach to expectation formation.

4. Calibration and steady state

The model given by Eqs. (1) through (7) and the expectation formation equations links investment, cash flow, debt, and interest rates in a macroeconomic model that embodies Minsky’s views and is consistent with much empirical evidence. Because the model is dynamic and nonlinear we analyze its properties by simulations. We present the dynamic pattern of several macroeconomic variables and explore how these patterns change with variations in key parameters. This analysis shows that the model produces Keynes–Minsky cycles. We also check some of our results analytically with a linearized version of the model (see Supplementary information).

Although this model is not designed to reproduce actual macroeconomic fluctuations, we choose parameter values that are justified by empirical research and observation of the U.S. economy. Each time period represents a quarter and reported statistics are annualized. The effect of expected growth on the investment–output ratio ($\eta_1$) is 0.15 in our simulations. This value follows from empirical accelerator models. The key parameter relating cash flow to investment, $\eta_2$, has been widely studied in recent empirical literature. We use a benchmark value of 0.35.

We assume that the steady-state unemployment rate ($u^*$) is 4%. In (4) the parameter $\sigma_1$ is the slope of the Phillips curve. Its benchmark value of 0.05 implies that each percentage point of unemployment above the steady-state level over a quarter reduces inflation by 0.05% points (0.2% points for a full year). The hysteresis effect ($\sigma_2$) is set to 0.15 so that an increase in the unemployment rate of 1% point reduces quarterly inflation by 0.15% points. The constant wage share in total output ($\omega$) is 0.80.

Steady-state real output growth ($g^*$), which equals labor productivity growth ($\tau$), is 3% at an annual rate. For the model to have a well-defined steady state, the steady-state real interest rate ($r^*$) must be less than $g^*$. We therefore set $r^* = 0.01$ and consider the impact of raising this parameter value in the simulations discussed in the next section. Steady-state inflation ($\pi^*$) is 2% per year, implying that the steady-state nominal interest rate ($R^*$) is 3%.

We assume that the aggregate marginal propensity to consume out of a permanent increase of income is 0.8. Half of the consumers base consumption on forecasted income and half use lagged income to determine consumption spending, so that $\lambda_1 = \lambda_2 = 0.4$.

Because we choose the steady-state real growth rate, inflation rate, and the interest rate exogenously to match realistic values, we need to solve for the steady-state debt service and investment rates that make aggregate demand and aggregate supply consistent. We solve for steady-state

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14 The estimated accelerator effect, summed over distributed lags, in Chirinko et al. (1999) ranges from 0.05 to 0.21. The dependent variable in these regressions is the investment–capital ratio rather than the investment–output ratio used in our simulation model. This substitution is justified because the U.S. capital–output ratio is approximately one.

15 Many studies, such as Fazzari et al. (1988), estimate the impact of cash flow on investment separately for heterogeneous samples of firms. Chirinko et al. present estimates for a pooled sample of U.S. firms across both manufacturing and service industries. These estimates are more appropriate for our aggregate purposes. The values of $\eta_2$ from this study, summed over distributed lags, range from 0.27 to 0.49. Our benchmark value is in the middle of this range.


17 This value is consistent with evidence for the United States and a mid range of values for European countries. See McMorrow.
investment from (3)
\[ i^* = 1 + g^* - \lambda_1(1 + g^*) - \lambda_2. \]
The steady-state debt ratio is derived from (2) by equating all ratios to their steady-state values
\[ d^* = \frac{i^* - (1 - \omega)(1 + g^*)}{g^* - r^*}, \]
where \( r^* \) is the constant real interest rate that gives a nominal interest rate of \( R^* = (1 + \pi^*)(1 + r^*) - 1 \). Intuitively, a higher investment ratio, higher wage share, or higher interest rate in the steady state necessitates more borrowing for a given \( i^* \) and thus increases \( d^* \).

To make the steady-state inflation rate consistent with steady-state investment and growth, the intercept in the investment equation is
\[ \eta_0 = i^* - \left[ \eta_1 g^* + \eta_2 (1 + g^*)(1 - \omega) - \eta_2 d^* \frac{R^*}{(1 + \pi^*)} \right]. \]
This completes the specification of the steady state.

5. Simulation results

Fig. 1 shows the benchmark simulation results for real growth for 100 periods. The simulation begins in periods 1 and 2 with all variables set at their steady-state values for the benchmark parameters. In the third period, the investment–output ratio receives a temporary positive (demand) shock of 0.005, which is 2.4% of steady-state investment. The figure shows results for both the income (or output) growth rate (\( g_t \), solid line) and the ratio of debt to income (\( d_t \), dotted line). Immediately after this shock, real growth jumps from the steady-state value of 3% to 5% in period 3. Although the shock disappears in the fourth period, it initiates a cyclical process, evident in the figure, with a period of roughly 36 quarters. The results in Fig. 1 suggest that our benchmark calibration produces reasonable cycles. After the initial effects of the temporary shock dissipate, real growth fluctuates between 2 and 4% at annual rates. Unemployment ranges from 3 to just over 5% and annualized inflation from just over 0 to 4%. While its amplitude tends to decline over time with the benchmark parameter values, the cycle is very persistent. The annualized growth rate at the initial trough of the cycle is 0.8 percentage points below the steady-state value of 3%. It takes 125 years for the cycle amplitude to decline to half this value. We therefore interpret these cycles as endogenous in the sense that a single temporary shock to the steady state generates empirically significant cycles indefinitely. Simulations run for thousands of periods, however, confirm that the system eventually converges to steady state for the benchmark parameters.\(^\text{18}\)

5.1. Minsky cycles

What features of the model cause the cycle? The most obvious candidate is the cash flow term in the investment function. The parameter \( \eta_2 \) determines the strength of the effect of the

\(^{18}\) In Supplementary information we present a linearized version of the model evaluated around the steady-state values. This model produces simulation patterns similar to the nonlinear model discussed in the text. We can evaluate the stability properties of the linearized model analytically for given parameter values. With the benchmark parameters, the linearized model yields complex eigenvalues with a maximum magnitude of 0.9988, supporting our contention that the cycles are very persistent.
flow of internal finance on investment. As $\eta_2$ declines from its benchmark value of 0.35 to 0, the amplitude of the fluctuations declines monotonically. When $\eta_2 = 0$, the positive temporary shock to investment raises aggregate demand and output by more than the magnitude of the shock, due to the Keynesian multiplier effects in both the consumption and investment functions. The effect of the shock persists because of its impact on expectations, but it decays quickly. Growth converges to within four decimal places of its steady-state value after just 10 periods, and there are no cycles. Symmetrically, a rise in $\eta_2$ above its benchmark values increases the amplitude of the cycle. Indeed, if $\eta_2$ is greater than about 0.45, the cycles become unstable asymptotically. The internal finance effect on investment is therefore clearly responsible for the cyclical fluctuations seen in Fig. 1. Further support for this conclusion comes from the eigenvalues of the linearized model (see Supplementary information). When $\eta_2$ is set at its benchmark value the eigenvalues of the linearized model are complex, indicating cyclical behavior. The eigenvalues become real when $\eta_2$ is set to 0, indicating no cyclical behavior.

A more detailed look at the cycle shows that its detailed characteristics correspond to the kind of dynamic financial processes described by Minsky. The initial shock raises output and investment. As unemployment falls, inflation and nominal interest rates rise through the Phillips curve wage equation. The resulting increase in nominal interest rates pushes debt service upward. The positive shock also causes expected growth rates to rise and gives some initial positive inertia to investment through the accelerator, even though the shock is removed after just one period. For a while this positive inertia keeps investment high and unemployment low and further increases inflation, interest rates, and debt service. When the initial positive demand effects of the temporary shock dissipate, debt service remains above its steady-state level because of the recent investment build up and high nominal interest rates. High debt service lowers internal cash flow, which eventually depresses investment below its steady state. Growth slows and unemployment begins to rise, but inflation continues to increase because unemployment is below $u^*$; inflation, nominal interest rates, and debt service continue to increase. When the unemployment rate hits $u^*$, the increase in nominal interest rates leaves a legacy of debt service substantially above the steady-state level. This pushes the economy into a slump with unemployment rising above steady state. Higher unemployment causes a decline in inflation, nominal interest rates, and debt service. The decline
in debt service is eventually large enough to cause investment to grow, which turns the growth rate around and begins the recovery phase of the cycle.

By assuming a constant real interest rate, the only variation in the rate of debt service in our model comes from changes in the expected inflation rate. Keen (1995) argues that the debt service interest rate in Minsky’s analysis rises with higher debt and financial fragility. Jarsulic constructs a model in which the interest rate depends on the supply and the demand for credit, and credit supply would presumably be affected by financial fragility. If we were to add these feature to our model, the cyclical instability would be magnified.

Many discussions of the Minsky cycle emphasize the stock of debt in addition to the flow of debt service. The debt stock in our model behaves consistently with Minsky’s descriptive analysis: the debt–income ratio begins to rise as the economy comes out of a trough and investment rises. Near the first growth trough in our benchmark simulation \( d \) equals 0.596, and it rises to 0.605 during the subsequent expansion. This increase makes internal cash flow more sensitive to interest payments as the expansion continues, which helps create the conditions for the next downturn. The ratio of the debt stock to income, plotted in Fig. 1, clearly shows that the turning points in debt lead the growth cycle, consistent with Minsky’s causative interpretation of debt for macroeconomic fluctuations. While the debt stock relative to output rises during much of the expansion phase of the cycle, the peak debt–output ratio declines across cycles for our benchmark parameter values. With different parameter values, however, the peak debt–output ratio explodes in this model like in other papers that emphasize debt-deflation phenomena. In particular, raising the size of the investment–cash flow coefficient (\( \eta_2 \)) from its benchmark value of 0.35 to 0.45 leads to explosive debt cycles.

In summary, the basic dynamic process is driven by (1) the Phillips curve effect of unemploy-
ment on inflation, (2) the effect of changing inflation on inflation expectations and nominal interest rates, (3) the impact of nominal interest rates on debt service, and (4) the effect of debt service on cash flow and investment. The cycles are fundamentally financial. Moreover, this model exhibits nonneutrality of inflation because nominal interest rates affect debt service, cash flow, and real investment.

5.2. Wage and price flexibility

The discussion to this point identifies the importance of inflation dynamics and the spillover of inflation to nominal interest rates and the financing of investment. At larger values of \( \sigma_1 \) (the slope of the Phillips curve) the amplitude of the cycle increases and its period shortens. Table 1 gives statistics for several variables at the first trough and peak after the initial shock. Statistics are reported both for the benchmark model with \( \sigma_1 = 0.05 \) and an alternative model with \( \sigma_1 = 0.07 \). The growth and inflation rates are annualized, and the period at which the troughs and peaks are reached are given after each statistic.

A larger \( \sigma_1 \) increases the model’s volatility because it accelerates the inflation-interest rate-debt service dynamics discussed above. With a higher \( \sigma_1 \), the initial shock causes inflation and debt service to rise more quickly as unemployment falls. The larger debt service overhang then makes the subsequent trough deeper. But inflation declines faster in the trough, and the economy recovers more quickly.

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19 In a Keynes–Kaldor model with a more passive role for debt and without financial constraints on investment, Shaikh (1989) finds that debt turning points lag the output cycle. See Fig. 1 in Shaikh (p. 76).

20 See, for example, Taylor and O’Connell, Delli Gatti et al. (1993), Keen (1995), and Andresen.
Table 1
Cyclical implications of different wage flexibility parameters ($\sigma_1$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>$\sigma_1$</th>
<th>Growth at $t$</th>
<th>Inflation at $t$</th>
<th>Unemployment at $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trough</td>
<td>0.05</td>
<td>0.0212</td>
<td>0.0028</td>
<td>0.0525</td>
</tr>
<tr>
<td></td>
<td></td>
<td>at $t=14$</td>
<td>at $t=31$</td>
<td>at $t=23$</td>
</tr>
<tr>
<td>Peak</td>
<td>0.05</td>
<td>0.0388</td>
<td>0.0368</td>
<td>0.0281</td>
</tr>
<tr>
<td></td>
<td></td>
<td>at $t=32$</td>
<td>at $t=49$</td>
<td>at $t=42$</td>
</tr>
<tr>
<td>Trough</td>
<td>0.07</td>
<td>0.0196</td>
<td>-0.0056</td>
<td>0.0545</td>
</tr>
<tr>
<td></td>
<td></td>
<td>at $t=13$</td>
<td>at $t=28$</td>
<td>at $t=21$</td>
</tr>
<tr>
<td>Peak</td>
<td>0.07</td>
<td>0.0432</td>
<td>0.0516</td>
<td>0.0223</td>
</tr>
<tr>
<td></td>
<td></td>
<td>at $t=30$</td>
<td>at $t=45$</td>
<td>at $t=37$</td>
</tr>
</tbody>
</table>

Note that $\sigma_1$ is the standard measure of the wage and price flexibility of the economy. A higher value of $\sigma_1$ implies that wages (and prices through the markup) respond faster to the unemployment gap in the Phillips curve ($u - u^*$). Our result that the economy is less stable with a higher value of $\sigma_1$ contrasts sharply with the New Keynesian macroeconomics perspective, in which demand driven fluctuations are caused by nominal rigidity, and more flexible wages and prices should be stabilizing. An increase in $\sigma_1$ represents more flexible wages, but the amplitude of the cycle increases in our model.21

The results are different for the hysteresis effect in the wage inflation equation, $\sigma_2$. Increasing the response of wage (and therefore price) inflation to the change in the unemployment rate stabilizes the model, reducing the amplitude and the period of the cycles. For example, the first cyclical peak in unemployment occurs in the benchmark model at $t = 23$ and an unemployment rate of 5.25%. If we raise $\sigma_2$ from the benchmark value of 0.15 to 0.25, the unemployment peak comes earlier ($t = 20$) and at a lower value ($u = 4.96\%$). The explanation for this effect is somewhat subtle. The effect of $\sigma_1$ on inflation lags investment and output growth because the level of the unemployment rate is a lagging indicator of the growth cycle. Consider the cyclical peak. Although growth has peaked, the unemployment rate remains below $u^*$, so the standard Phillips curve effect causes a rise in inflation, nominal interest rates, and therefore debt service, even though investment and output growth have begun to decline. This “hangover” of accelerating inflation destabilizes the model. In contrast, the trough of the hysteresis effect is exactly in phase with the peak of the growth cycle of output and investment. When growth peaks, the change in the unemployment rate reaches its low point and begins to rise. This increase in the change in unemployment reduces inflation through the hysteresis effect and thus mitigates the rise of inflation, nominal interest rates, and debt service. The larger is $\sigma_2$, the greater this stabilizing influence, reducing the amplitude of the cycle. This effect can be very powerful. Doubling the benchmark value of $\sigma_2$ to 0.30 causes all meaningful cycles to die out within 100 periods, in contrast to the benchmark results for which the cycles are strongly persistent.

5.3. Debt, interest rates, and income distribution

Consistent with the conclusion that the debt and financial effects on investment generate the cycles in our model, an increase in the real interest rate makes the economy more volatile and shortens the cycle period. In our benchmark simulations, the first trough of the growth cycle occurs

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21 See Fazzari et al. (1998) and Palley (2005) for a discussion of the theoretical and empirical research on this issue. Modern new-classical models driven exclusively by supply-side factors are often viewed as the limiting case when wage and price adjustment become instantaneous. This limit need not be approached, however, if greater wage and price flexibility is destabilizing.
at $t = 14$ with a growth rate of 2.12%, and the subsequent peak is at $t = 32$ with growth of 3.68%. If we double the benchmark real interest rate to 2%, the first growth trough and subsequent peak occur earlier and at larger amplitudes (trough at $t = 12$ with growth of 0.72%; peak at $t = 24$ and growth of 4.44%). Higher real interest rates raise the importance of debt service, causing cash flow to respond more to cyclical changes in nominal interest rates. This change magnifies and accelerates the cycles.

Our results thus far are based on the simplifying assumption that debt rolls over every period, so that all debt is serviced at the current nominal interest rate. In reality, interest expenses are likely less responsive to interest rate changes because of long-term debt contracts. A full analysis of longer term debt is beyond the scope of this paper; it would require changing all the debt equations and the steady-state analysis. For this reason, we cannot completely explore the phenomenon of debt deflation emphasized by Minsky, following Fisher (1933). Debt deflation is more relevant to long-term debt with rigid nominal terms. But we can develop some intuition for the impact of lengthening debt contracts with a simple change to the model. The assumption that debt service depends on the average of the past two interest rates introduces some inertia into the effective interest rate that enters the cash flow calculation. One might expect this change to dampen the fluctuations, but in this model the opposite is true. Debt service inertia does initially slow the rise in debt service and the subsequent decline in cash flow, but the rise in debt service is responsible for dampening the expansion after the initial positive shock. By reducing this dampening effect, inertia in debt service allows the expansion to proceed further and adds to the amplitude of the cycle. In a sense, longer term debt reduces the “discipline” of debt accumulation on the investment boom, which allows the boom to persist longer, adding to the amplitude of the cycle. These results are symmetric for cycle troughs. This result again shows the benefits of formalization; it would be difficult to predict this effect from descriptive analysis alone.

Finally, we consider the impact of changing the wage share. The results are predictable from our analysis of interest rates and debt service. An increase in the wage share raises the steady-state level of indebtedness because firms must borrow more to maintain a given rate of investment with a lower profit share. Higher wage shares thus make the model less stable, increasing the amplitude and raising the frequencies of the cycles. A reduction in the wage share has the opposite effect. We conclude that the link between debt, cash flow, and investment in the empirically based investment equation employed here necessarily implies a link between income distribution and macroeconomics dynamics.

5.4. Expectations and bounded rationality

Expectations enter the model through the forecast of growth, which affects cash flow and the accelerator, and the forecast of inflation, which affects nominal wage and price dynamics and nominal interest rates. The benchmark results discussed to this point assume a naive forecasting model that projects this period’s value into the next period. Fig. 2 presents scatter plots of the expected versus actual values for inflation (first 200 observations). The graph for expected versus actual values for growth is similar. The forecasts are clearly correlated with the actual values generated by the simulation. Expectations quickly converge close to actual values. The correlation between expected and actual values is 0.949 for growth and 0.986 for inflation. The assumed forecast models seem consistent with the model structure, even though the forecasts rely on a “boundedly rational” behavioral rule rather than detailed structural knowledge of the economy.
We now consider more sophisticated forecasting behavior. Suppose that agents use a more general autoregressive model to forecast a variable $x$ (either growth or inflation)

$$\hat{x}_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2}.$$  

The AR(2) form of this rule admits the possibility that the forecast could capture the cyclical behavior of the simulations, and we allow agents to learn from the data about the parameter values of this forecast rule. To implement this procedure, the simulation is run for 50 periods with the naive forecasting rule. In subsequent periods, the forecasting parameters $\alpha_1$ and $\alpha_2$ are estimated from an OLS regression on the previous 50 periods of data. We begin the analysis with an AR(1) model to facilitate comparison with the naive rule (that is, we set $\alpha_2 = 0$ for both the growth and inflation forecasts). The estimated values of $\alpha_1$ remain very close to unity, ranging from 0.980 to 1.017 for periods 50 through 200. This result further confirms the consistency of the naive forecasting rule and the actual values, which imposes a value of unity for $\alpha_1$. It is also interesting to compare the effect of different forecasting rules on the cycles. Table 2 presents some statistics for the benchmark naive expectation model, the AR(1) forecasting model with learning, and the AR(2) model with learning. All statistics are computed from the simulation results of period 50 through 200. Although the more sophisticated learning models improve the correlation of the forecasted and actual values relative to naive expectations, the amplitude of the real growth cycles increases when agents use more complex forecasting. We conclude that the cycles produced by our benchmark model are not the result of a simple expectations specification.
Table 2
Comparison of forecasting models

<table>
<thead>
<tr>
<th>Forecasting model</th>
<th>Minimum $g$</th>
<th>Maximum $g$</th>
<th>Correlation ($g, \hat{g}$)</th>
<th>Correlation ($\pi, \hat{\pi}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naive</td>
<td>2.20</td>
<td>3.84</td>
<td>0.949</td>
<td>0.986</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.72</td>
<td>4.76</td>
<td>0.987</td>
<td>0.987</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.44</td>
<td>4.84</td>
<td>0.980</td>
<td>0.992</td>
</tr>
</tbody>
</table>

6. Conclusion

We have analyzed the macroeconomic implications of a dynamic model with the following key features: (1) an investment function that emphasizes financial effects, especially the empirically robust impact of cash flow, (2) a Keynesian macroeconomic framework that endogenously determines cash flow, (3) a dynamic labor market model that generates inflation from an expectations-augmented Phillips curve and hysteresis. Both the specification of investment and the parameter values chosen for the simulations reflect extensive recent empirical work with microeconomic data. The results are therefore based on the actual investment behavior of U.S. firms. This paper extends the implications of the microeconomic evidence to macroeconomic phenomena.

Simulation results validate Minsky's descriptive analysis of macro cycles arising from financial influences on investment. The dynamics of the model are inherently cyclical in the sense that conditions in the boom systematically lead to a downturn, and, symmetrically, high unemployment creates conditions that cause a recovery. Debt, interest rates, and inflation play key roles in the cyclical process. High inflation in the boom leads to high interest rates; these increase debt service, lower cash flow, and eventually constrain investment. When investment growth declines, aggregate demand and output follow. Low inflation and low interest rates in the slump reduce debt service, raise cash flow, and eventually cause investment growth to recover. We have demonstrated that these cyclical phenomena arise in the model as the result of the link between investment and cash flow. As Minsky himself has written, “[o]ne simple assertion – that investment has to be financed by capitalist-retained profits – has profound effects” (Delli Gatti et al. 1996, p. 408).

The model presented in this paper emphasizes a fundamental nonneutrality of money and finance. Real investment depends on real cash flow, and real cash flow depends on debt service for firms that have financed past investment with borrowing. Debt service is driven by nominal interest rates. In this model higher inflation raises nominal interest rates, which implies that inflation affects real investment and plays a central role in cyclical dynamics. To the extent that monetary policy affects inflation, this channel creates a nonneutrality of money. More broadly, any link between monetary policy and interest rates will affect real investment.

These observations suggest important possible extensions of our work to the analysis of the transmission mechanism for monetary policy. Policy-induced changes in interest rates could alter the dynamics of the macroeconomy through their effect on debt service. This channel is an alternative to the role of interest rates as intertemporal prices. The idea that monetary policy induces short-run substitution between capital and other factors of production is often emphasized in monetary transmission explanations, but the empirical evidence for a substantial interest rate elasticity of capital investment is weak. The link between interest rates, debt service, and investment through cash flow effects may provide a more empirically robust channel for a monetary transmission mechanism.

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22 See Chirinko et al. and Chirinko for further discussion of this point.
The results of this model may also have other implications for monetary policy. For example, advocates of inflation targeting suggest that nominal interest rates should respond more than point-for-point with changes in the inflation rate to cause real interest rates to rise when inflation accelerates and fall when inflation declines. This recommendation is based on a mainstream understanding of the monetary transmission mechanism that relies on price stickiness and substitution effects between capital and other productive factors caused by changes in the real interest rate. In our model, however, a larger response of nominal interest rates to inflation than the point-for-point specification employed in our simulations would likely raise the volatility of debt service and induce more unstable cycles. These topics deserve further exploration in an extended model that explicitly incorporates monetary policy.

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Appendix A. Supplementary Data


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