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The Model of an Expanding Economy

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THE MODEL OF AN EXPANDING ECONOMY

THE various models of a continuously expanding capitalist economy, set up, for instance, by Marx, by Cassel and, in recent times, by Mr. Harrod and Professor Domar,¹ all have their origin in a simple piece of arithmetic. When a constant proportion of income is added to capital every year and capital bears a constant ratio to income, then income expands continuously at a constant proportional rate. Thus, when 10% of net income is invested every year, and the stock of capital is five years' purchase of net income, then the stock of capital, the rate of investment per annum, consumption per annum and net income per annum all expand cumulatively at 2% per annum.

The various models which have been set up are based on widely different assumptions, and are arrived at by widely different arguments, but it is no accident that they all yield the same result, for the various assumptions and arguments are merely various ways of giving an economic application to the same piece of arithmetic.²

To what use can the model be put? The meaning of a proposition depends very much upon what it denies. In this respect the model is two sided. On the one hand, it shows that there is no inherent logical impossibility in conceiving of a capitalist system enjoying continuous expansion—it contradicts the view that there is an inescapable necessity for capitalism to run down. On the other hand, the model shows that certain special conditions are required for continuous expansion, and so

¹ Karl Marx, *Capital*, Volume II, Part III. (References are made to the English edition of Volume II, published by Swan Sonnenschein, and of Volume III, published by Kerr.) Gustav Cassel, *Theory of Social Economy*, Chapter I, § 6. R. F. Harrod, *Towards a Dynamic Economics*. E. D. Domar, "Expansion and Employment," *American Economic Review*, March 1947. A summary of some recent literature is provided by Harrod, "March on the Trade Cycle," *Economic Journal*, June 1951.

² In spite of a superficial resemblance, Mr. Hicks' conception of equilibrium growth does not belong to this family (*A Contribution to the Theory of the Trade Cycle*, Chapter V) for the ratio of capital to output plays no part in it. Output is perfectly elastic to demand (p. 61), that is, capital equipment is available *ad lib*. "Autonomous investment," like pyramid building, absorbs savings without adding to productive capacity. In short, Mr. Hicks transfers Keynes' short-period analysis into the long period without introducing the essential point of the long period—the growth of productive capacity resulting from investment. He purports to have been influenced by Mr. Harrod's *Dynamic Economics* (p. 7), but he evidently failed to notice what it was about.

it contradicts the view that there is, in general, an automatic tendency for capitalism to keep going.

Cassel lays the main emphasis upon the first aspect; the rest upon the second.

To see the use to which the model is put we must examine the various superstructures of assumptions and arguments that have been based on the arithmetic. First, how are the quantities concerned measured?

The arithmetic makes sense if we apply it to output in real terms. Marx reckons in *value*, that is, labour-time.¹ To arrive at real output it is necessary to multiply *value* by output per man-hour, which is rising through time when technical progress is taking place.

Cassel assumes that output per head is constant (there is no technical progress) so that he has no difficulty in reckoning in terms of real output. Harrod assumes constant prices, and reckons in terms of money. In effect, he takes the money value of output, corrected for changes in prices.

Whatever measure we choose we cannot avoid an index-number problem when relative wage-rates and relative prices alter. The arithmetic represents income simply as a number, and it can be applied only when it is a reasonable abstraction to treat output as though it were homogeneous, that is, with constant relative prices of commodities. When changes in relative prices and relative wages are important, a more complicated analysis has to be developed.

None of our authors gives a very perspicuous account of how capital is measured, but it seems clear that the quantity of capital, at a moment of time, means all the goods in existence at that moment, valued at their prices in terms of a unit of final output, for this is the quantity which is increasing at a constant proportional rate when the conditions of the model are fulfilled.

What about employment? It is an essential characteristic

¹ So long as the rate of exploitation (the ratio of profits to wages) is uniform throughout the economy (wages and profits being the only categories of income) and is constant through time, this comes to the same thing as using the money-wage per man-hour as the unit of account. When the organic composition of capital is different in different lines of production, the ratio of capital to labour is different, and if the rate of profit on capital tends to be the same in all lines (as Marx assumes) the rate of exploitation cannot be uniform. This gives rise to the "problem of transforming *values* into prices"—the *pons asinorum* of Marxian theory, similar to the "adding-up problem" in marginal-productivity theory. Cf. Sweezy, *Theory of Capitalist Development*, Chapter VII. See also his preface to the edition published by Kelley of Böhm-Bawerk, *Karl Marx and the Close of his System*.

of the model that output increases in proportion to the stock of capital. Therefore, if output per man-hour is rising (with technical progress) at a faster rate than the stock of capital, the number of hours' work done in a year is falling through time. This (unless the available supply of labour is shrinking) entails either growing unemployment or a falling number of hours worked per man-year. If output per man-hour is rising in a smaller proportion than the stock of capital, employment is growing, which entails either that population is growing or that there is an indefinitely large reserve army of labour, in open or disguised unemployment, to be taken into service. (Alternatively, hours worked per man-year may be increasing, but this has obvious limits.)

Cassel assumes no rise in output per man-hour and full employment; therefore he requires population to be growing at the same rate as the stock of capital.¹

Domar assumes full employment, although he allows for rising output per head. This involves him in contradictions or in assumptions about hours of work and the rate of growth of population which he does not in fact specify. Either his model is intended to be radically different from the others or the introduction of full employment into it was simply a mistake.

Harrod is rather vague about employment, while for Marx the existence of a reserve army is one of the central features of the model.

Employment, as such, does not appear in the arithmetic. The conditions of the model concern only the accumulation of capital. The basic condition, that the ratio of output to capital is constant, is satisfied if (1) technical progress is *neutral* in Harrod's sense; this means that, when capital is reckoned in terms of the cost in wage-units of the stock of capital goods, capital per unit of labour is constant, and capital per physical unit of output is falling at the rate at which output per man-hour is rising (that is, capital-saving innovations are being introduced to the same extent as labour-saving innovations);² and (2) profit (interest *plus* net profit) per unit of capital is constant. These two conditions entail that prices in terms of wage-units fall at the pace at which output per man-hour rises (if money prices are

¹ Rather, he looks at it the other way round. He assumes that population is increasing at a steady rate, and he postulates that the community carries out investment at a sufficient rate to maintain capital per head at a constant level.

² Though capital in terms of wage-units is constant, physical capital is increasing; thus horse-power per man-hour is likely to be rising as technical progress goes on.

constant, the money wage per hour rises with output per hour). The ratio of output to capital, measured in terms of product, is constant. The wage per hour in terms of product rises with output per hour. The relative shares in proceeds of wages and profits (which are taken to be the only categories of income) are constant. (In Marx's language, the rate of exploitation is constant.)

Marx makes great use (and the rest no doubt would concur) of the division of the stock of capital between the two main sectors of industry—that producing investment goods and that producing consumption goods. The model requires that the division between these sectors, both of the stock of capital existing at any moment and of the investment currently going on, corresponds to the division of output between investment and consumption. Productive capacity in each sector is expanding at the same proportional rate as the total stock of capital, and therefore at the same rate as income, investment and consumption.

Next, to satisfy the conditions of the model, the proportion of saving in net income must correspond, at any moment, to the division of total productive capacity between investment and consumption goods, and must remain constant through time. This provides that effective demand expands at the same pace as total output. (The condition that saving is a constant proportion of income is consistent with the condition that the relative share of profit in net income is constant, since it is then not unplausible to postulate a given long-run propensity to save in the economy as a whole.)

Two further conditions are necessary. The stock of capital in each sector must be continuously maintained and adapted to new techniques as it grows. This condition is fulfilled if annual renewals bear a constant proportion to the stock of capital, and amortisation funds, providing at the required rate for wear and tear and obsolescence, taken as a whole, are continuously re-invested as they accrue.

Finally, the gestation period of capital goods must be constant, so that a given rate per annum of investment entails a given growth per annum in the stock of capital available for use.

No doubt it is possible to construct models in which a failure in one of these conditions is compensated by a variation in another—for instance, in which a growth through time of capital per unit of output is offset by an appropriate rate of rise in the proportion of income saved—but this involves complicated relationships between the quantities involved, and the present

argument is confined to the simple model in which all the conditions are fulfilled.

In the following numerical example, the stock of capital is equal to five years' purchase of net income; annual renewals are 10% of the stock of capital in each sector. Investment is 10% of net income. Capital is divided between the sectors in the ratio of consumption to net investment *plus* renewals. The

	Stock of capital.			Annual output.				
	Investment industries.	Consumption industries.	Total.	Renewals.	Consumption.	Investment.	Net income.	Gross income.
Year 1	200	300	500	50	90	10	100	150
Year 2	204	306	510	51	91·8	10·2	102	153
Year 10 (approx.)	240	360	600	60	108	12	120	180

“ year ” is an arbitrary length of calendar time. It must be long enough relatively to the gestation period of capital goods to make it a reasonable approximation to take the investment made in one “ year ” equal to the addition to capital available for use in the next.¹

¹ The example has to be slightly altered in order to be set out in the form which Marx used. His terminology obliges him to make the stock of capital (pre-existing capital, *c*, *plus* the wages fund, *v*) equal to the cost of production of a year's output (annual replacement of capital, *c*, and the year's wages bill, *v*). He usually takes the rate of exploitation to be unity (*v*, wages, equal to *s*, profits) so that, in our example, *v* would be 50. We must therefore put *c* at 450, and consequently gross income at 550. The other quantities are the same as those set out above. (On this basis, organic composition of capital is equal to 9, and the rate of profit on capital 10% per annum.) The gross output of Department I, renewals plus net investment, is 460; while the gross output of Department II, consumption goods, is 90. To avoid tiresome fractions, multiply all the quantities in the example by 11.

Then we have, in the first year :

	<i>c.</i>	<i>v.</i>	<i>s.</i>	Total.
Department I	4140	460	460	5060
Department II	810	90	90	990
Total	<u>4950</u>	<u>550</u>	<u>550</u>	<u>6050</u>

Each quantity increases at the rate of 2% per annum. Marx left his own numerical examples in a state of confusion (*Capital*, Vol. II, pp. 591-610). They were reconstructed by Rosa Luxemburg in better shape, but they are still rather awkward in the form in which she set them out; *Accumulation of Capital*, Chapter VI, pp. 115 *et seq.* See also Introduction p. 18. (References are to the English edition, published by Routledge.)

The specified conditions ensure that output expands continuously provided that investment is maintained at the required continuously expanding rate. This is as far as arithmetic can take us. We must now inquire what motive force may be conceived to keep the economy running along the rails which the conditions have laid out.

In Marx's scheme capitalists are subject to a strong pressure to accumulate. Saving is made only for the purpose of investment, and (apart from crises) all savings are invested as they are made. For Cassel saving *is* investment. But for those who have revived the model under the influence of Keynes the existence of thriftiness by no means guarantees that investment will be carried out.¹ They must therefore pose the question: How can perpetual accumulation be conceived to occur?

Domar regards continuous investment as requiring a kind of collective faith. Each capitalist finds it worth while to invest at the appropriate rate provided that all the others do so, and so long as each believes that the others will continue, he continues himself.

Harrod relies upon the "acceleration principle." The increase in the rate of output which is taking place "induces" the increase in stock of capital which makes it possible. This is backed up by the view that so long as capitalists collectively keep the stock of capital expanding at the required rate, they are "satisfied," and continue to do so. (Neither argument is at all clearly explained.²)

Since he is tied up in the "acceleration principle," Harrod has to regard his system as chronically unstable. Any chance increase in output above the rate appropriate to the conditions of the model "induces" a higher rate of investment, and causes a boom which cannot last, and so precipitates a slump.

Another solution of the problem is to graft Marshall's analysis of long- and short-period supply price on to the model. When an economy is expanding at the rate appropriate to the given conditions, all prices are equal to long-period average costs (including in cost, profit on capital at the given rate) and all capital equipment is working at its designed capacity. In each sector conditions of rising short-period supply price obtain, so

¹ Nor does the existence of human needs. One of the confusing points in Mr. Harrod's scheme of ideas is that he seems to identify needs with prospective profits, so that a growth in population automatically induces profit-seeking capitalists to make investment (Southern Italy?).

² Mr. Harrod has elaborated his view in the article referred to above, but still leaves it somewhat mysterious.

that any increase in output relatively to capacity would be accompanied by a rise in price above long-period average cost. The capitalists expect the rate of profit to continue in the future to rule at the present level.

Now, if we postulate that the capitalists' expectations of future profits have great inertia and do not react to passing events, the system can be regarded as being in equilibrium from the short-period point of view. A chance increase in consumption would cause the output of consumption goods to rise above designed capacity, prices to rise above normal costs and so profits to rise above their long-run level. But since this state of affairs is not expected to last, investment is not stepped up, and no "acceleration" occurs. Similarly, a chance increase in investment does not raise expected future receipts (in spite of a rise at the moment, due to the operation of the short-period multiplier). But the prices of capital goods have risen above the normal long-run level, the rate of profit to be expected on funds invested at these prices is less than the accustomed rate, and so, we may suppose, investment is checked. If investment chanced to fall, the price of capital goods would fall, the rate of profit to be expected on funds invested at those prices would rise, and investment would pick up again. Thus, the postulate that expectations do not vary with current events may be considered to endow the system with short-period stability, and (combined with faith in future profitability of capital) to provide a presumption that the rate of investment tends to be maintained at a level which continuously corresponds to the gradually growing capacity of the investment-good industries.

The foregoing argument is intended to show that perpetual steady accumulation is not inherently impossible. We now turn to the arguments intended to show that the conditions required by the model are unlikely to be found in reality. First, consider the assumption, so frequently made, consciously or tacitly, in economic theory, that the economy must always be tending towards a position of static equilibrium.

An economy with constant population and unchanging technique, with capital equipment working at capacity, which has settled down to consuming the whole of its net income, fulfils the specified conditions. It is a limiting case of the model, with the rate of accumulation at zero.¹

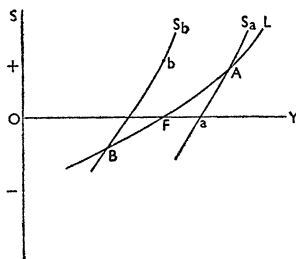
¹ This—simple reproduction—was the only example of the model which Marx worked out satisfactorily. (His examples of expanded reproduction are full of inconsistencies, see above p. 46, note.)

But an economy in which accumulation has been taking place up to the present, and which now finds itself with a stationary population and a fixed body of technical knowledge, is in a very different pickle. Investment cannot continue for long at a constant rate of profit; accumulation is tending to come to an end, and as it falls off, a slump will occur.

This is the setting of the vision of the Day of Judgment which Pigou attributed to Keynes,¹ and of the argument as to whether a falling rate of interest and a rising value of money can maintain a given level of employment in stationary conditions.² However, the notion of a static state is not more

¹ "Mr. J. M. Keynes's *General Theory of Employment, Interest and Money*," *Economica*, May 1936. It is true that some passages in the *General Theory*, especially Chapter 17, can be taken to suggest that this was Keynes's view. But it is doubtful if he had anything that can properly be called a view on this question. He was impatient of the notion of long-period equilibrium and never brought his mind to bear upon it.

² This argument is illustrated in the following diagram. The axes measure saving, S , and income, Y , both in real terms. At each point on the L curve, which represents the long-period relation between saving and income, the stock of capital is appropriate to income. Thus at the point A the stock of capital, say a , is that of which A represents the capacity output; the point B represents capacity output of capital β ; and so forth. At the point F net saving is zero.



Now, if the economy exists in a stationary environment, with given, unchanging techniques, population, tastes and rate of interest, positive net investment cannot continue for any length of time, for, if it did, capital, after a while, would become redundant. Thus the only possible position of full stationary equilibrium is at F . (If the L curve does not cut the Y axis, that is, if saving is positive at all levels of income, there is no point of equilibrium short of universal extinction.)

The model of a "pure" trade cycle in a trendless economy (Kalecki, *Essays in Economic Fluctuations*) exhibits income cycling round the point F , disinvestment in each slump wiping out the increment of capital created in each preceding boom.

Each point on the L curve is cut by an S curve showing the short-period relation between saving and income with given capital equipment. The S curves are steeper than the L curve, since a rise in income above the normal capacity output of a given stock of capital is accompanied by a rise in prices above long-period costs, an increase in profits and consequently a rise in the ratio of saving to

realistic than the notion of continuous expansion, and this objection to the model may be dismissed as a case of the pot calling the kettle black.

Next, consider Marx's point of view. His general picture of the process of capitalist development throws up three main objections to the conditions of the model. The first is that in an unplanned private-enterprise economy there is no reason to expect the capitalists to hit off the right proportions of investment in various sectors of the economy.¹ The capital stock is always getting out of gear with the main division of demand between consumption and capital goods,² and with subdivisions of demand for particular commodities, so that gluts and scarcities of particular goods frequently occur. This makes smooth development impossible.

The second objection is rather vaguely sketched by Marx;³ it is more clearly set out by Sismondi,⁴ Rosa Luxemburg⁵ and

income. In the diagram S_a represents incomes produced with capital a and S_b incomes produced with capital β .

Take a case where income happens to be at the point b . The economy is now facing the Day of Judgment. Positive investment is going on at the moment, but this will not last, and the economy cannot make the transition from b to F without passing through a cataclysmic slump. The problem then is whether, by lowering the rate of interest and raising the real value of the stock of money, it is possible to move the point F to the right, and flatten the S curves, in such a way as to make a smooth transition from b to F possible.

Now suppose that the stock of capital happens to be a . This is even worse than the Day of Judgment conceived by Pigou. Any income greater than that shown by the point a entails positive net investment, and cannot be sustained for long. But at the point a where savings are zero, output is below the capacity of the stock of capital; therefore disinvestment must be taking place, income must be below a , and the stock of capital must be dwindling towards that appropriate to F . If the economy is subject to a trade cycle, it pursues a spiral course, the net disinvestment in each slump exceeding the net investment in the preceding boom.

When the environment is not stationary, but technical progress combined with population growth make steady accumulation possible, then if the conditions of the model of steady expansion are fulfilled at A and if capitalists are willing to make continuous investment at the appropriate rate, expansion is taking place through time. A line rises from the page through the dimension of time, with its root at A , set at a north-easterly angle to the plane of the diagram indicating the proportional rate of expansion per annum in income and in saving.

¹ E.g., *Capital*, Volume III, p. 141.

² Rosa Luxemburg pushed this argument further than it will go. She believed that saving out of profits can be invested only in the sector of the economy where the profits were made, so that, unless the ratio of savings in each sector is just right to begin with, the system will immediately jam, with a surplus of production in one sector and a deficiency in the other. *Op. cit.*, p. 337.

³ E.g., *Capital*, Vol. III, p. 293.

⁴ *Nouveaux Principes d'Economie Politique*.

⁵ *Accumulation of Capital*.

Hobson.¹ In their view, real wage-rates fail to rise in proportion to productivity, while profits are largely saved, so that the demand for consumption goods fails to expand as fast as the stock of capital, and accumulation cannot continue to be profitable.

The third objection to be found in Marx's analysis is based on the opinion that technical progress normally takes forms which raise the ratio of capital to output. This violates a basic condition of the model. Marx assumes that the rate of exploitation (which governs the share of profit in proceeds) cannot rise sufficiently to compensate for the increase in capital per unit of output, so that the rate of profit tends to fall over the long run, and the capitalist system is caught in a "contradiction" which sooner or later will bring it to destruction.² This is a weak point in Marx's argument, for it is hard to understand how the share of labour in national income can remain constant in face of a rising ratio of capital to output.

If we assume that there is a normal rate of profit on capital (obtainable when effective demand is such as to keep output just at the level corresponding to capacity) which tends to remain constant through time, then a rising ratio of capital to output entails a rising share of profits in total income. A rising capital ratio then has two contrary effects. On the one hand, it means that as time goes by an ever-larger amount of investment is required to create a given increase in capacity; on the other hand, it means that the proportion of saving in income is rising. These two effects might balance, so that accumulation could continue smoothly. This leads us back to the unexplored field of compensated models, in which a deviation from one of the basic conditions of the simple model is offset by an appropriate deviation from another.

When the increase in required investment is greater than corresponds to the rise in thriftiness, then (provided that capitalists do want to make a continuous increase in capacity) the economy undergoes a secular boom. In the reverse case it dwells in a chronic slump.

However, if in fact technical progress on the whole is more or less *neutral*, this problem is not important either way.

In Domar's view the main objection to the conditions required by the model concerns the motive power which keeps accumulation going. A failure of confidence, or a mere tendency for capitalists each to wait and see what the others will do, brings investment to a halt.

¹ *Economics of Unemployment.*

² *Capital*, Vol. III, Chapter 13.

Harrod's bugbear is a modified form of the Day of Judgment. He conceives of the maximum physically possible rate of increase in output, given by the rate of increase of employable population and the rate of increase in output per head due to technical progress (this he calls the *natural* rate of growth—an unnatural use of language). He expects that in the future this maximum possible rate of growth will fall short of the rate corresponding to the rate of accumulation which has been going on in the past, so that chronic slump conditions will set in unless policies are devised either to reduce thriftiness or to keep the ratio of capital to output rising.

All these views point to circumstances (which may or may not be realised) in which the model would break down.

Even when there is no systematic failure in any of the conditions required by the model, an over-riding objection remains. History and geography present a developing economy with all sorts of chances and changes—some favourable, some unfavourable to accumulation—so that development cannot follow a steady course for long, even if all the conditions are present to start with.¹ And the very fact that actual development is erratic destroys the basic conditions for smooth development.

First, the stocks of specific equipment of various kinds and the supplies of particular types of labour, in existence at any moment, have been moulded by the past history of demand and are usually out of gear with current demand. This destroys the basic condition of the model that productive capacity in the various sectors of industry is adjusted to the division of demand between their products.

Second, when the capitalists know that unpredictable disturbances are liable to occur, the inertia of the economy is destroyed. When the present state of affairs alone is certain, it has an undue influence upon behaviour. Thus, when output expands, for any reason, relatively to capacity, capitalists have a tendency to behave as though they expected the consequent high level of profit to be maintained in the future, and to plan investment accordingly.² While investment is going on, profits rule all the higher; but the increase in capacity which is being created is doomed to bring the rate of profit below the level which caused it to be planned. Thus accumulation can take

¹ This point of view is supported by T. C. Schelling, "Capital Growth and Equilibrium," *American Economic Review*, December 1947.

² R. M. Goodwin makes an illuminating comparison between expectations that have this characteristic and the operation of a thermostat. (Chapter 22 of Alvin Hansen's *Business Cycles and National Income*, p. 437.)

place only in a series of booms interrupted by slumps. If the economy has not developed smoothly in the past, it is incapable of doing so in the future.¹

From this it seems to follow that it is a mistake to look for a theory of the trade cycle conceived in terms of oscillations around a trend of steady growth, for an economy in which steady growth is possible differs in its internal structure from one which is subject to oscillations. The connection between the cycle and the trend is both more intimate and more complicated than any that has yet been set out in a systematic theory.

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¹ Mr. Harrod conceives the "warranted rate of growth" which fulfills the conditions of the arithmetical model as a path which the economy is constantly crossing and re-crossing as it advances. But if the above argument is correct, unless the economy is actually on the path, the path does not exist.