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A Keynes-Kalecki Model of Cyclical Growth with Agent-Based Features

Mark Setterfield*

Department of Economics
Trinity College
Hartford, CT
06106, USA
mark.setterfield@trincoll.edu

and

Andrew Budd

Department of Economics
Trinity College
Hartford, CT
06106, USA

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Abstract

We construct a Keynes-Kalecki model of cyclical growth with agent-based features. Our model is driven by heterogeneous firms who, confronting an environment of fundamental uncertainty, revise their "state of long run expectations" in response to recent events. Model simulations generate fluctuations in the rate of growth that are both aperiodic and of no fixed amplitude. We also study the size distribution of firms resulting from our simulations, finding evidence of a power law distribution that we have no reason to anticipate from the basic structure of our model. Finally, we reflect on the potential advantages of combining aggregate structural modelling with some of the methods and practices of agent-based computational economics.

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1. Introduction

The neo-Kaleckian model of growth and distribution is a well established feature of Post-Keynesian macrodynamic analysis. Since its introduction by Del Monte (1975), Rowthorn (1981) and Dutt (1984),[1] the neo-Kaleckian model has been refined and extended in a variety of ways, to include (*inter alia*) financial variables (Dutt, 1992; Dutt and Amadeo, 1993; Lavoie, 1992, 1995; Hein, 2006), analyses of the interaction of growth, distribution and inflation (Dutt, 1987; Lavoie, 2002; Cassetti, 2002) and even to incorporate the effects of advertising and conspicuous consumption (Dutt, 2007).

Despite all this, one theme that has received scant attention in the neo-Kaleckian literature is the role of historical time and uncertainty in shaping the economy's growth path. Under conditions of uncertainty, economic outcomes (including growth) can be affected by changes in the "state of long run expectations" (SOLE) – that is, second order features of the decision making process, such as confidence and animal spirits, that cannot be described in closed form, but that nevertheless impinge on behaviour independently of the best forecast of actual future events that decision makers are able to procure (Gerrard, 1995; Dequech, 1999).[2] Explicit acknowledgement that historical time and uncertainty are part of the fabric of the economy can certainly be found in the neo-Kaleckian literature (see, for example, Lavoie, 1992, pp.282-4). But by-and-large, neo-Kaleckians have chosen to adopt the modelling strategy of Keynes (1936) who, according to Kregel (1976), sought to "lock up without ignoring" the effects of uncertainty on behaviour and hence economic outcomes by assuming a *given* SOLE. In analytical terms, this provides a form of model closure that has, in turn, permitted the use of an

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[1] See Blecker (2002) for a recent survey of core issues in the neo-Kaleckian literature.
[2] See, for example, Taylor and McNabb (2007) for a recent empirical assessment of the impact of business confidence – a component of the state of long run expectations – on the economy's growth path.

equilibrium methodology in neo-Kaleckian analysis. This, together with the attendant method of comparative statics (or dynamics), has been used to good effect to demonstrate the main results of the neo-Kaleckian theory of growth and distribution.

From a Post-Keynesian perspective, however, permitting variability in the SOLE is a necessary and important step in the development of Keynesian macrodynamics (Kregel, 1976). The purpose of this paper is to take up this challenge in the confines of an otherwise canonical neo-Kaleckian growth model. The paper builds on Setterfield (2003), who describes a model in which variations in the SOLE affect investment behaviour. The model is formally open and hence admits no closed form solution, but is shown to suggest the possibility of cyclical growth. In this paper, we: (a) extend the analysis of Setterfield (2003) by permitting heterogeneity amongst firms (in particular, with respect to changes in their SOLE), thus introducing agent-based features into the analysis; (b) simulate the resulting model to show more clearly the aperiodic growth cycles to which Setterfield alludes; and (c) explore other features of the model economy (including the size distribution of firms) that are not obvious from its basic construction, and that might be considered emergent properties of its operation.

The remainder of the paper is organized as follows. Section 2 describes the model on which the paper is based, with particular attention paid to the way in which agent-based features are incorporated into what is initially an aggregate structural model. Section 3 reports our simulation results, and finally section 4 concludes.

2. A Keynes-Kalecki Model of Cyclical Growth

i) An initial structural model

We begin with a structural model of the following form:

$$g_t^i = \alpha_t + g_r r_t^e + g_u u_t^e \qquad [1]$$

$$g_t^s = s_\pi r_t \qquad [2]$$

$$r_t = \frac{\pi u_t}{v} \qquad [3]$$

$$g_t^s = g_t^i \qquad [4]$$

$$r_t^e = r_{t-1} \qquad [5]$$

$$u_t^e = u_{t-1} \qquad [6]$$

$$\alpha_t = \alpha_t(u_{t-n}, u_{t-n}^e) \quad , \quad n = 1,2 \qquad [7]$$

where g^i is the rate of accumulation and g^s is the rate of growth of savings, α denotes the SOLE, r^e and r are the expected and actual rate of profits, respectively, u^e and u are the expected and actual rates of capacity utilization, respectively, π is the profit share and v is the fixed capital-output ratio. The model stated above is replicated from Setterfield (2003), and comprises what Lavoie (1992, chpt.6) describes as the canonical neo-Kaleckian growth model (equations [1]—[6]) augmented by a SOLE reaction function (equation [7]). Hence equation [1] is a standard neo-Kaleckian investment function, equation [2] is the Cambridge equation, and equation [3] is true by definition. Note that, since the capital-output ratio v is fixed by assumption, the rate of accumulation described in equation [1] is equivalent to the economy's rate of growth. Equation [4] insists that the growth of savings adjusts to accommodate the rate of accumulation in each period, whilst equations [5] and [6] describe the adjustment of expectations between periods. Finally, equation [7] states that the SOLE – which includes the confidence that firms place in their expectations and their animal spirits, and hence the willingness of firms to act on the basis of their expectations – depends on expected and actual events in the recent past.[3]

Combining equations [1]—[6] to produce reduced-form expressions for g^i and u and combining these expressions with equation [7], we arrive at the following system of equations:

$$\alpha_t = \alpha_t(u_{t-1}, u_{t-2}, u_{t-3}) \quad [7]$$

$$g_t^i = \alpha_t + \left(g_u + \frac{g_r \pi}{v} \right) u_{t-1} \quad [8]$$

$$u_t = \frac{v}{s_\pi \pi} g_t^i \quad [9]$$

In Setterfield (2003), the implicit function in [7] is rendered explicit in the manner described in Table 1 below, with c assumed constant and:

$$\varepsilon_t \sim (\mu_{\varepsilon t}, \sigma_{\varepsilon t}^2)$$

The basic idea in Table 1 is that firms revise their SOLE in a manner that depends on: (i) a comparison of the difference between actual and expected events to the value of a conventionally determined "acceptable" margin of expectational error, c; and (ii) an adjustment parameter (ε) that is influenced by the convention $\mu_{\varepsilon t}$, from which decision makers can deviate at will (hence $\sigma_{\varepsilon t}^2 \neq 0 \; \forall \; t$).[4]

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[3] See Kregel (1976).

[4] The convention μ_ε is described as time-dependent on the basis that, although conventions are relatively enduring, they can (and do) change, and in novel ways. It is this latter feature (novelty) that explains the absence of any equation of motion that purports to explain *how* μ_ε changes over time.

See Setterfield (2003, pp.326—7) for further discussion of the process of revising the state of long run expectations.

[TABLE 1 GOES HERE]

Outcomes in the model described above result from the recursive interaction of equations [7]—[9]. Using conventional analytical techniques, Setterfield (2003, 327—31) shows that the model has the capacity to produce cumulative increases (or decreases) in the rates of growth and capacity utilization, that may occasionally be punctuated by turning points. He thus alludes to the capacity of the model to produce growth cycles, that are aperiodic and of no fixed amplitude. Part of the purpose of this paper is to more clearly demonstrate the existence of these cycles by utilizing simulation techniques.

ii) Introducing agent-based features into the model

As intimated above, part of our motivation for simulating the model developed in this paper is to more clearly demonstrate its outcomes, and in particular the model's description of a growth path that is subject to endogenously generated aggregate fluctuations. But a second advantage of the simulation method that we can also exploit is that is that it eliminates the need for simplifying assumptions designed to permit the derivation of a tractable analytical solution to a model. Put differently, models designed for simulation can be as complicated as available computing capacity allows. In what follows, we use this advantage to introduce "agent-based features" into our model. Specifically, we replace the single representative firm implicit in the structural model developed thus far with a multiplicity of heterogeneous firms.

So-called agent-based computational economics (ACE) is a fast growing sub-field in economics.[5] One of the basic ambitions of ACE is to construct dynamic economic

[5] See, for example, Markose et al (2007), 1801-03) and Tesfatsion (2006) for (respectively) brief and more extensive overviews of this sub-field.

models that feature multiple, heterogeneous agents. In some quarters, the impetus for this ambition derives from a desire for a "second generation" microfoundations project in macroeconomics – one that properly recognizes the substance of the SDM theorems in Walrasian economics and thus eschews the notion of "microfoundations" that rest on a single, representative agent (see, for example, Kirman, 1989; 1992).[6] As such, the ACE project is avowedly "bottom up" in its approach to model building, beginning with (heterogeneous) individual agents and looking for macroscopic phenomena – at whatever level of aggregation – to arise from their interaction (see, for example, Markose et al, 2007, p.803). The approach taken in this paper is, however, rather different. It involves disaggregating certain features of an aggregate structural model in order to incorporate some amount of agent heterogeneity. It is for this reason that we refer to the model in this paper as having "agent-based features", rather than as an ACE model *per se*.

Our introduction of agent-based features into the model described earlier focuses exclusively on firm behaviour, with respect to the revision of the SOLE in response to expectational disappointment. We distinguish between different types of firms along two broad dimensions. First, we differentiate between "aggressive adapters" and "cautious adapters". Aggressive adapters revise their SOLEs in response to small discrepancies between u and u^e. In terms of the contents of Table 1, they set a low value of the convention c. Aggressive adapters are also characterized by short reaction periods. In other words, there need only be a discrepancy between u and u^e for a brief period of

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[6] It can be argued that this second generation microfoundations project shares certain ontological affinities with aggregate structural modelling in macroeconomics. See Setterfield (2006).

calendar time in order for this discrepancy to trigger a change in the SOLE.[7] Cautious adapters, meanwhile, display the opposite characteristics: they revise their SOLE only in response to large discrepancies between *u* and u^e (i.e., they set high values of *c*) observed over longer intervals of calendar time (i.e., they have long reaction periods).

Second, we differentiate between firms whose fortunes – and hence their SOLEs – are more sensitive to macroeconomic events, and firms whose fortunes and SOLEs are less sensitive to macroeconomic events. More specifically, we envisage all firms as revising their SOLEs in response to a mixture of *both* their own individual experience *and* aggregate economic outcomes. The more sensitive to macroeconomic events a firm is, the greater will be the weight it attaches to aggregate economic outcomes (relative to individual experience) in the process of revising its SOLE.[8]

Based on these considerations, we replace equations [7]—[9] of the structural model above with:

$$\alpha_{jt} = \alpha_j(u_{jt-n}, u_{t-n}) \quad , \quad n = 1,2,3 \qquad [7a]$$

$$g^i_{jt} = \alpha_{jt} + \left(g_u + \frac{g_r \pi}{v} \right) u_{jt-1} \qquad [8a]$$

$$u_{jt} = \frac{v}{s_\pi \pi} g^i_{jt} \qquad [9a]$$

for *j* = 1, ..., 100, and with [7a] rendered explicit as in Table 2 below.

[TABLE 2 GOES HERE]

[7] The concept of a reaction period in the adjustment of firms' expectations is due to Harrod – see Asimakopulos (1991, chpt.7) for further discussion. The reaction period concept is not formally represented in Table 1.

[8] Note that this creates feedback from macroeconomic outcomes to microeconomic (firm) behaviour, thus avoiding the "one way street" favoured by reductionist approaches to macroeconomics, according to which macro outcomes are affected by micro behaviour, but the converse does not apply. It is also central to the conception of agent interaction in our model, as is explained below.

In Table 2, $\varepsilon_{jt} \sim (\mu_{\varepsilon t}, \sigma^2_{\varepsilon t})\ \forall j$, and the conventions c_j are now modelled as:

$$c_j = \beta_j \sigma_u^2$$

where $0 < \beta_j \leq 1$ and σ_u^2 is the variance of the aggregate capacity utilization rate. We then use the values of β_j, k_j and κ_j to distinguish between the different types of firms outlined above – aggressive adapters (low β_j and k_j), cautious adapters (high β_j and k_j), firms that are more sensitive to aggregate economic outcomes (low κ_j) and firms that are less sensitive to aggregate economic outcomes (high κ_j). The precise values of these parameters and their correspondence to the types of firms discussed above is described in detail in section 2(iii)c below.[9]

Before proceeding, however, note that the recursive interaction of [7a]—[9a] is subject to an important constraint that is not considered by Setterfield (2003), but that must inform our simulations. Specifically, since $u \in [0\ 1]$, we can identify from equation [9a] upper and lower bounds to the growth rate, given by:

$$g^i_{\max} = \frac{s_\pi \pi}{v}$$

for $u_j = 1$, and:

$$g^i_{\min} = 0$$

for $u_j = 0$. These "limits to growth" can be incorporated into our simulation model by insisting, following the calculation of g^i_{jt} during each iteration, that:

[9] Notice that k_j, β_j, κ_j, and ε_j are the only agent-specific parameters in our model. Parameters such as g_u and g_r in equations [7a]—[9a] are common to all firms. Ultimately, then, our model retains many features of the single representative firm implicit in our original aggregate structural model, introducing agent heterogeneity only into the SOLE reaction function. We focus on equation [7a] as the essential basis for distinguishing between agents of different types because revisions to the SOLE are the key "driver" of aggregate fluctuations in our model.

$$g^i_{jt} > 0 \;\Rightarrow\; g^a_{jt} = \min[g^i_{jt}, g^i_{\max}]$$

$$g^i_{jt} < 0 \;\Rightarrow\; g^a_{jt} = \max[g^i_{jt}, g^i_{\min}]$$

where g^a_{jt} denotes the rate of growth that is actually used in the calculation of u_{jt}. In order to ensure that our simulations are consistent with the logical bounds on u, we therefore add to our model the equation:

$$g^a_{jt} = \max\left[0, \min\left(g^i_{jt}, \frac{s_\pi \pi}{v}\right)\right] \qquad [10]$$

and replace [9a] with:

$$u_{jt} = \frac{v}{s_\pi \pi} g^a_{jt} \qquad [11]$$

so that outcomes in our model are now described by the recursive interaction of equations [7a], [8a], [10], and [11].

iii) Setting parameter values and initial conditions

In order to proceed, we need to establish the values of the parameters in equations [8a], [10] and [11], set the initial values of certain variables, and operationalize equation [7a].

a) Setting parameter values

Referring first to equations [8a] and [9a], and drawing on Lavoie and Godley (2001-02) and Skott and Ryoo (2007), we set:[10]

[10] The values taken from Lavoie and Godley (2001-02) are not reported in the article itself, but were provided in a private correspondence. Note that the value of g_r actually set by both Lavoie and Godley (2001-02) and Skott and Ryoo (2007) is 0.5. We have adjusted this parameter value very slightly to

$g_r = 0.49 \qquad g_u = 0.025$

We also set:

$\pi = 0.33 \qquad v = 3.0$

which, together with their implications for the rate of profits, are broadly congruent with the stylized facts of long run growth, as originally identified by Kaldor (1961).

This leaves us with the parameter s_π. Lavoie and Godley (2001-02) set the corporate retention rate at 0.75, and (on p.291) the household saving rate (regardless of the form of household income) at 0.2. Total saving out of profit income, S, is therefore given by the sum of corporate retained earnings and household saving out of distributed earnings, or in other words:

$$S = 0.75\Pi + (0.2)(0.25\Pi)$$

where Π denotes total profits. The propensity to save out of profits $s_\pi = S/\Pi$ is therefore given by:

$$s_\pi = 0.75 + 0.25(0.2) = 0.8$$

b) Initial conditions

Note that in the event that we replace equation [7] with:

$$\alpha = \bar{\alpha} \qquad [7b]$$

equations [1]—[6] can be solved for the steady-state rates of growth and capacity utilization:

$$g^* = \frac{s_\pi \pi \bar{\alpha}}{\pi(s_\pi - g_r) - g_u v} \qquad [12]$$

somewhat better calibrate our model (which is different from theirs) to the stylised facts of growth and capacity utilization.

$$u^* = \frac{v\bar{\alpha}}{\pi(s_\pi - g_r) - g_u v} \qquad [13]$$

Skott and Ryoo (2007) set $\bar{\alpha} = 0.0075$. Using this parameter value, together with those noted earlier, we can numerically evaluate equations [12] and [13] to get:

$$g^* = 0.0725$$

$$u^* = 0.8242$$

The computed value of u^* reported above can now be used as a reference point for setting the initial values of u and u_j that we require for our simulation exercise. Hence we set:

$$u_{jt-1} = u_{t-1} = u^* = 0.8242$$

and:

$$u_{jt-2} = u_{t-2} = u_{t-1} - \sigma_u^2 = 0.6857$$

where $\sigma_u^2 = 0.1385$ is the variance of u calculated from US capacity utilization data.[11]

c) <u>Operationalizing equation [7a]</u>

As intimated above, equation [7a] is rendered explicit by Table 2, with:

$$c_j = \beta_j \sigma_u^2 = 0.1385\beta_j$$

Consistent with our setting $u_{jt-1} = u_{t-1} = u^* = 0.8242$, we set $\alpha_{jt-1} = \bar{\alpha} = 0.0075$ (which is the value of α consistent with our computed steady state value of u). The variables ε_{jt} are set as random draws from a normal distribution with mean $\mu_\varepsilon = 0.0015$ and variance

[11] As will become clear in the discussion of operationalizing equation [7a] below, this will ensure that $u_{jt-1} - u_{jt-2} = u_{t-1} - u_{t-2} = \sigma_u^2 \geq c_j \ \forall j$ initially.

We used monthly data on total industry capacity utilization in the US 1967—2007 taken from the Board of Governors of the Federal Reserve System to compute the variance of u reported above.

$\sigma_{\varepsilon}^{2} = 0.0005$, moments that have been chosen in accordance with the magnitude of the parameter $\bar{\alpha}$. Note the system closure implicit in this formulation – for the sake of simplicity, both the mean and the variance of $\varepsilon_j \ \forall j$ are treated as time-invariant, unlike their original formulation in Setterfield (2003). Finally, we choose the values of β_j, k_j and κ_j to distinguish between the different types of firms described earlier, as follows:

- $\beta_j = 0.5$ and $k_j = 1$ denotes "aggressive adapters" – firms with a greater inclination to be encouraged/discouraged by short-term results, and a shorter reaction period.

- $\beta_j = 1$ and $k_j = 3$ denotes "cautious adapters" – firms that are less inclined to be encouraged/discouraged by short-term results, and that have longer reaction periods.

- $\kappa_j = 0.9$ denotes firms who consider themselves less affected by macroeconomic events, and thus attach less weight to aggregate economic outcomes when revising their SOLEs.

- $\kappa_j = 0.5$ denotes firms who consider themselves more affected by macroeconomic events, and thus attach more weight to aggregate economic outcomes when revising their SOLEs.

Ultimately, then, our model distinguishes between four different types or classes of firms, as follows:[12]

$j = 1, ..., 25$: $k_j = 1, \beta_j = 0.5, \kappa_j = 0.9$

$j = 26, ..., 50$: $k_j = 1, \beta_j = 0.5, \kappa_j = 0.5$

$j = 51, ..., 75$: $k_j = 3, \beta_j = 1, \kappa_j = 0.9$

[12] Note that, with reference to the calculations in Table 2, for $j = 51, ..., 100$ (i.e., firms for which $k_j = 3$) we set $\alpha_{jt} = \alpha_{jt-1}$ for: (i) any value of t that is not a multiple of 3; *and* (ii) any value of t that *is* a multiple of 3, but for which none of the conditions of expectational disappointment in Table 2 are fully satisfied. The latter is necessary to prevent a behavioural "black hole" during early iterations of the model, given that we have only specified the values of $u_{jt-1} = u_{t-1} = 0.8242$ and $u_{jt-2} = u_{t-2} = 0.6857$ in the process of specifying initial conditions.

$j = 76, ..., 100$: $k_j = 3, \beta_j = 1, \kappa_j = 0.5$

Recall that even within these types or classes of firms, the value of ε_{jt} will vary between individual firms. Hence our model ultimately features a population of one hundred different firms, the dynamics of our model depending on the heterogeneous behavioural responses of these firms to disappointed expectations.

iv) Determination of aggregate outcomes

Simulating equations [7a], [8a], [10], and [11] will produce one hundred different values of g^i_{jt} and u_{jt} (one for each firm) at the end of each period. But of course our interest is ultimately in g^i_t and u_t – and in fact, we *need* to know the latter in order make the calculations described in Table 2 and thus continue with the next iteration of our simulation. As such, we proceed to calculate the aggregates g^i_t and u_t as follows. We begin by assuming that all firms initially have the same capital stock, which we normalize so that $K_j = 1 \ \forall j$ initially. Then for any subsequent period t:

$$K_t = \sum_{j=1}^{100} (1 + g^i_{jt}) K_{jt-1} \qquad [14]$$

and:

$$g^i_t = \frac{K_t - K_{t-1}}{K_{t-1}} \qquad [15]$$

Finally, the value of u_t can then be calculated from equation [9].

v) Summary

Our simulations proceed as follows. Given the initial conditions and parameter values outlined above, every k_j periods we establish the value of ε_{jt} for each individual firm and, using α_{jt-1}, calculate α_{jt} in accordance with the criteria in Table 2. Next, we numerically evaluate equations [8a], [10] and [11] to produce growth and utilization rates for each of our individual firms. Finally, we numerically evaluate equations [14], [15] and [9] to produce the growth and capacity utilization rates for the aggregate economy. The simulation then moves forward one period and the process described above starts again.

Before discussing our simulation results, it is worth drawing attention to one final feature of our model: the nature of agent interaction. Agent-based simulations are typically dependent on the notion of locality. That is, one agent must be within a certain proximity of another agent in order for the two agents to interact. This notion of locality is usually conceptualized in terms of a grid of cells. Our model, however, does not depend on proximity to facilitate the interaction of agents. Instead, each firm engages in its own individual decision making process, through which it revises its SOLE for the next period (or set of periods) based on its own past performance and the performance of the aggregate economy. It is each firm's reference to the latter (in the form of the aggregate rate of capacity utilization, and as a result of $\kappa_j \neq 1 \; \forall j$ in Table 2) that causes individual agents to interact with one another in our model. Put differently, instead of the "direct" interaction between individual agents typical of ACE models, our model exhibits "indirect" agent interaction, resulting from the sensitivity of individual firm behaviour to aggregate economic outcomes that are a product of the actions of *all* agents.

3. Results and discussion

Our simulation was implemented using the open source Repast (Recursive Porous Agent Simulation Toolkit) toolkit, developed at the University of Chicago. The version of Repast that we used was written in the Java programming language. More information about Repast is available online at http://repast.sourceforge.net.

i) Aggregate outcomes

Figures 1 and 2 illustrate the aggregate rates of growth and capacity utilization from a representative run of our model. After about 50 periods, the behaviour of the model stabilizes, the economy experiencing aggregate fluctuations about average rates of growth and capacity utilization of 7.5% and 83.4 %, respectively.[13] This is the behaviour anticipated by Setterfield (2003, 327—31). Recall that there are no (fixed) equilibrium rates of growth or capacity utilization towards which the economy is automatically drawn (or that it is compelled to orbit). Instead, the behaviour of the economy in Figures 1 and 2 bears out Keynes's (1936) claim that even in the absence of such "anchors", a capitalist economy in which expectations are formed under conditions of fundamental uncertainty is likely to fluctuate for long periods of time at levels of economic activity that are below potential, but without the system ever collapsing completely. Put differently, rather than displaying classical stability, the economy displays *resilience* (Holling, 1973).[14]

[FIGURES 1 & 2 GO HERE]

[13] The latter is close to the average rate of capacity utilization in the US over the past 60 years (82.4%).
[14] The concept of resilience focuses on the *durability* of a system and hence its capacity for longevity. The key question posed by this concept is: can the system under scrutiny reproduce itself in a sufficiently orderly manner to ensure that it persists over time?

The fluctuations depicted in Figures 1 and 2 are aperiodic and of no fixed amplitude, but certain regularities are, nonetheless, evident from these Figures. First, they show booms generally lasting considerably longer than recessions. Second, the longest peak-peak cycle depicted in Figures 1 and 2 lasts for about 25 periods – which can be interpreted, in calendar time, as an interval of about 12 years.[15] These features of the aggregate fluctuations in Figures 1 and 2 are broadly in keeping with those of the US business cycle.

ii) Firm-specific outcomes and the size distribution of firms

The aggregate regularities noted above are, however, not typical of the experience of all individual firms. Figure 3, which shows the total number of idle firms, provides the first indication of this. Figure 3 draws attention to an important feature of our model. Although it does not formally involve firm exit, the model does provide for the possibility of "pseudo exit" in the sense that firms can become idle (their rate of capacity utilization falling to zero) at any point in time. By the same token, although the model does not formally involve firm entry, it provides for "pseudo entry", since the SOLE reaction function in Table 2 allows for the possibility of currently idle firms becoming economically active again in the future. In this way, although the population of firms in our model is fixed, the ability of firms to transition into and out of a state of economic activity provides for pseudo entry and exit. And as is illustrated in Figure 3, this type of behaviour is actually observed over the course of our simulations.

[FIGURE 3 GOES HERE]

[15] This interpretation is based on the observations that: the capital stock expands/contracts in our model between periods; the capital stock is usually assumed to be constant in the short run; and the short run is conventionally conceived as a period of about 6—9 months.

Indeed, Figure 3 shows an *increasing* number of firms becoming inactive over time, providing *prima facie* evidence that the aggregate economy is becoming dominated by an ever smaller number of firms over time.[16] This is borne out by Figures 4—7, which illustrate the size distribution of firms (as measured by the quantity of capital that firms own) at various points during our representative simulation.[17] The distributions in Figures 4—7 are suggestive of power laws of the form:

$$p(x) \sim x^{-\beta} \qquad [16]$$

where x denotes the size of the capital stock owned by firms. Power laws (and in particular, the Pareto distribution) are thought to characterize numerous size distributions in economics (Reed, 2001).[18] They are empirically well established as features of the size distribution of firms (Steindl, 1965; Ijiri and Simon, 1977) and the size distribution of wealth (Pareto, 1897) – both of which are effectively being represented in Figures 4—7.

[FIGURES 4—7 GOE HERE]

In order to subject the power law hypothesis to further scrutiny, we first estimate the scaling parameter β in equation [16] for the size distribution of firms in each period of our representative simulation, using the maximum likelihood technique outlined by Clauset et al (2007, pp.4-6).[19] We then determine the goodness of fit of our estimated power law to the original data by computing the Kolmogorov-Smirnov (KS) statistic:

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[16] Note that, although economically inactive firms retain their capital (which does not depreciate), their inactivity means that their (constant) stock of wealth will become progressively smaller relative to the capital stock of the economy as a whole.

[17] In order to construct the size distributions in Figures 4-7, several functions were written to automatically "bin" all of the firms from each period based on the relative size of their capital stocks and the maximum permitted number of bins. The maximum bin number was set to 12 for the execution of this analysis.

[18] The Pareto distribution is sometimes referred to as the "Pareto principle" or the "80-20 rule" (according to which 20% of the population owns 80% of society's wealth).

[19] The actual relationship estimated is $p(x) = Bx^{-\beta}$ where B is a constant. The power law analysis was executed using the plfit.r library, which was written by Aaron Clauset of the Santa Fe Institute and

$$D = \max_{x \geq x_{\min}} \left| S(x) - P(x) \right|$$

where $S(x)$ is the cumulative distribution function (CDF) of the data for all observations that satisfy $x \geq x_{\min}$, $P(x)$ is the CDF of our estimated power law for $x \geq x_{\min}$, and $x_{\min}$ is the lower bound of the estimated power law (Clauset et al, 2007, pp.8, 11). The KS statistic measures the maximum distance between the CDFs of the data and our estimated power law relationship – so the higher is D, the worse is the goodness of fit of the power law. Bearing this in mind, the KS statistics for each of the 250-plus periods of our representative simulation are illustrated in Figure 8.

[FIGURE 8 GOES HERE]

Excluding the first few periods, the values of the KS statistics reported in Figure 8 appear uniformly low throughout our representative simulation. This lends support to the claim that the size distribution of firms generated by our model conforms to a power law. Of course, this is something of a value judgment: there is no established critical value of the KS statistic above which it is conventional to reject the hypothesis that the power law is a good fit to the data. It is possible to calculate a p-value to quantify the probability that a data set was drawn from a particular (estimated) power law distribution. As explained by Clauset et al (2007, pp.11-12), this involves a Monte Carlo procedure in which we would need to generate $1/4\varepsilon^2$ synthetic data sets, where ε is the difference between the estimated p-value and its true value that we are willing tolerate. While this is well within the possibilities of modern High Performance Computing, it still leaves us with the problem of subjectively choosing a critical p-value that we deem sufficiently small to

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University of New Mexico. This library, and more information about it, is available at http://www.santafe.edu/~aaronc/powerlaws/.

reject the hypothesis of a power law. Moreover, it is important to bear in mind that the evidence that real-world size distributions conform to power laws is not incontrovertible. For example, Clauset et al (2007, pp.16-20) *reject* the hypothesis that the size distribution of wealth (specifically, the aggregate net worth of the richest individuals in the US in 2003) conforms to a power law. It seems, then, that the best we will ever be able to say is that there is some evidence that the size distribution of firms generated by our model conforms to a power law, just as there is some evidence that this same size distribution conforms to a power law in real-world data.

Nevertheless, even this tentative result is interesting in the context of this paper. The cyclical behaviour of the growth and utilization rates discussed in the previous sub-section is more or less predictable based on the underlying structure of our model (see, for example, Setterfield, 2003, pp.327—31). In this instance, the process of simulation serves to better illustrate a property of the model that is already understood to (potentially) exist. However, nothing in the structure of our model pre-empts or in any way suggests that we are likely to observe a size distribution of firms that conforms to a power law. This feature of our model – which also appears to be a feature of real-world size distributions of firms – emerges spontaneously from our simulation results.

One final feature of Figure 8 that merits discussion is the apparent tendency of the value of the KS statistic to drift upwards over time. Interpreted literally, this suggests that the goodness of fit of the power law declines as our simulation progresses. However, there may be a simple explanation for this. The increasing value of the KS statistic may be explained by the decreasing number of "bins" into which firms are sorted as our simulation progresses. In order to properly estimate a power law, there can be no empty

(0 sized) bins in the histograms in Figures 4-7. It is therefore necessary to choose the largest number of bins that will result in each of the individual bins containing at least one firm. However, as the number of small firms grows, and the gap between the very large firms and the very small firms becomes larger, it is necessary to use fewer and fewer ever larger bins to prevent the emergence of empty bins.[20] This reduces the number of data points that we have, resulting in a poorer quality fit for the power law reflected in a higher value of *D* in Figure 8. If this explanation is correct, it provides us with a compelling reason to use many more firms in future simulations, in order to improve the spectrum or breadth of our data and thus increase the accuracy of our analysis of the size distribution of firms.

4. Conclusion

The purpose of this paper has been to construct and simulate a Keynes-Kalecki model of cyclical growth with agent-based features. Based on the propensity for decision makers confronted by fundamental uncertainty to revise their "state of long run expectations" in response to short-run events, it has been shown that the economy can experience aggregate fluctuations in its rate of growth that are aperiodic and of no fixed amplitude. While this observation merely corroborates and better illustrates the results of an earlier study based on a similar model, the incorporation of agent heterogeneity into our model allows us to also explore other features of the economy – most notably, the size distribution of firms. We have shown that there is evidence to suggest that the size distribution of firms produced by our simulation model – like the size distribution of firms in real-world economies – conforms to a power law. Unlike the observation of

[20] This is evident from inspection of Figures 4-7.

cyclical growth, this outcome is not at all obvious from the basic construction of our model, and might instead be considered an emergent property of its operation.

Perhaps the most interesting feature of our model, however, is methodological. Markose et al (2007, p.1803) list four prominent features of the "ACE revolution" in economics, two of which ("heterogeneous (instead of homogenous) decision processes as a characteristic of socio-economic systems and the statistical non-Gaussian properties of their macro-level outcomes; [and] adaptive and evolutionary dynamics under limited information and rationality") are exhibited by the model developed above. And yet ours is not an ACE model *per se*, but rather an aggregate structural model with "agent based features": it involves disaggregating a structural model rather than the "bottom up" approach characteristic of ACE; and it involves indirect interaction (which does not depend on locality) rather than locality-dependent direct interaction amongst heterogeneous agents. The methodological question that these observations prompt is: are aggregate structural models with agent-based features a potentially useful but relatively under-exploited frontier of the increases in computing power that have facilitated the development of ACE? Our tentative answer to this question is affirmative. First, the results presented in this paper suggest that exploitation of this frontier offers obvious advantages for aggregate structural modellers – namely, it presents the opportunity to generate results (regarding the size distribution of firms, for example) that conventional aggregate structural models cannot, by their very nature, produce. Second, exploitation of the same frontier may well be advantageous to the development of ACE. This claim stems from observations such as that of Tesfatsion (2006, p.??), that "it is not clear how well ACE models will be able to scale up to provide empirically and practically useful models of large scale systems with many thousands of agents".[21] The point to be made here is that the approach taken in this paper – which clearly *does* yield recognizable macroeconomic results – may represent a useful compromise between aggregate structural modelling and "bottom up" ACE modelling, either at this particular stage in the development of the latter or possibly even in the long term.

[21] Similar reservations have been expressed by Hartley (2001) who, in his review of Gallegati and Kirman (1999), questions "whence comes our certainty that it is possible to build tractable models of the macroeconomy from the ground up? Maybe the real lesson of the book is that it may not be possible to build such models, that we can certainly build better microeconomic models than those used in the representative agent literature, but that such models do not directly translate into macroeconomics" (Hartley, 2001, pp.F146-7).

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Table 1: Revisions to the state of long run expectations in response to disappointed expectations.

Nature of Disappointment	Value of α_t
$u_{t-1} - u_{t-2} \geq c$	$\alpha_t = \alpha_{t-1} + \varepsilon_t$
$u_{t-1} - u_{t-2} > -c$ And $u_{t-2} - u_{t-3} \leq -c$	
$u_{t-1} - u_{t-2} \leq -c$	$\alpha_t = \alpha_{t-1} - \varepsilon_t$
$u_{t-1} - u_{t-2} < c$ And $u_{t-2} - u_{t-3} \geq c$	

Table 2: Agent-based revisions to the state of long run expectations in response to disappointed expectations.

<table>
<tr><th>Nature of Disappointment</th><th>Value of α_{jt}</th></tr>
<tr><td>$\frac{\kappa_j}{k_j}\sum_{i=1}^{k_j}(u_{j,t-i} - u_{j,t-1-i}) + \frac{(1-\kappa_j)}{k_j}\sum_{i=1}^{k_j}(u_{t-i} - u_{t-1-i}) \geq c_j$</td><td rowspan="2">$\alpha_{jt} = \alpha_{jt-1} + \varepsilon_{jt}$</td></tr>
<tr><td>$\frac{\kappa_j}{k_j}\sum_{i=1}^{k_j}(u_{j,t-i} - u_{j,t-1-i}) + \frac{(1-\kappa_j)}{k_j}\sum_{i=1}^{k_j}(u_{t-i} - u_{t-1-i}) > -c_j$
And $\frac{\kappa_j}{k_j}\sum_{i=1}^{k_j}(u_{j,t-k_j-i} - u_{j,t-k_j-1-i}) + \frac{(1-\kappa_j)}{k_j}\sum_{i=1}^{k_j}(u_{t-k_j-i} - u_{t-k_j-1-i}) \leq -c_j$</td></tr>
<tr><td>$\frac{\kappa_j}{k_j}\sum_{i=1}^{k_j}(u_{j,t-i} - u_{j,t-1-i}) + \frac{(1-\kappa_j)}{k_j}\sum_{i=1}^{k_j}(u_{t-i} - u_{t-1-i}) \leq -c_j$</td><td rowspan="2">$\alpha_{jt} = \alpha_{jt-1} - \varepsilon_{jt}$</td></tr>
<tr><td>$\frac{\kappa_j}{k_j}\sum_{i=1}^{k_j}(u_{j,t-i} - u_{j,t-1-i}) + \frac{(1-\kappa_j)}{k_j}\sum_{i=1}^{k_j}(u_{t-i} - u_{t-1-i}) < c_j$
And $\frac{\kappa_j}{k_j}\sum_{i=1}^{k_j}(u_{j,t-k_j-i} - u_{j,t-k_j-1-i}) + \frac{(1-\kappa_j)}{k_j}\sum_{i=1}^{k_j}(u_{t-k_j-i} - u_{t-k_j-1-i}) \geq c_j$</td></tr>
</table>

Figure 1: The Aggregate Rate of Growth

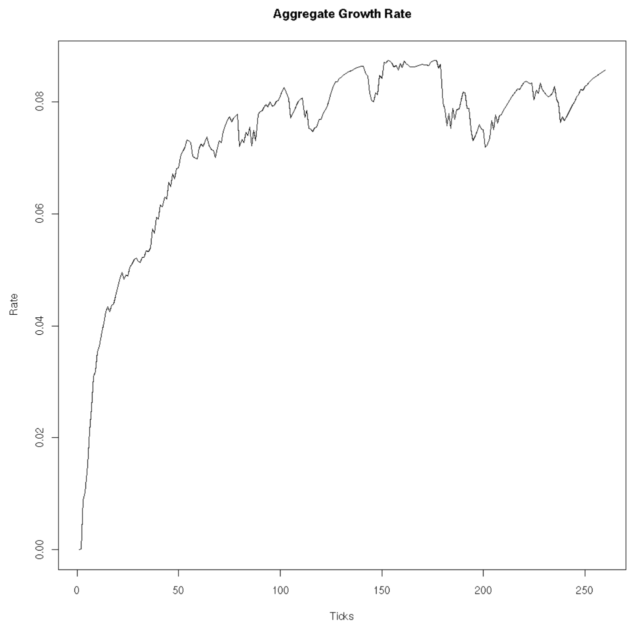

Figure 2: The Aggregate Rate of Capacity Utilization

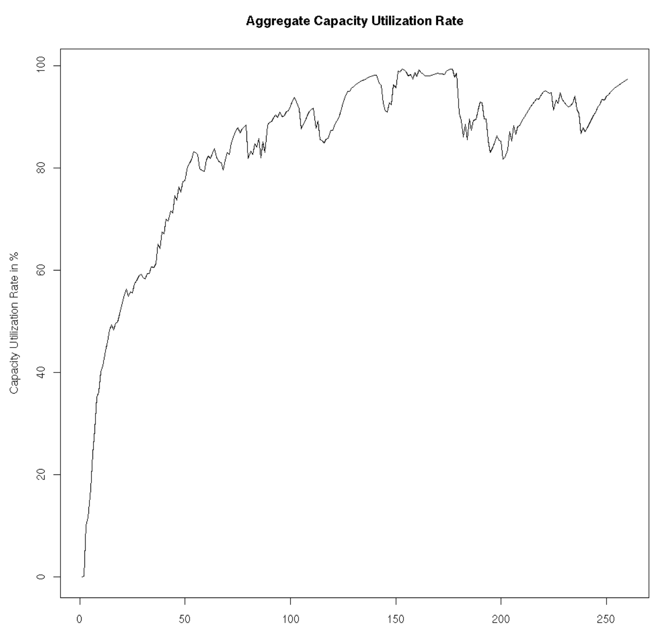

Figure 3: Number of Idle Firms in the Economy

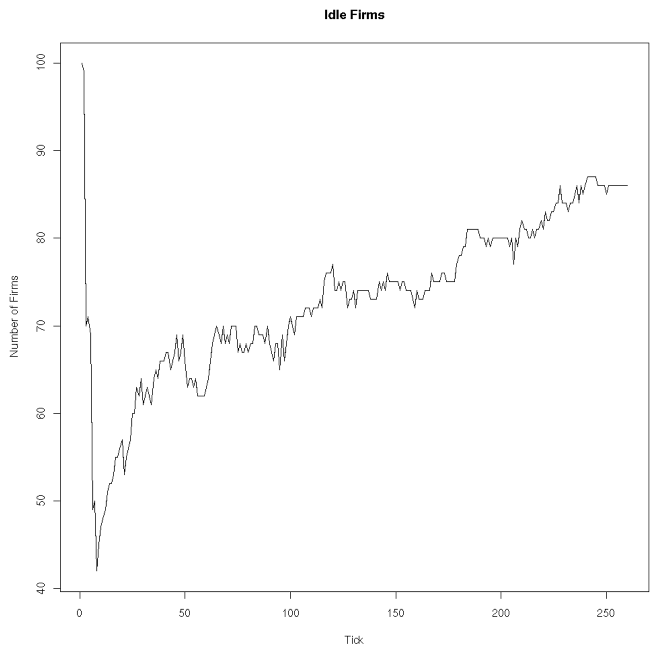

Figure 4: Size Distribution of Firms in Period 18

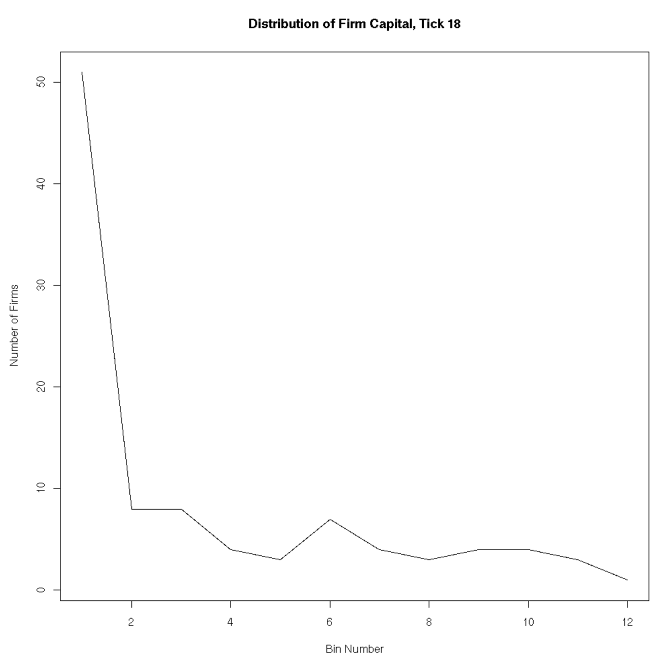

Figure 5: Size Distribution of Firms in Period 75

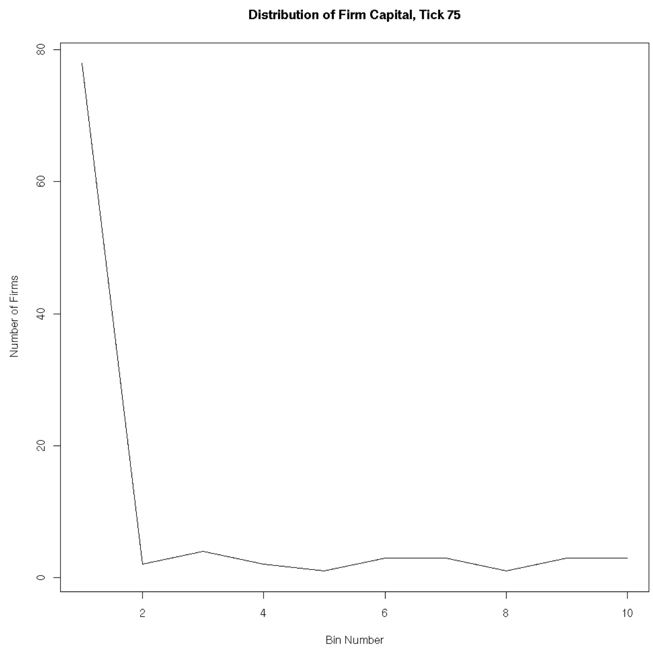

Figure 6: Size Distribution of Firms in Period 150

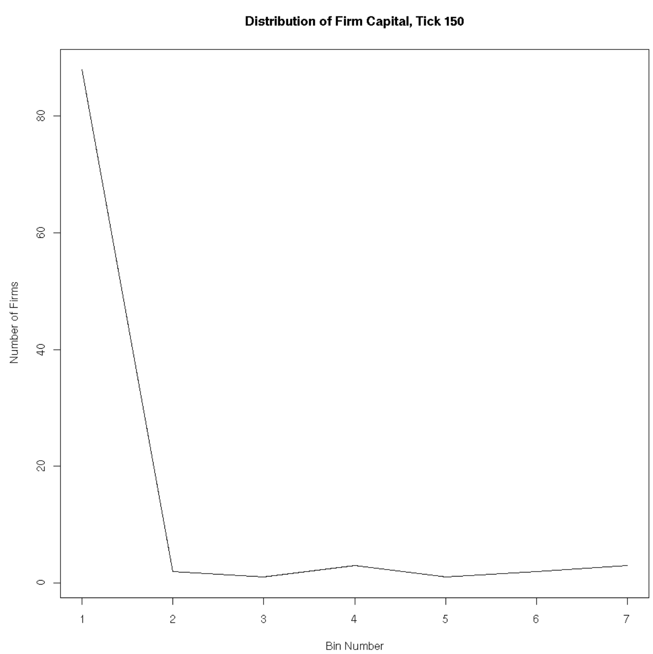

Figure 7: Size Distribution of Firms in Period 225

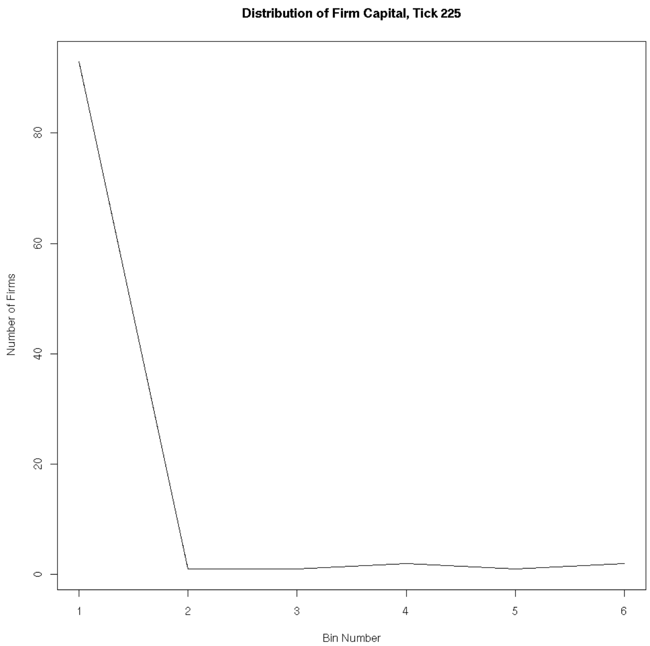

Figure 8: Goodness of Fit of Estimated Power Laws Over Time