# Growth, Income Distribution and Public Debt A Post Keynesian Approach

João Basilio Pereima Neto<sup>\*</sup> José Luis Oreiro<sup>\*\*</sup>

Abstract: The objective of this paper is to evaluate the long-run impact of changes in fiscal policy and income distribution over the degree of capacity utilization, within the context of a high public debt so that risk premium, paid by the government to the capitalists/income earners, is an increasing function of the level of public debt. In order to do that, a macro dynamic model of Post Keynesian inspiration is presented, whereby (i) capitalists obtain income from either the profits achieved from the productive use of existing capital stock or from interest derived from the acquisition of government bonds; and (ii) the rate of return (required) in government bonds is an increasing function of the degree of indebtedness of public sector. The model shows that the economy does not follow a linear response to the fiscal shocks in the sense that an expansionary fiscal policy could have positive or negative effects over the level of capacity utilization. It is also shows that the effect of changes in distribution of income over the level capacity utilization is dependent on the level of public debt. If the economy is operating with a high level of public debt, an increase in profit share will result in an increase in the degree of capacity utilization, thus defining a profit-led accumulation regime in the long run.

Key-words: Economic Growth, Income Distribution, Fiscal Policy, Public Debt, Risk Premium.

**JEL Classification**: E12

## July, 2008

<sup>\*</sup> Assistant Professor of Economics at Federal University of Parana (UFPR), Curitiba, Brazil. E-mail: joaobasilio@ufpr.br.

<sup>\*\*</sup> Associate Professor of Economic at University of Brasilia (UNB), Brazil. E-mail: <u>jlcoreiro@terra.com.br</u>. Web-page: <u>www.onkx.com/oreiro</u>.

## **1** Introduction

This paper presents a dynamic model of economic growth, in which the degree of indebtedness of the public sector plays an important role in determining the growth dynamics, as it can modify the nature of accumulation regime. The economic theory has traditionally emphasized the pro-cyclical role of fiscal deficits, established through mechanisms such as the Keynesian multiplier, even in the presence of some crowding out effects that may attenuate the impacts of an expansionary fiscal policy.

The debate surrounding the crowding out effects between monetarists and Keynesians, which occurred during the 1970s, led to divergent conclusions regarding the effects of an expansionary fiscal policy that is financed by the emission of public debt and the effects it has on the level of employment in the short and in the long-run. Since it is widely understood, that the monetarist position, defended especially by Friedman (1972), suggested that a fiscal expansion financed by the emission of public bonds would generate a so strong wealth effect in demand of money, increasing interest rates, so that aggregate demand and, consequently, the level of real output, would remain more or less constant, resulting in the thesis of an inefficacy in the fiscal policy in the long-term. The Keynesian position - defended by Blinder and Solow (1973) - was that wealth effects are present not only in the LM curve, but also in the IS curve and that the latter are stronger than the in such a way that the final result of a fiscal expansion would produce an increase in the aggregate demand and hence the level of real output.

The issues regarding the long-run effects of public debt over the real economy was recently revisited by You and Dutt (1996) in the context of a Post Keynesian model of growth and income distribution. The authors concluded that an increase in the level of public debt (as a ratio to capital stock) in the long-term will cause an increase in the rate of growth of real output and, at the same time, a decrease in the wage share. However, the model ignores the sensibility of investment to interest rate, since this variable is exogenously determined, so that an important crowding out mechanism is not taken into account in this model. The hypothesis regarding exogeneity in the interest rate implies that the money supply is completely horizontal which suggests an acceptance of the horizontalist theory of demand for money, based on Kaldor and Moore (1988).

However, the exogeneity of the interest rate does not seem to be a reasonable hypothesis, especially in the context of monetary economies such as the modern capitalist economies, since they possesses more than two assets for which private wealth can be accumulated. In fact, the issuing of bonds that provide interest payments by the private and public sector, used to finance their consumption and investment expenditures, rules out the equality between interest rate and

profit rate, an assumption usually made in context of models of balanced growth<sup>2</sup>. Given the existence of financial assets, it is very likely that the behavior of the interest rate within the economy becomes influenced not only by the money demand and supply, as it is traditionally supposed, but also by the subjective evaluations about the risk of capital or income losses of these assets. Kalecki (1971) had already drawn attention to a theory of interest rate that would take into account the so called principle of increased financial risk, which is nowadays fully considered by the literature on corporate finance (Brealey and Myers, 1996). This principle establishes that the investors and creditors demand a higher interest rate as the degrees of leverage increases.

In the model developed below, the government will be the only agent that issues debt bonds, making it possible to analyze not only the macroeconomic dynamics that results from public debt, but also the effectiveness of fiscal policies.

The inclusion of these elements in a model of growth and distribution of income shows that the economy can present differentiated regimes of accumulation depending on the degree of leverage or indebtedness in which it is operating. This introduces the possibility, which has not been considered by the Post Keynesian models of growth, such that the long-run effects on the degree of indebtedness depend on the level of public debt. In fact, You and Dutt (1996) reaches the conclusion that the degree of capacity utilization in short-term and the rate of growth in the longterm are *always* positively correlated with the level of public debt (as a ratio to capital stock).

However, recent studies<sup>3</sup> have demonstrated that the macroeconomic dynamics does not respond in a linear way to expansionary and contractionary fiscal policy. Many of the works have found empirical evidence of nonlinear responses in the economy to the fiscal measures that affect the relation between debt and GNP. They also found that, amongst the various channels, the effect it had on the expected interest rates and the effect of income and wealth on consumption, together with the investment and savings made by the agents were extremely important.

In the following, we will present a model of growth and income distribution that includes these nonlinear mechanisms. In section 2, the basic structure for the theoretical model is presented. Section 3 is devoted to the attainment of the short-run equilibrium of the model. Section 4 presents the long-run equilibrium and the analysis of stability. Section 5 presents the long-run effects of change on the fiscal policy, while section 6 analyses the long-run effects of changes in the functional distribution of income based over the degree of capacity utilization. Finally, section 7 summarizes the paper's conclusions.

 $<sup>^{2}</sup>$  This principle is found, for example, in the reduced version of the Cambridge equation attributed to Pasinetti (1974), a result obtained by the hypothesis that there is only one asset in the economy, which is physical capital.

<sup>&</sup>lt;sup>3</sup> Regarding this, see Giavazzi and Pagano (1990), Alesina and Perotti (1995,1997), IMF (1996), McDemott and Wescott (1996), Alesina and Ardagna (1998), Alesina *et al.* (1999) and Blanchard and Perotti (1999).

## 2. The model structure

Let us consider a closed economy, which has government and is a mono-producer. Without an external sector, the economy produces only one product, which is allocated for both consumption and investment. There are only two factors of production, capital (K) and labour (L), which are combined in fixed ratios, in a sense that the production function, in the absence of technological progress, can be expressed by a Leontieff type in the form:

$$X = \min\left[K, L/a\right] \tag{1}$$

Where: X is the production and income level and a is the unitary requirement of labour. In this case, the amount of labor employed is a direct function of the production level and can be expressed by the equation:

$$L = aX \tag{2}.$$

The total income (X) generated during the productive process is distributed between wages and profits such as specified in the equation (3):

$$X = \frac{W}{P}L + rK \tag{3}$$

Where: W/P represents the real wage and *r* is the rate of return on capital.

Following the tradition of a classical political economy and Marx, the interest rate is not considered as part of the income generated during the productive process. In the model presented here, the hypothesis is easily justified by the fact that the interest rate is a simple transference of tax resources (current and future) from government to the owners of public bonds (the capitalists), not being part of the "added value" generated by the economy in a certain period of time.

Dividing (3) for *K* and defining the real wage as  $V = W/P^4$ , the degree of productive capacity utilization as u = X/K, the profit share as  $m = rK/X^5$ , we can then express (3) as follows:

<sup>&</sup>lt;sup>4</sup> And by the definition (2), L = a.X

<sup>&</sup>lt;sup>5</sup> As K/X is the inverse of the degree of use of the productive capacity, it is possible either to express the participation of the profits in the income as m = r/u or to define de profit rate as r = mu.

Where: isolating m (participation of the profits in the income) and Va (participation of the wages in the income), we obtain by income side a result as follows:

$$m = 1 - Va \tag{3b}$$

$$Va = 1 - m \tag{3c}$$

Furthermore, we can assume that the real wage V is determined by the subsistence level of the work force, so that it can be considered an exogenous variable of the model and represented by  $\overline{V}$ . Since the economy under scrutiny is deprived of technological progress, the parameter *a* can also be considered as a constant. Thus, the participation of profits (and of the wages) in the income is determined by the following equation:

$$m = 1 - \overline{Va} \tag{3d}.$$

With regards to the expenditures, the income of the economy is distributed between consumption (C), investment (I) and public expenses (G):

$$X = C + I + G \tag{4}$$

The total consumption is determined by the consumption of workers and capitalists. Following Kalecki (1954), Kaldor (1956) and Robinson (1956, 1962), we assume that the workers spend all their income on consumption; while the capitalists save a constant fraction of their income, which is obtained either in the form of profits over the stock of existing capital or interests on bonds that they own. Finally, the government charges a tax  $\tau$  only on the income that is obtained through profits and interests. The wages are exempt from taxation. Thus, the consumption function is given by:

$$C = VaX + (1 - s_c)(1 - \tau)(rK + i\frac{D}{P})$$
(5)<sup>6</sup>

<sup>&</sup>lt;sup>6</sup> Despite the fact that the interests are a simple transference of current and future tax revenues from the government to the capitalists, the capitalists consider the income obtained from the interests as part of their available income for finance their consumption expenses. This is the case because we assume that the government is a typical *Ponzi* agent; so part of the government expenses with the payment of interest is funding with the emission of a new debt by the government. Thus, part of the interest paid to the capitalists today is, indeed, funded by the taxes that will be charged from the capitalists on the future. Assuming that the current generation of capitalists does not care about the well-being of the future generations and/or is not able to foresee the moment at which the government will have to increase the taxes for the payment of its debt; it follows that the income that they obtain in the form of interests will be considered as

where:  $\tau$  is the tax rate,  $s_c$  is the propensity to save by the capitalists, r is the rate of profit on capital, i is the nominal interest rate and D is the nominal stock of debt and and P is the level of prices.

The specification for the investment function follows the conclusions by Steindl (1952), Spence (1977) and Cowling (1982), and hence we assume that the decision regarding investment by the firms depends, amongst other things, on the rate of productive capacity utilization, due to the strategy of creating barriers for potential new entrants. Thus, in an oligopoly market, the firms keep a surplus capacity level as a means of reacting rapidly to demand changes, which prevents stimulating the entrance of new firms.

Besides that, investment in fixed capital is also negatively correlated to the real interest rate, such as in Keynes (1936), and therefore the higher the interest rate, the lower the amount of investment in the economy. Thus, the investment function assumes the following form:

$$I = I_{\alpha} + \beta X - \phi(i - \pi) K$$
(6).

Dividing it by K, we obtain the equation for the growth rate of capital:

$$g = I/K = \alpha + \beta u - \phi(i - \pi)$$
(6a)

Where: g is the growth rate of capital,  $\alpha$  is the autonomous investment as a ratio of the capital stock,  $\beta$  is a parameter that measures the sensitivity of investment to the productive capacity utilization,  $\phi$  measures the sensitivity of investment to the real interest rate, *i* is the nominal interest rate and  $\pi$  is the inflation rate.

Differing from previous works in the Post Keynesian tradition<sup>7</sup>, we choose to rescue the role played by the interest rate as an explicative variable of investment in our model. In fact, many models in the Post Keynesian tradition have assumed an exogenous and constant interest rate. This has allowed them to introduce a profit rate as an explicative variable based on the behavior determined by the decisions of firm's investments. A recent example of this approach is given by

part of their available income, affecting then their expenses of consumption. As a corollary of this argument, it follows that the form of financing the government expenses has an effect on the decisions about expenses of the economic agents, thus the Ricardian equivalence is not valid in the model under consideration.

<sup>&</sup>lt;sup>7</sup> On that, we differ from other ways to represent the investment function. According to Robinson (1956, 1962), Kalecki (1971), Rowthorn (1981) and Dutt (1984, 1990), the investment positively depends on the profit rate. For Bhaduri and Marglin (1990), the investment monotonically depends on profit share in income. And, more recently, Lima (1998) makes the investment to depend not linearly, but in a quadratic form, on the wage participation in the income.

You and Dutt (1996). In their model, they assume an exogenous and constant interest rate and take the profit rate as an explicative variable of the investment function.

The hypothesis of an exogenous and constant interest rate is endorsed by the so called horizontalist view of the monetary endogeneity, as developed by Kaldor (1982) and Moore (1988). According to this approach, the commercial banks meet all the demands for credit by means of a constant interest rate, which is determined through a fixed mark-up over the cost of the resources obtained in the inter-banking market (Rousseas, 1992, p.85).

However, the horizontalist approach to money and credit has been criticized by Post Keynesians authors. The main criticism is that it ignores the commercial banks *liquidity preference* (Carvalho, 2005, p. 58-62). Indeed, if the banks were to provide all the credit demanded by a constant interest rate, then, in as far as money and credit supply would increase, the banks themselves would have less liquidity, since the relationship between reserves and demand deposits would be reduced, which then increases the risk of illiquidity for banks. If they had a preference for liquidity, as any other agent, then they would only accept an increase in the illiquidity risk, if they were compensated through higher profitability. Hence, an increase in the interest rate that they charged on loans would be recommended.

Another boundary within the Kaldor-Moore approach is that it does not consider the issue regarding the limits of indebtedness. With regards to these limits, Kalecki (1954) argues that the companies with a higher degree of leverage have greater costs on capital following extreme increases in their liabilities, consequently, a greater compromise towards a short run solvency. In an extreme case, the companies may be unable to get new loans. A simple way to formalize this argument, such as in Bresser and Nakano (2002) and Oreiro (2002, 2004), is to assume that the interest rate paid on the debt is positively influenced by the level of indebtedness of the company, which makes it an endogenous variable. Thus, we can determine the interest rate on bonds according to the following equation:

$$i = \rho \delta$$
  $\rho > 0$  (7)

Where  $\rho$  is a fixed and positive parameter and  $\delta$  is the degree of public indebtedness that can be defined as:

$$\delta = \frac{D}{P.K}$$
(8)<sup>8</sup>.

<sup>&</sup>lt;sup>8</sup> Actually, the degree of indebtedness should be expressed by a relation between the real stock of debt and gross domestic product (D/PX). For the purpose of modeling, we are using as *proxy* the definition of the indebtedness degree as the relation between real debt and capital stock, given that the rate of economic growth is gotten from the relation investment (I) and capital supply (K).

In the economy under consideration, we assume that the firms determine the prices of their products based on a fixed mark-up over the direct unitary costs of production. The mark-up effectively practiced by the firms may however be smaller than the mark-up that the companies would wish for. The *desired mark-up* is determined by the long-run strategic decisions of the companies (cf. Kalecki, 1954, p.17). The effective mark-up must be seen as a commitment solution between the desired mark-up and the existing competition conditions within the economy (cf. Possas and Dweck, 2005, p. 12); that is to say, that the companies can fix, in the short-run, a mark-up that is smaller than the desired one with an aim to, for example, attain a greater market-share.

In this context, the inflation is derived from the attempt of firms to line up the effective mark-up with the desired mark-up. Thus, if the effective mark-up is lower than the desired mark-up; then the firms must increase the prices of their products in the long-run as a strategy to reach the desired mark-up. As the profit share is given by z/1+z, where z is the effective rate of the mark-up; then the firms will increase the prices charged for their products whenever the desired participation of the profits in the income (such as determined by the desired rate of mark-up) is greater than the effective participation of the profits in the income (such as determined by the effective rate of mark-up). That is:

$$\pi = \frac{P}{P} = \varepsilon (m^f - m) \tag{9}$$

Where:  $m^{f}$  is the participation of the profits in the income that is *desired* by the capitalists (by the firms), and *m* the effective participation.

#### 3. The behaviour of the economy in the short-run

In the short-run, the public debt is considered to be a constant ratio of the capital stock. As the real wage is constant and exogenous, it follows that the profit share in income is also constant, suggesting a fixed mark-up rate. Thus, the production level is determined by the effective demand, which is given by the equations (5) to (9). Replacing these equations in (4), dividing the resultant expression by *K*, and defining u = X/K and  $\gamma = G/K$ , we get the following equation:

$$u = u^* = \frac{1}{\lambda(m)} \left[ \left( (1 - s_c)(1 - \tau)\rho\delta^2 - \phi\rho\delta \right) + \alpha + \phi\varepsilon(m^f - m) + \gamma \right) \right]$$
(10)

Where:  $u^*$  is the degree of the productive capacity utilization of short run equilibrium and  $\lambda(m) = \{ [1 - (1 - s_c)(1 - \tau)]m - \beta \}$  is assumed to be bigger than zero<sup>9</sup>.

Substituting (7) and (10) in (6a), we obtain the expression for growth rate of capital supply within the short-run equilibrium of the economy under consideration. This is given by:

$$g^* = \alpha + \phi \varepsilon (m^f - m) + \beta u^* - \phi \rho \delta \tag{11}$$

Based on the equations (10) e (11), we can appraise the effects of exogenous changes in the functional distribution of income, in government expenses and public sector level of indebtedness over the degree of capacity utilization and on the rate of growth in the short-run equilibrium.

Differentiating (10) e (11) to *m*, we get the following expressions:

$$\frac{\partial u^*}{\partial m} = -\frac{1}{\lambda(m)} \left\{ \phi \varepsilon + \left[ 1 - (1 - s_c)(1 - \tau)u^* \right] \right\} < 0$$
(12a)

$$\frac{\partial g^*}{\partial m} = -\phi \varepsilon + \beta \left(\frac{\partial u^*}{\partial m}\right) < 0$$
(12b)

The expression (12a) shows that an increase in the profit share (*m*) will cause a reduction in the degree of capacity utilization in the short-run equilibrium. This is because an increase in the participation of profits in income will reduce the effective demand for two mechanisms. The first mechanism is the traditional Kaleckian one, that is: a redistribution of income in favour of capitalists which will reduce the aggregate consumption expenses since workers' propensity to consume is bigger than the capitalists'. The second mechanism is kind of a Mundell-Tobin effect in the context of a model of growth and distribution. An increase in the actual profit share will reduce the difference between this variable and the desired participation of profits on income by capitalists, which leads to a reduction in the inflation rate. For a given nominal interest rate, there will be an increase in the real interest rate, which will lead the capitalists to invest less, thus reducing the effective demand and a degree of use in productive capacity.

<sup>&</sup>lt;sup>9</sup> This hypothesis is necessary to guarantee the stability of the short-run equilibrium position. In economic terms, this hypothesis establishes that the sensitivity of the capitalists saving to a variation of the degree of use of the productive capacity is bigger than the sensitivity of the investment to changes in the utilization degree. It is worth emphasizing that this hypothesis is usually adopted in the context of Post Keynesian models of growth and distribution.

The expression (12b) shows that in an economy, such as that being considered, a regime of accumulation of wage-led type prevails, since a reduction in the participation of profits in income (that is, an increase in wage share) will result in an increase in the rate of growth of capital stock.

The short-run effects of a fiscal expansion, that is, an increase of the government expenses as a ratio of capital stock, can be appraised by the following expressions:

$$\frac{\partial u^*}{\partial \gamma} = \frac{1}{\lambda(m)} > 0 \tag{13a}$$

$$\frac{\partial g^*}{\partial \gamma} = \frac{\beta}{\lambda(m)} > 0 \tag{13b}$$

The equations above show that a fiscal expansion will produce an increase of the degree of capacity utilization and of the rate of growth of the capital stock of the short-run equilibrium in the economy under consideration, like in the traditional Keynesian models.

Finally, the effects of an increase of the public sector indebtedness as a ratio of the capital stock can be appraised by the following expressions:

$$\frac{\partial u^*}{\partial \delta} = \frac{2(1-s_c)(1-\tau)\rho\delta - \phi\rho}{\lambda(m)}$$
(14a)

$$\frac{\partial g^*}{\partial \delta} = \frac{\beta}{\lambda(m)} \{ 2(1 - s_c)(1 - \tau)\rho \delta - \rho \phi \} - \rho \delta$$
(14b)

The signs of expressions (14a) e (14b) are ambiguous, depending on the value of the public sector indebtedness as a ratio of the capital stock. Based on (14a), we can conclude that the sign of this partial derivative will be positive if the following condition is satisfied:  $\delta > \frac{\phi}{2(1-s_c)(1-\tau)} = \delta^*;$  while it will be negative, on the contrary. In this context, the relation between the degree of use of the productive capacity and the indebtedness of the public sector as a ratio of the capital supply is nonlinear, being able to be visualized by the Figure 1 below:



In figure 1, we observe that for low levels of indebtedness of public sector as a ratio of capital stock, an increase of  $\delta$  will reduce the short-run equilibrium level of capacity utilization; while for high levels of indebtedness, the inverse effect occurs. This results from the fact that variations of  $\delta$  generate effects of positive and negative signs over aggregate demand. On the one hand, an increase of  $\delta$  discourages the aggregate demand in as far as it generates an increase on the interest rate paid on the public bonds; hence, increasing the opportunity of cost on the investment in fixed capital. On the other hand, the increase of  $\delta$  has a wealth effect and a positive income effect on the consumption for the capitalists, since the income interests are an important part of the available income for the capitalists. In this context, the figure 1 shows that the first effect tends to be stronger than the second one for low values of the level of indebtedness of the public sector; while for higher variable values, the second effect tends to be stronger than the first one.

Finally, we observe in the expression (14b) that the sign of the partial derivative will be positive if the following condition is satisfied:  $\delta > \frac{(\beta + \lambda)\phi}{\beta 2(1 - s_c)(1 - \tau)} = \delta^{**}$ , while on the contrary it will show negative.

#### 4. The behaviour of the economy in the long-run.

In the long-run, the indebtedness of the public sector as a ratio to capital stock is an endogenous variable; being affected by the primary deficit of the government, by the rate of growth of capital stock and by the inflation rate. Differentiating  $\delta$  to time from (8), we obtain the following expression:

$$\frac{d\delta}{dt} = \frac{D}{PK} - (\pi + g)\delta$$
(15).

The debt of the public sector changes with time, based on the following differential equation:

$$\dot{D} = P(G-T) + i.D \tag{16}$$

The first part of the equation (16) represents the primary deficit of the government, therefore emphasizing differences between the expenses and tax revenues of the government, apart from the payment of interests over the existing debt. The second part of represents the financial charges (interests) on the total debt of the public sector.

The real value of the taxes charged by the government is determined by the following equation:

$$T = \tau (mu + i\delta)K \tag{17}$$

Replacing (17) with (16) and the resultant of (15) and then executing some mathematical transformations, we have the following dynamic equation to  $\delta$ :

$$\frac{d\delta}{dt} = \gamma + (1 - \tau)i\delta - \tau mu - (\pi + g)\delta$$
(18).

Substituting (9), (10) and (11) in (18), we get the following expression:

$$\frac{\partial \delta}{\partial t} = -\frac{\beta}{\lambda(m)} [(1-s_c)(1-\tau)\rho] \delta^3 - \left\{ \frac{1}{\lambda(m)} [\tau m(1-s_c)(1-\tau)\rho + \phi\rho\beta] - [(1-\tau) + \phi\rho] \right\} \delta^2 - \left\{ \frac{1}{\lambda(m)} [\beta(\alpha + \gamma + \phi\varepsilon(m^f - m) - \tau m\rho\phi)] + \alpha + (1+\phi)\varepsilon(m^f - m) \right\} \delta + \left\{ \gamma - \frac{\tau m}{\lambda(m)} [\alpha + \gamma + \phi\varepsilon(m^f - m)] \right\}$$
(19)

The equation (19) is, indeed, a third degree polynomial differential equation, which can be rewritten as follows:

$$\frac{d\delta}{dt} = A\delta^3 + B\delta^2 + C\delta + D \tag{20}$$

Where:

$$A = \frac{\beta}{\lambda(m)} \left[ (1 - s_c)(1 - \tau)\rho \right]$$
<sup>(21a)</sup>

$$B = -\left\{\frac{1}{\lambda(m)} \left[\tau m(1-s_c)(1-\tau)\rho + \phi \rho \beta\right] - \left[(1-\tau) + \phi \rho\right]\right\}$$
(21b)

$$C = \left\{ \frac{1}{\lambda(m)} \left[ \beta \left( \alpha + \gamma + \phi \varepsilon \left( m^{f} - m \right) - \tau m \rho \phi \right) \right] + \alpha + (1 + \phi) \varepsilon (m^{f} - m) \right\}$$
(21c)

$$D = \left\{ \gamma - \frac{\pi m}{\lambda(m)} \left[ \alpha + \gamma + \phi \varepsilon (m^f - m) \right] \right\}$$
(21d).

In the long-run equilibrium, the public debt as a ratio of the capital stock will be constant though time, that is:  $d\delta/dt = 0$ . Thus, the equation (20) is reduced to a polynomial of the third degree type:

$$A\delta^3 + B\delta^2 + C\delta + D = 0 \tag{22}$$

The roots of this polynomial equation are the values of long-term equilibrium of public debt as a ratio of the capital stock. As it is a polynomial of the third degree, we know that three roots satisfy the cited equation. However, we are only interested in the real positive roots, given that a negative root would denote a situation in which the government was a net creditor of the private sector. Therefore, the possibility exists for only three distinct situations regarding the hypothesis of three different real roots: the equation presents one, two or three positive roots. If two equal roots occur, the graph will be intercepted on the horizontal axis in only two points. Graphically, we would have the following representations<sup>10</sup>, with the hypothesis that the equation would be such that there are three distinctive real roots, as shown in figure 2:

<sup>&</sup>lt;sup>10</sup> Perhaps it is important to emphasize that the sine format of the curve depends on the existence of opposing signs between the parameter A and B, with A < 0 and B > 0. The bigger the value of B, the greater is the amplitude of the undulation. In economic terms, more accented undulations favor the occurrence of a bigger interval between the roots as well as increase the domain of the debt/capital relation in which stable equilibria are observed (points where the curve cuts the horizontal axis).

Figure 2 – Distinct Real Roots of the Equation of the Degree of Indebtedness



In example (a), the positive root is given by  $\delta^3$ ; although it is stable, it is the only possibility of equilibrium with regards to the debt/capital relationship being greater than zero. The example (b) shows two points of equilibrium, the smaller one ( $\delta^2$ ) being unstable, while the bigger one ( $\delta^3$ ) being stable. Furthermore, the stable level of indebtedness ( $\delta^3$ ) will be situated in a higher level in relation to example (a). Finally the situation (c) enables the economy to find three equilibrium points, with the stable points occurring in a low level and another high one for indebtedness. Furthermore, we can include some additional conditions to the parameters regarding the attainment of a configuration that allows us to reproduce the situation (c).

Based on the *Theorem of the Decomposition and Girard Relationships*, we find that the roots of a third degree polynomial obey the following properties in the parameters:

$$\delta_1 + \delta_2 + \delta_3 = -\frac{B}{A}$$

$$\delta_1 + \delta_2 + \delta_3 = -\frac{C}{A}$$
(23a)

(23b) 
$$(23b)$$

$$\delta_1 \delta_2 \delta_3 = -\frac{D}{A} \tag{23c}$$

Moreover, it is possible to establish the following conditions with regard to the parameters, occurring in situations (a), (b) or (c) represented above, knowing beforehand that the parameter A is definitely negative:

Table 1 – Necessary Conditions

Parameter	1 Positive Root	2 Positive Roots	3 Positive Roots
	(a)	(b)	(c)
А	(-) (-) (-)	(-) (-)	(-)
В	(+)(-)(-)	(-) (+)	(+)
С	(+)(+)(-)	(+) (-)	(-)
D	(+)(+)(+)	(-) (-)	(+)

In the expression (20) above, only the sign of the coefficient A is known for certain, since we know that in the equation (10) that  $\lambda(m) > 0$  and therefore A < 0. All the other coefficients have ambiguous signs. To solve the ambiguity we must impose additional restrictions to the values of the parameters.

In this context, the coefficient B is positive if the following condition is satisfied:

$$B > 0 \Longrightarrow \lambda(m) > \frac{\rho \left[\tau m(1 - s_c)(1 - \tau) + \phi \beta\right]}{\left[(1 - \tau) + \phi \rho\right]} = \lambda^*$$

$$(24)^{11}.$$

The coefficient C requires a more careful analysis, though it has only a positive term, which could lead us to conclude that C is possibly negative. As the expression of the parameter is the same for all the domain of the function, we can use some of the equilibrium points in which  $m^f = m$ . Thus, the expression can be rewritten as follows:

$$C < 0 \Rightarrow \frac{1}{\lambda(m)} \left[ \beta \left( \alpha + \gamma - \tau m \rho \phi \right) \right] + \alpha > 0$$
<sup>(21c')</sup>

Solving it for  $\gamma$ , we obtain the condition for that C < 0:

$$C > 0 \Longrightarrow \gamma > \frac{\tau m \rho \phi}{\beta} - \alpha \left( 1 + \frac{\lambda(m)}{\beta} \right)$$
(21c'').

Finally, adopting the same procedure for the coefficient D, such that  $m^f = m$  in equilibrium, we obtain the following inequality:

<sup>&</sup>lt;sup>11</sup> It is possible to demonstrate easily that this condition can be satisfied if the sensitivity of the investment to changes of the degree of the use of the productive capacity is low, or if the participation of the profits in the income is high.

$$D > 0 \Longrightarrow \gamma - \frac{\pi m}{\lambda(m)} [\alpha + \gamma] > 0$$
(21d').

Solving for  $\gamma$ , we obtain the condition for that D > 0:

$$D > 0 \Longrightarrow \gamma > \frac{\tau m \alpha}{\lambda(m) - \tau m}$$
(21d'').

Moreover, to have three positive real roots it is necessary that their product  $\delta_1 \delta_2 \delta_3 > 0$ . In order to obey Girard's third relation, which establishes that the product  $\delta_1 \delta_2 \delta_3 > -D/A$ , it is possible to find that the imposed conditions on the parameters -D/A indeed suggests that the result of the relationship is greater than zero, for A < 0 and D > 0.

Satisfying all these requirements, we can represent the dynamic equation for the degree of indebtedness in the long-run as follows, representing the situation (c) previously shown. So we have the following A < 0, B > 0, C < 0 and D > 0, in which the polynomial has 3 positive roots. Thus, we



can visualize the fixed points of (20) through figure 3 represented below.

It is observed that the economy has three values of long-run equilibrium level of public debt as a ratio of capital stock, that is:  $\delta_L^I$  (equilibrium with low debt),  $\delta_M^2$  (equilibrium with medium debt) and  $\delta_H^3$  (equilibrium with high debt). It is also observed that the equilibrium with medium debt is unstable, while the equilibrium points with low and high debt are stable. From that, it follows that the initial value of debt as a ratio of the capital supply is greater than  $\delta_M^2$ ; the economy will present a transient dynamics characterized by the rise of public debt as a ratio of the capital supply and an increase in the nominal and real interest rate, thus defining a vicious circle of increase in the debt/rise of interests/increase of the debt.

Finally, it is important to explain why the situation (c) with three positive real roots was chosen as representative of the economy under study. The situations with one root (a) and two roots (b) were discarded because of the following reasons. From a numerical simulation, with different values for the parameters, it is possible to find that the parameters A < 0, B < 0, C > 0 e D > 0, the

case (a), is only reproduced for economically unrealistic values for the parameters. Moreover, the economy would be such that only one equilibrium with a positive level indebtedness would be obtained, which does not seem to be a realistic situation and therefore we have discarded the case (a). However the difference between the situation (b), with two positive roots, and the situation (c), with three positive roots is limited to the position of the stable equilibrium point with low indebtedness. If the parameters are such that the case (b) prevails, then one of the stable equilibrium will occur with a negative value for debt-capital relation, that is, a creditor government, instead of a resources borrower, a situation that is not empirically verifiable. The other stable equilibrium would only be possible with a very high degree of indebtedness. But if the parameters values are such that three real positive roots are obtained, so the macroeconomic dynamics will be able to reflect, in fact, a more realistic situation in which the economy can operate, with two positive and stable degrees of indebtedness, one low and one high. The (b) and (c) cases and the necessary conditions imposed on the parameters, such as in table 2, are fully feasible<sup>12</sup>.

In the following, we will focus on case (c) to analyze the long-run dynamics and the implications of economic policies in the context of different regimens of indebtedness. The qualitative conclusions are valid for both cases.

### 5. Long-Run Effects of Changes in Fiscal Policy

The next step in our analysis consists of determining the long-run effect of variation in the government expanses as a ratio of capital stock and in the profit share over the level of capacity utilization and over the rate of growth of capital stock. The difference regarding the analysis made in section 3 is that now we will take into account the impact of these variations over the level of indebtedness of the public sector and, therefore, the indirect effects of these changes on the variables under consideration.

For that, we will initially appraise the impact of changes in the fiscal policy and in the distribution of income on long-run equilibrium values of the degree of indebtedness of the public sector. One way to analyze that, without having to appeal to the numerical calculation of the roots of the expression (22), is to appraise the impact of changes in the variables under consideration for the position of locus  $d\delta/dt$ , in order to appraise graphically the effect over the fixed points of the locus under consideration. Variations in the fiscal policy and in the share of profits will make that the equilibrium points of the locus  $d\delta/dt$  move from the right to the left, as well as alter the distance between them, according to the type of the variation considered.

<sup>&</sup>lt;sup>12</sup> The proof of this statement is made by means of numerical simulations, which can be obtained with authors by request.

Going back to equation (18), we can rewrite it emphasizing the degree capacity utilization (u) and also as a function of  $\gamma$ . Doing this we have:

$$\frac{d\delta}{dt} = \dot{\delta} = \gamma + (1 - \tau)i\delta - \tau m u(\gamma) - (\pi + g)\delta$$
(18b)

Differentiating (18b) to  $\gamma$ , we obtain the following expression:

$$\frac{\partial \dot{\delta}}{\partial \gamma} = \frac{\lambda(m) - \tau \, m - \beta \delta}{\lambda(m)} \tag{25a}.$$

The result is ambiguous. Based on expression (25a) we can conclude that  $\partial \dot{\delta} / \partial \gamma > 0$  if the following condition is satisfied:

$$\delta^c = \delta = \frac{s_c(1-\tau)m}{\beta} - 1 > 0 \tag{25b}.$$

That is, the effect of a change in the fiscal policy over the position of locus  $d\delta/dt$  will depend on whether the indebtedness of the public sector as a ratio of the capital supply is smaller or bigger than a certain critical value  $\delta^{c}$ . For degrees of indebtedness lower than this value the derivative (25a) is positive, then a fiscal expansion will dislocate the locus  $d\delta/dt$  to above. On the other hand, for levels of indebtedness bigger than this critical value, a fiscal expansion will dislocate this locus to below. Given that the equation of the movement of the long-run degree of indebtedness has three roots, then the critical value  $\delta^{c}$  will be able to be located in four different points, as follows:

Table 2 – Positions of the Critical Value $\delta^{c}$	

Case I	$\delta^{c} > \delta^{3}_{H}$
Case II	$\delta^2_{M} < \delta^c < \delta^3_{H}$
Case III	$\delta^1_L < \delta^c < \delta^2_M$
Case IV	$\delta^{c} < \delta^{1}_{L}$

## Case I - $\delta^c > \delta^3_H$

In the case where the critical value of the public indebtedness level is bigger than the value of this variable in the initial equilibrium with high indebtedness  $(\delta^3_H)$ , then  $\partial \dot{\delta} / \partial \gamma > 0$ , thus a displacement to above of the entire locus  $d\delta/dt$ , as represented by Figure 4 below:

Figure 4 – Case I –  $\delta^{c} > \delta^{3}_{H}$   $d\delta/dt$   $\delta_{L}^{1'}$   $\delta_{M}^{2'}$   $\delta_{H}^{3}$   $\delta_{H}^{3}$   $\delta_{H}^{3}$   $\delta_{H}^{3}$   $\delta_{H}^{3}$   $\delta_{H}^{2}$   $\delta_{H}^{3}$   $\delta_{H}^{3}$  $\delta_{$ 

In figure 4 we observe that a fiscal expansion generated an increase of the public indebtedness as a ratio of capital stock in the long-run equilibrium (from  $\delta^3_H$  to  $\delta^3'_H$ ). A reduction of the distance also occurs between the points of low  $\delta'_L$  and medium  $\delta'_M$  indebtedness, which in economic term means an increase of the expenses, in this situation, reduces the region of stability with low indebtedness and increases the possibility of the economy to enter the high equilibrium earlier. Remember, as shown in figure 3, the  $\delta'_M$  is instable. The room for the adoption of an expansionary fiscal policy diminishes.

Case II -  $\delta^2_M < \delta^c < \delta^3_H$ 

In the case where the critical value of the indebtedness level is between the medium and high values of equilibrium, the effect of an increase of government expenses has different effects as the degree of initial indebtedness is below (to the left) or above (to the right) this critical value. If it is below  $\partial \dot{\delta}/\partial \gamma > 0$ , then the curve will dislocate to above. If it is above  $\partial \dot{\delta}/\partial \gamma < 0$ , then the curve is dislocated below. These two movements cause a twist in the locus  $d\delta/dt$  around the point A, which is the limit between the two regions, as demonstrated in figure 5 below:

In this case, a fiscal expansion increases the degree of indebtedness of low equilibrium  $\delta^{1'}_{H}$  and reduces the medium equilibrium to  $\delta^{2'}_{M}$  and, as before, reduces the interval of low stability in the economy. On the other hand, it causes a **reduction** in the degree of indebtedness of high equilibrium of  $\delta^{3}_{H}$  to  $\delta^{3'}_{H}$ .



Case III -  $\delta^1_L < \delta^c < \delta^2_M$ 

In the case that the critical value level of indebtedness lies between the low and medium equilibrium, an increase in the expenses has different effects according to the initial degree of indebtedness shown above (to the left) or below (to the right) this is the critical value, such as in case 2. If it is below  $\partial \dot{\delta} / \partial \gamma > 0$ , then the curve moves up. If it is above  $\partial \dot{\delta} / \partial \gamma < 0$ , then the curve moves down. These two movements cause a twist of the locus  $d\delta / dt$  around point A, which is the



limit between two regions, as it is shown in figure 6.

The difference of this case from case II is related to the point of unstable equilibrium  $\delta_{M}^{2}$ . This point is now placed in a higher level. However, the point of low equilibrium of the economy also moved up. In economic terms, case III is preferable to case II due to bigger distance between  $\delta_{L}^{\prime} e \delta_{M}^{\prime}$ , which means that there is more room for the execution of expansionary fiscal policies with increases of government expenses, before the equilibrium point  $\delta_{M}^{2}$  is exceeded and the economy starts a transient trajectory to the direction of a higher equilibrium. The economic cost of this is that the low equilibrium also increases to  $\delta_{L}^{I'}$ . Case IV -  $\delta^c < \delta^1_L$ 

Finally we have case IV, where the critical value of the indebtedness level is below of the low equilibrium. In the case where the critical value of the level of public indebtedness is lower than the value of this variable in the initial equilibrium with low indebtedness  $(\delta^{I}_{L})$ , then  $\partial \dot{\delta} / \partial \gamma < 0$ . So there is a moving down of the entire locus  $d\delta/dt$ , such as represented by figure 7 below. This is the best one of the worlds. The low stable equilibrium point diminishes for  $\delta^{1'}_{L}$ , at the same time that the medium stable equilibrium point increases for  $\delta^{2'}_{M}$ , which makes the difference between them bigger than all the other cases. Additionally, the high stable equilibrium point also diminishes for  $\delta^{3'}_{H}$ .

How is it possible that a fiscal expansion generates a reduction in the public debt as a ratio of the long-run capital supply? This counter-intuitive result can be explained by the fact that in the case IV above, a fiscal expansion generates a very strong expansion of the accumulation of capital and in the level of capacity utilization (and, therefore, of the tax revenue of the government) so that debt as a ratio capital stock is reduced.



Although this result is a logical possibility of the model presented here, we have to keep in mind that it is not very probable that the same would be observed in the real world. This is because for minimal and realistic values of the parameters  $s_c$ ,  $m, \tau e \beta$ , the critical value of  $\delta$  must be very high, so that case VI would be discarded as a simple theoretical curiosity. As for case III, though a fiscal expansion increases the long-run degree of indebtedness as expected, it is not feasible either,

since it requires a  $\delta^{\varepsilon}$  lower than that which can be effectively obtained with plausible values for the parameters<sup>13</sup>. So therefore, case I and case II are situations that are much closer to reality.

#### 5.1 SHORT AND LONG RUN FISCAL MULTIPLIER

We now need to analyze the effects of a fiscal expansion over the level of capacity utilization in the long-run equilibrium, so that it is possible to calculate the long-term fiscal multiplier. For that, we must differentiate the equation (10) to  $\gamma$ , taking into consideration, however, the effects of  $\gamma$  on  $\delta$ . We have, then, that:

$$\left. \frac{\partial u^*}{\partial \gamma} \right|_{CP} = \frac{1}{\lambda(m)} > 0 \tag{13a}$$

$$\frac{\partial u^*}{\partial \gamma}\Big|_{L^p} = \frac{1}{\lambda(m)} \left\{ 1 + 2(1 - s_c)(1 - \tau)\rho(\delta - \delta^*)\frac{\partial \delta}{\partial \gamma} \right\}$$
(26).

In the expression (26) we observe that if  $\delta > \delta^*$ , that is, if the economy is operating in a regime of high public indebtedness, the long-run fiscal multiplier will, certainly be positive. It must also be observed that the long-run fiscal multiplier - given by the equation (26) - is greater than the short-run fiscal multiplier - represented by the equation (13a). From that, it follows that a fiscal expansion will have a bigger impact on the aggregate demand and the level of economic activity in the long-run than in the short-run in economies that operate in a regime of high public indebtedness.

## **6** Long-run effects of a change in the income distribution

We will now analyze the long-run effects of a change in the functional distribution of the income, more specifically, the effect of an increase of the participation of profits in income. For

<sup>&</sup>lt;sup>13</sup> Indeed, taking  $s_c = 0.75$ ;  $\tau = 0.20$ ; m = 0.40 and  $\beta = 0.10$ , we obtain a critical value of  $\delta$  equal to 2,40. Assuming a degree of utilisation of productive capacity equal to 0,85 and a capital-product relation equal to 2,5; this critical value of  $\delta$  implies a critical value for the public debt as a ratio of the GDP of 5.10 or 510%. To get this value suffices to remember that:  $\delta = \frac{(D/P)}{Y} \frac{Y}{Y^*} \frac{Y^*}{K} = \frac{d u}{\sigma}$ . Where: *d* is the public debt as a ratio of the GDP, *u* is the degree of the use of the productive capacity,  $\sigma$  is the capital/product relation and  $Y^*$  is the potential product. There is no case in the real

of the productive capacity,  $\sigma$  is the capital/product relation and Y' is the potential product. There is no case in the real world of a sovereign government that has a public debt as a ratio of the GDP greater than 200%, so the effective value of  $\delta$  must be a lot smaller than the critical value of this variable, taking then case III and this as a simple theoretical curiosity.

that, we must initially appraise the impact of a variation in m over the values of long-run equilibrium of public debt as a ratio of capital stock.

From equation (18), substituting  $\pi$ , and recalling that u and g also depend on m, we have:

$$\dot{\delta} = \frac{d\delta}{dt} = \gamma + (1 - \tau)i\delta - \tau m u(m)^* - \left\{\xi\left(m^f - m\right) + g(m)\right\}\delta$$
(27)

Differentiating (27) to m, we get:

$$\frac{\partial \dot{\delta}}{\partial m} = -\tau \, u - \tau m \frac{\partial u^*}{\partial m} + \xi \delta - \frac{\partial g}{\partial m}$$
(28)

Remembering that the partial derivative  $\partial g/\partial m$  was already calculated in (12), and then substituting, we have:

$$\frac{\partial \dot{\delta}}{\partial m} = \left\{ \left(1 + \phi\right)\varepsilon - \beta \frac{\partial u^*}{\partial m} \right\} \delta - \tau \left[ m \frac{\partial u^*}{\partial m} + u^* \right]$$
(29)

Defining  $\eta_{u,m} = -\frac{m}{u^*} \frac{\partial u^*}{\partial m}$  as the elasticity of the degree of capacity utilization with respect to the participation of the profits in the income (cf. Oreiro, 2004, p.46), the expression (29) can be rewritten as follows:

$$\frac{\partial \dot{\delta}}{\partial m} = \left\{ (1+\phi)\varepsilon - \beta \frac{\partial u^*}{\partial m} \right\} \delta - \tau [1-\eta_{u,m}]$$
(30).

Assuming that  $\eta_{u.m} < 1$ , and given that  $\partial u^* / \partial m < 0$  as shown in (12a), in order the partial derivative  $\partial \dot{\delta} / \partial m > 0$ , it is necessary that  $\delta$  is above a certain critical value given by  $\delta^{cc}$ . If  $\delta$  is below the critical value, the derivative will be negative. The critical value, which is obtained equalizing (30) to zero and solving it to  $\delta$ , is given by:

$$\delta^{cc} = \frac{\tau(1 - \eta_{u,m})}{\left[(1 + \phi)\varepsilon - \beta \frac{\partial u^*}{\partial m}\right]}$$
(31).

That is, the effect of an increase on the participation of profits on the income of the position of locus  $d\delta/dt$  will depend on whether the public sector debt as a ratio of the capital stock is smaller or bigger than a certain critical value  $\delta^{ec}$ . For degrees of indebtedness bigger than this

critical value, an increase of the participation of the profits in the income will move the locus  $d\delta/dt$  up. Otherwise, to degrees of indebtedness smaller than this critical value, an increase of *m* will move the locus down.

In contrast to the case analyzed in the previous section, which referred to a fiscal expansion, the critical value of  $\delta$  for a change in the participation of the profits in the income must be very low<sup>14</sup>. Thus, the economically relevant case is the one that corresponds with a situation in which:  $\delta^{cc} < \delta_L^1$  or even  $\delta_L^1 < \delta^{cc} < \delta_M^2$ , discarding both possibilities in which  $\delta^{cc}$  is either above  $\delta_M^2$  or above  $\delta_H^3$ .

## Case I - $\delta^{cc} < \delta^1_L$

In this context, as we can see in figure 8 below, the three equilibrium levels of indebtedness



are above the critical value, which means, taking the results obtained in (30) and (31), that the partial derivative will be  $\partial \dot{\delta} / \partial m > 0$ , therefore, in this situation an increase of the participation of the profits in the income causes the locus  $d\delta/dt$  to move up in all the extension above  $\delta^{cc}$ .

The consequence of this is that the low stable equilibrium point increases to  $\delta^{I'}_{L}$ , the medium unstable point diminishes to  $\delta^{2'}_{M}$ , so that the distance between them is small, which diminishes, therefore, the range of the stability of the economy.

**Case II -**  $\delta_L^1 < \delta^{cc} < \delta_M^2$ 

In this case, the values below  $\delta^{cc}$ , the partial derivative will be negative, as before, and for values above it, it will be positive, as represented in the following figure 9:

<sup>&</sup>lt;sup>14</sup> Assuming  $s_c = 0.75; \tau = 0.20; m = 0.40 \ e \ \beta = 0.10$  as we did before, and  $\eta_{u,m} = 0.5 \ e \ \partial u^* / \partial m = -0.5$ , we get  $\delta^{cc} = 0.1667$ .



The curve suffers then a counter-clockwise rotation around point A, which delimits the two regions. A consequence of this is the low stable equilibrium point reduces to  $\delta^{I'}{}_{L}$ , and the medium stable point diminishes to  $\delta^{2'}{}_{M}$ , so that the distance between them depend on how much each one of the points withdrew. In the graph above, as  $\delta^{cc}$  is closer to  $\delta^{2'}{}_{M}$ , this means that the relative displacement to left from the stability point  $\delta^{2'}{}_{M}$  will be smaller than  $\delta^{1'}{}_{L}$ , increasing then the interval of stability of the debt-capital relation. The inverse would occur if  $\delta^{cc}$  were closer to  $\delta^{1'}{}_{L}$ . Anyway, given that  $\delta^{2'}{}_{M}$  moves to the left, there will be less room for the execution of expansionary fiscal policies without breaching the barrier of sustainable debt.

# 6.1 LONG-RUN EFFECTS OF AN INCREASE IN "m" ON THE DEGREE OF THE CAPACITY UTILIZATION (u)

In the short-run, as we have seen before, in equation (12a), the effect of an increase in profit share has a negative effect over the degree of productive capacity utilization, thus configuring a regime of accumulation of waged-led type, as reproduced below.

$$\frac{\partial u^*}{\partial m}\Big|_{CP} = -\frac{1}{\lambda(m)} \Big\{ \phi \varepsilon + \Big[ 1 - (1 - s_c)(1 - \tau) u^* \Big] \Big\} < 0$$
(12a)

In the long-run, the effect of an increase in the participation of the profits can be appraised differentiating the equation (10) to *m*, taking into account the effects of changes of the participation of the profits in the income on  $\delta$ . This procedure results in:

$$\frac{\partial u^*}{\partial m}\Big|_{L^p} = -\left\{\frac{1}{\lambda(m)}\right\} \left[\phi\varepsilon + \left(1 - (1 - s_c)(1 - \tau)\right)u^*\right] + \left\{\frac{1}{\lambda(m)}\right\} \left[2\left(1 - s_c\right)\left(1 - \tau\right)\rho\frac{\partial\delta}{\partial m}\left(\delta - \delta^*\right)\right]$$
(32).

As it can be observed, the first term of the expression (32) is nothing but the short-run effect of a variation of the participation of the profits in the income over the degree capacity utilization, which - on the basis of the equation (12a) - is negative. The second term presents the indirect effect of changes in the functional distribution of income over capacity utilization. The sign of this indirect effect depends, however, on the regime of indebtedness in which the economy is. If the economy is operating in a *regimen of low indebtedness*, then the indirect effect will be negative, thus strengthening the direct or short-run effect of changes on the income distribution. If the economy is operating in a *regime of high indebtedness*, then the indirect effect would be positive, which in turn could cause that an increase in profit share could produce a rise in the level of capacity utilization in the long-run. This result will be more likely as higher is the level of indebtedness of the public sector. As summary of this result, it follows that if the economy is operating in a regime of high indebtedness; then the regime of accumulation will be of *profit-led type*.

# 7 Final Comments

As demonstrated, the short and long-run economic dynamics differ from the traditional Keynesian models when the interest rate is make an endogenous variable that responds to the degree of indebtedness, since it introduces a region in which the occurrence of persistent public debt, in as much as it provokes disequilibrium in the debt stock and is able to alter the regime of accumulation. In the short-term, the effectiveness of public expenses depends on the initial conditions and the degree of indebtedness in which the economy finds itself.

If, on the one hand, expenses financed with emission of bonds can provoke an increase in the aggregate demand, either via consumption or via public investment, then on the other hand, the existence of a risk premium on the public debt has a negative effect on private investments, so that there is a point where the latter effect is greater than the former and throws the economy into a region where the indebtedness degree produces a vicious cycle for fiscal policies. This behavior, as demonstrated, could be an extension on the traditional Keynesian models, for which the positive effects of fiscal policies are always expansionary. Here we demonstrated that there can be a difference with inverse dynamics. Moreover, we have also demonstrated that this mechanism, in the presence of the hypothesis that the workers do not save, leads to a concentration income process in the hands of capitalists as far as they possess the prerogative of continuing the process of wealth accumulation, even in the case of reducing the activity level. Currently, the accumulation of wealth occurs in the financial environment of the economy, so that the addition of profits and income-interests, when collated with the addition of wages in the economy, makes this perverse relationship abundantly clear.

#### REFERENCES

- Badhuri, A. Marglin, S. (1990). Unemployment and the real wage: the economics basis for contesting political ideologies. Cambridge Journal of Economics, 14(4).
- Bresser-Pereira, Luiz. C; Nakano, Yoshiaki. (2002). *A strategy of development with stability*. In : Brazilian Journal of Political Economy, v.22, n.3, jul/set/2002, p. 146-77.
- Cowling, K. (1982). Monopoly Capitalism. London, Macmillian.
- Dutt, A. K. (1984). *Stagnation, income distribution and monopoly power*. Cambridge Journal of Economics, 8(1), pp.25-40.
- Dutt, A. K. (1990). *Growth, distribution and uneven development*. Cambridge: Cambridge University Press.
- Kaldor, Nickolas. (1956). Alternative Theories of Distribution. *Review of Economics Studies*. Vol 23, nº 2.
- Kalecki, Michal. (1954). *Theory of Economic Dynamics*. São Paulo: Nova Cultural (*The Economists*), Translation 1977.
- Kalecki, Michal. (1971). Select essays on the dinamics of the capitalist economy. Cambridge: Cambridge University Press.
- Keynes, John Maynard. (1936). *The General Theory of Employment, Interest and Money*. Macmillian Press: Cambridge.
- Lima, Gilberto Tadeu. (1998). A Non-linear dynamics of capital accumulation, distribution and conflit inflation. Papers and Proceedings of XXVI National Meeting of the Brazilian Association of Economics. Jul/1998, Brazil.
- Oreiro, José L. (2004). "Endogeneous Premium Risk, Multiple Equilibria and Public Debt Dynamics". *Revista de Economia Contemporânea* Vol 8, Nº 1. Universidade Federal do Rio de Janeiro.
- Robinson, Joan. (1956). The accumulation of capital. London: Mcmillian.
- Robinson, Joan. (1962). Essays in the theory of economics growth. London: Mcmillian.
- Rowthorn, B. (1981). Demand, real wages and economics growth. Studi Economici, 18 pp-2-53.
- Spence, M. (1977). Entry, investment and oligopolistic pricing. Bell, Journal of Economics, 8(2).
- Steindl, J. (1952). *Maturity and stagnation in Americam Capitalism*. New York: Monthly Review Press.
- You, Jong-II; Dutt, Amitava K. (1996). "Government debt, income distribution e growth." *Cambridge Journal of Economics*, vol 20, (335-351).