

# Flexible Inflation Targeting, Real Exchange Rate and Structural Change in a Kaldorian Model with Balance of Payments Constrained Growth\*

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**Abstract:** The present article contributes to the literature of Kaldorian growth models with balance of payments constraint introducing some changes in the basic structure of Santana and Oreiro 2018 model. First, we will assume that monetary policy is conducted under a *Flexible Inflation Targeting Regime* where the goal of the monetary policy is not only to achieve a certain target for inflation; but also stabilize capacity utilization at its potential or target level. Second, we will assume that potential or target level of capacity utilization depends on the lagged value of this variable. This hypothesis captures the *hysteresis effect of lower output growth over potential output*, which is a phenomenon well documented in recent literature (Cerra, Fatás and Saxena, 2020). Finally, we will assume that inflation expectations depend on the credibility of the Central Bank. In the case where Central Bank is fully credible than inflation expectations are equal to the target inflation. Although the model was developed for a *small open economy*, it is designed for a *mature economy* in the sense of Lewis (1954): labor supply is inelastic and real output growth had to be equal to the natural growth rate in the long run. The natural growth rate is, however, endogenous because productivity growth depends both on output growth and employment rate. The natural growth rate adjusts itself to the actual growth rate of real output, determined by the balance of payments constraint, due to changes in the level of employment. In order for a balanced growth path to exist, it was also necessary to make the autonomous component of investment demand an endogenous variable in the long-run, as done by Lavoie (2016). In the long run equilibrium with structural change, it was shown that a *decrease* in the target inflation could *increase* the growth rate of real output, the capacity utilization, the rate of employment and the manufacturing share in GDP, becoming a driver of the process of structural change. Since monetary policy can affect the productive structure of the economy as well as the growth rate of real output and the rate of employment; then it is not possible to separate macroeconomics from economic development, which is core of the so-called structuralist development macroeconomics, the theoretical basis of the Brazilian New-Developmentalism School.

**KEYWORDS:** Monetary Policy; Balance of Payments Constrained Growth; Structural Change; Real Exchange Rate.

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## 1. Introduction

The balance of payments constrained growth model (BPCG models, hereafter), pioneered developed by Anthony Thirlwall (1979), holds two fundamental problems. Firstly, they fully disregard the cumulative causation mechanism, so relevant to Kaldorian growth models. Indeed, assuming constant terms of trade then productivity gains induced by economic growth have no effect over the dynamics of the system, in a such way they become, strictly, irrelevant. However, in this case, the system no longer has any adjustment mechanism between aggregate supply and demand. This deficiency was observed by Palley (2002) for whom the balance of payments constrained growth model would be inconsistent in the extent that only in a “happy coincidence” would be possible the equality between the growth rate compatible with the balance of payments equilibrium and natural growth rate, i.e., the one that keeps the unemployment rate constant over the time. In this way, the balance of payments constrained growth models does not, in general, compatible with a balanced growth path.

Last but not least, the balance of payments constrained growth models fully neglected the relationship between the real exchange rate and the long-term growth. Indeed, in those models the long-term equilibrium growth rate depends on the ratio of export and import income elasticities multiplied by rest of the world growth rate. Thus, real exchange rate variations are assumed irrelevant to the long-term growth either because empirical evidence shows that finding that export and import price elasticities are low, in a such way that the impact of a real devaluation of exchange rate over the growth path of exports and imports is low; either because the terms of trade do not show a systematic trend to appreciation or depreciation in the long-term.

In recent years, an interesting literature has been developed about the relation between real exchange and economic growth. The Razin and Collins (1997) seminal paper indicated to the existence of important non-linearities in the relationship between exchange rate misalignment - defined as a lasting deviation of the real exchange rate with respect to some reference value, determined by the "fundamentals"- and the real output growth in a sample of 93 developing and developed countries between 1975-1993. Indeed, the empirical results showed that while only large overvaluations of real exchange rate are associated with a slower economic growth in the long term, even moderated undervaluation of the real

exchange rate have a positive effect on economic growth. Rodrik (2008), analyzing the development strategies adopted by a group of countries, noted that an important factor for the ignition of a process of sustained growth of the real output is the maintenance of an undervalued and stable real exchange rate.

More recently, Gabriel et al (2020) analyzed the effects of manufacturing and of the real exchange rate (RER) on real per capita income growth. They use dynamic panel models and the calculation of output and employment multipliers for a diversified sample of countries from 1990 to 2011. Three important results can be highlighted. Two novel results were obtained. First, the authors provide new evidence that manufacturing is the most important tradable sector for achieving greater real per capita income growth for developing countries. Second, the greater a country's gap in relation to the technological frontier, the greater the positive effect of an undervalued RER on the real per capita income growth rate. Finally, the manufacturing industry's output multipliers and employment multipliers in the developing countries are higher than those in developed ones, in all years analyzed.

For the Brazilian case, Oreiro et al (2020) analyzed the determinants of the deindustrialization of the Brazilian economy in the period between 1998 and 2017, which was considered by the authors a typical example of 'premature deindustrialization' in the sense that the major reason for the fall in the manufacturing share has not been the increase in per-capita income but rather real exchange rate overvaluation. In the Brazilian case, real exchange rate overvaluation results both from an appreciation of the real effective exchange rate, and an increase in the equilibrium value of the real exchange rate, the "industrial equilibrium exchange rate" of the new developmentalist literature. The elimination of the real exchange rate overvaluation requires not only the adoption of a macroeconomic policy regime in which some kind of real exchange rate targeting is adopted, but also industrial policies designed for increasing the economic complexity of the Brazilian economy and, hence, to reduce the equilibrium value of the real exchange rate. Therefore, the absence of a connection between the level of the real exchange rate and the long-term growth in the context of the balance of payments constrained growth models becomes theoretically unacceptable.

Some of these deficiencies in the BPCG models are addressed by Santana and Oreiro (2018) who developed a Kaldorian growth model that (i) incorporates the balance of payments constraint, eliminating the inconsistency presented on balance of payments

constrained growth models; (ii) establishes a mechanism by which the level of the real exchange rate may affect the long-term growth of capitalist economies.

Santana and Oreiro 2018 model incorporate some innovations introduced by Oreiro (2009) into the structure of Kaldorian growth models, such as the conduction of monetary policy in an Inflation Target Regime, nominal interest rate determined by a Taylor rule, a floating exchange rate regime and imperfect capital mobility. In contrast to the Oreiro 2009 model, however, Santana and Oreiro 2018 model assume a balance of payments constraint in which the growth rate of international capital inflows is a positive function of the differential between the domestic interest rate and the international interest rate plus the country risk premium. In this context, the differential between the domestic and international interest rates (plus the risk premium) will also determine the rate of depreciation (or appreciation) of the nominal exchange rate.

Another important innovation introduced in the model was the hypothesis that the Kaldor-Verdoorn coefficient - which captures the sensibility of the rate of growth of labor productivity with respect to the rate of growth of the real output - depends on the manufacturing share on output. This hypothesis allowed to introduce into the model the possibility of structural change, which is understood as a dynamic process by which the manufacturing share of output changes over time. In this way, it was possible to analyze the dynamic properties of the model both in the case where the productive structure is kept constant (case with no structural change), and in a situation in which it changes due to some economic process (case with structural change).

The structural change, in its turn, as defined by the exchange rate misalignment, that is, by the difference between the actual value of the real exchange rate and the level of the real exchange rate that would correspond to the "industrial equilibrium", in other words, the exchange rate level in which domestic firms that use state-of-art technologies are competitive in international markets (Bresser-Pereira, Oreiro and Marconi, 2014, 2015).

The present article contributes to this line of research introducing some changes in the basic structure of Santana and Oreiro 2018 model. First of all, we will assume that monetary policy is conducted under a *Flexible Inflation Targeting Regime* where the goal of the monetary policy is not only to achieve a certain target for inflation; but also stabilize capacity utilization at its potential or target level. Second, we will assume that potential or target level of capacity utilization depends on the lagged value of this variable. This

hypothesis captures the *hysteresis effect of lower output growth over potential output*, which is a phenomenon well documented in recent literature (Cerra, Fatás and Saxena, 2020). Finally, we will assume that inflation expectations depend on the credibility of the Central Bank. In the case where Central Bank is fully credible than inflation expectations are equal to the target inflation; otherwise, they are determined by a simple adaptive rule, which will introduce a level of inflation inertia in the model. In the long run equilibrium with structural change, it was shown that a *decrease* in the target inflation could *increase* the growth rate of real output, the capacity utilization, the rate of employment and the manufacturing share in GDP, becoming a driver of the process of structural change. Since monetary policy can affect the productive structure of the economy as well as the growth rate of real output and the rate of employment; then it is not possible to separate macroeconomics from economic development, which is core of the so-called structuralist development macroeconomics, the theoretical basis of the Brazilian New-Developmentalism School.

Although the model was developed for a small open economy, it is designed for a *mature economy* in the sense of Lewis (1954): labor supply is inelastic and real output growth had to be equal to the natural growth rate in the long-run. The natural growth rate is, however, endogenous because productivity growth depends both on output growth and employment rate. The natural growth rate adjusts itself to the actual growth rate of real output, determined by the balance of payments constraint, due to changes in the level of employment. In order for a balanced growth path to exist, it was also necessary to make the autonomous component of investment demand an endogenous variable in the long-run, as done by Lavoie (2016).

## 2. Model Structure

Let us consider a small open economy with a free-floating exchange rate regime and imperfect capital mobility, in which growth rate of exports (quantum) and imports (quantum) are given by:

$$(1) \quad \hat{x}_t = \chi_0(\hat{p}_t^* - p_t + \hat{e}_t) + \chi_1 \hat{z}_t$$

$$(2) \quad \hat{m}_t = \mu_0(\hat{p}_t - \hat{p}_t^* - \hat{e}_t) + \mu_1 \hat{y}_t$$

In which  $\hat{x}_t$  is the growth rate of exports (*quantum*) in the period  $t$ ,  $\hat{m}_t$  is the growth rate of imports (*quantum*) in the period  $t$ ,  $\hat{p}_t^*$  is the domestic rate of inflation in the period  $t$ ,  $\hat{p}_t$  is the rest of the world inflation in the period  $t$ ,  $\hat{e}_t$  is the rate of depreciation of the nominal exchange rate in the period  $t$ ,  $\hat{y}_t$  is the domestic income growth rate in the period  $t$ ,  $\hat{z}_t$  is the rest of the world income growth rate in the period  $t$ ,  $\chi_0$  is the price elasticity of exports,  $\mu_0$  is the price elasticity of exports,  $\mu_1$  is the income elasticity of imports.

We will assume the validity of Marshall-Lerner's condition, so that,  $|\chi_0 + \mu_0| > 1$ . Moreover, we will follow Moreno-Brid's (2003) model in which the Balance of Payments restriction in period  $t$  is given by:

$$(3) \quad \hat{e}_t + \hat{p}_t^* + \hat{m}_t = \theta_1(\hat{p}_t + \hat{x}_t) - \theta_2(\hat{p}_t + \hat{d}_t) + \theta_3(\hat{p}_t + \hat{f}_t)$$

In equation (3)  $\theta_3 \equiv (1 - \theta_1 + \theta_2)$ , in which  $\theta_1 = \frac{P_x}{ep^*m}$  is the ratio between the initial value of exports and the initial value of imports,  $\theta_2 = \frac{pr}{ep^*m}$  is the ratio between the initial value of external liability services and the initial value of imports.

Moreover,  $\hat{d}_t$  is the growth rate of services (interest and dividends) related to the external liabilities in the period  $t$ ,  $\hat{f}_t$  is the real growth rate of external capital flows in period  $t$ .

As in Santana and Oreiro (2018) two important points can be observed. Firstly, the constraint imposed above is deflated in terms of value paid by imports. Secondly, we are considering an economy with a net debt to the rest of the world, since  $\theta_2$  is a positive parameter and there is a negative signal before it.

Assuming capital mobility to be imperfect in Mundell's sense, the real rate of growth of external capital flows will be a function of the difference between the domestic interest rate and the international interest rate adjusted by the country-risk premium:

$$(4) \quad \hat{f}_t = f(i_t - i_t^* - r) \quad ; \quad 0 \leq f < \infty$$

In equation (4)  $f$  is the sensibility of the growth of external capital flows to the interest differential,  $i_t$  is the domestic interest rate in the period  $t$ ,  $i_t^*$  international interest rate and  $r$  is the country risk premium. This coefficient captures the level of capital controls of

the economy. If  $f = 0$  then the capital account is closed, and capital mobility is zero. Since  $f < \infty$  then capital mobility is necessarily imperfect or limited due to the existence of some kind of capital controls.

Considering an economy with imperfect capital mobility, the dynamics of the nominal exchange rate, depend on inflows and outflows of foreign capital; but also on the level or intervention of Central Bank in the foreign exchange market. By means of reserve accumulation, the Central Bank can reduce the impact of capital flows over the dynamics of nominal exchange rate. Thus, we will assume that the rate of change of the nominal exchange rate will be a negative function of the growth rate of the external capital flows:

$$(5) \quad \hat{e}_t = -\kappa \hat{f}_t, \quad 0 \leq \kappa < \infty$$

In equation (5)  $\kappa$  is the coefficient of sensibility of the rate of change of nominal exchange rate in relation to the growth rate of external capital flows. This magnitude of this coefficient depends on the level of Central Bank intervention on the exchange rate market by means of reserve accumulation. If  $\kappa = 0$  then the Central Bank acts as a market maker in the exchange rate market, buying all inflows of foreign capital and accumulating them as international reserves.

Using (4) in (5) we get:

$$(5a) \quad \hat{e}_t = -\kappa f(i_t - i_t^* - r)$$

Equation (5a) shows that the rate of change of nominal exchange rate is a negative function of the interest rate differential.

The determination of the nominal interest rate ( $i_t$ ) follows the Taylor's (1993) rule.

$$(6) \quad i_t = \beta_0 + \beta_1(\hat{p}_t - \hat{p}^T) + \beta_2(u_t - u^T)$$

According to equation (6) the nominal interest rate essentially depends on the deviation of the effective inflation rate ( $\hat{p}_t$ ) from the target based on the monetary authority and the output gap. The latter is the difference between the actual real level of capacity utilization,

$u_t$ , and the *target level of capacity utilization*<sup>1</sup> by monetary authorities,  $u^T$ . All  $(\beta_i)$  are positive.

In particular, the parameter  $(\beta_0)$  can be seen as the sum of the international interest rate and the country risk premium:

$$(6a) \quad \beta_0 \equiv i_t^* + r$$

The *target level of capacity utilization* is determined by equation (7):

$$(7) u^T = u^{T-1} + \tilde{\theta}(u_{t-1} - u^{T-1}); \quad 0 < \tilde{\theta} < 1$$

In equation (7) we can see that the target level of capacity utilization (their own estimates about normal rate of capacity utilization) is adjusted over time as the actual level of capacity utilization in previous period is different from their previous estimates. It is clear that in steady-state  $u^T = u^{T-1} = u^n = u$ , which means that target level of capacity utilization is a full endogenous variable.

Using (6) in (5a) we get:

$$(5b) \quad \hat{e}_t = -\kappa f[\beta_1(\hat{p}_t - \hat{p}^T) + \beta_2(u_t - u^T)]$$

We will consider a small open economy without government activities. In the short-run, firms operate with idle capacity, which means that output is demand determined. The aggregate demand is the sum of household consumption ( $C_t$ ), firms' investment ( $I_t$ ) and net exports of final goods ( $Nx_t$ ). So, the level of real output is given by:

$$(8) \quad y_t = C_t + I_t + Nx_t$$

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<sup>1</sup>This hypothesis deserves further explanation. Mainstream models generally take for granted that potential output is determined by the availability of factors of production (capital and labor) and by the level of technological knowledge measured by the so-called total factor productivity. When applied to capacity utilization, this "vision" would impose the existence of a potential or normal level of capacity utilization, which is exogenous and independent of aggregate demand. The same idea is presented in some heterodox growth models like the Sraffian Super-Multiplier Models developed by Freitas and Serrano (2015). Demand-led growth models, however, calls into question the idea that output is supply constrained in the long-run. Kaldor (1988) argued that the growth rate of capital accumulation, growth rate of labor force and the growth rate of labor productivity depends, in the long-run, on the rate of growth of autonomous demand, which means that natural growth rate is an endogenous variable demand determined. The endogeneity of natural growth rate was tested by Leon-Ledesma and Thirlwall (2002) for a sample of 15 OECD countries in the period of 1961 to 1985. The results clearly shows that natural growth rate is elastic to demand and output growth. Oreiro et al (2012) make the same econometric tests for the Brazilian economy in the period between 1980 and 2002 and obtained similar results. This means that "normal" or potential level of capacity utilization can be considered a dependent variable, influenced by the lagged values of effective capacity utilization.

Household income is composed by wages and profits. Following Kaldor (1956, 1966) we will suppose that propensity to save out of profits is zero. For sake of simplicity, we will take the propensity to save out of profits to be equal to one. So, the real consumption expenditure is given by:

$$(9) \quad C_t = V_t L_t = V_t \frac{u_t}{q_t} K_t$$

Where:  $V_t$  is the real wage rate;  $L_t$  is the number of workers employed,  $q_t$  is the level of labor productivity.

In the absence of depreciation of the capital stock, the aggregate investment is equal to the product of the capital stock ( $K_t$ ) with the desired growth rate of capital stock ( $g$ ). This rate, in turn, is influenced by autonomous investment and by the expected real interest rate:

$$(10) \quad I_t = gK_t = [\varphi_0 - \varphi_1(i_t - \hat{p}_t^e)]K_t$$

$$\text{Where: } g = [\varphi_0 - \varphi_1(i_t - \hat{p}_t^e)]$$

In which ( $\hat{p}_t^e$ ) is the expected inflation rate, ( $\varphi_0$ ) is the autonomous component of investment (positive) and ( $\varphi_1$ ) a sensitivity coefficient (positive) that captures the influence of the expected real interest rate on investment decisions.

Inflation expectations are formulated according to the agents' perception of the monetary authority's credibility. In a context in which monetary policy is *fully credible*, the inflation expectation converges to the inflation target established by the monetary authority - equation (10a).

$$(10a) \quad \hat{p}_t^e = \hat{p}^T$$

The net exports ( $Nx_t$ ) - as a proportion of the capital stock - depend on the actual level of capacity utilization ( $u_t$ ) and on the real exchange rate of the previous period ( $E_{t-1}$ ). The time lag of the real exchange rate over net-exports captures the J curve effect.

$$(11) \quad Nx_t = (\eta_0 - \eta_1 u_t + \eta_2 E_{t-1})K_t$$

In which ( $\eta_0$ ) is a positive parameter, which incorporates the other variables that affect net exports, such as world income.

In order to determine the actual level of capacity utilization let us substitute equations (9)-(11) in (8):

$$(12) \quad u_t = \left( \frac{\varphi_0 + \eta_0}{\pi_t + \eta_1} \right) - \left[ \frac{\varphi_1}{\pi_t + \eta_1} \right] (i_t - \hat{p}_t^e) + \left[ \frac{\eta_2}{\pi_t + \eta_1} \right] E_{t-1}$$

Where:  $u_t = \frac{y_t}{K_t}$  is the actual level of capacity utilization and  $\pi_t \equiv 1 - \frac{V_t}{q_t}$  is the profit share.

Substituting (6) in (12) we get:

$$(13) \quad u_t = \left( \frac{1}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) [(\varphi_0 + \eta_0 - \varphi_1 \beta_0) + \varphi_1 \beta_2 u^T + \eta_2 E_{t-1} + \varphi_1 \hat{p}_t^e - \varphi_1 \beta_1 (\hat{p}_t - \hat{p}^T)]$$

Equation (13) determines the short-run equilibrium value for the actual level of capacity utilization. Considering a *fully credible monetary policy* we had  $\hat{p}_t^e = \hat{p}^T$  so:

$$(13a) \quad u_t = \left( \frac{1}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) [(\varphi_0 + \eta_0 - \varphi_1 \beta_0) + \varphi_1 \beta_2 u^T + \eta_2 E_{t-1} + \varphi_1 ((1 - \beta_1) \hat{p}^T - \beta_1 \hat{p}_t)]$$

From (13a) we get:

$$\frac{\partial u_t}{\partial (\varphi_0 + \eta_0)} = \left( \frac{1}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) > 0 \quad (13b)$$

$$\frac{\partial u_t}{\partial u^T} = \left( \frac{\varphi_1 \beta_2}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) > 0 \quad (13c)$$

$$\frac{\partial u_t}{\partial E_{t-1}} = \left( \frac{\eta_2}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) > 0 \quad (13d)$$

$$\frac{\partial u_t}{\partial \hat{p}^T} = \left( \frac{\varphi_1 (1 - \beta_1)}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) > 0 \quad (13e)$$

$$\frac{\partial u_t}{\partial \hat{p}_t} = - \left( \frac{\beta_1}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) < 0 \quad (13f)$$

We have now to describe the supply side of the economy. Following Blecker (2002) we will suppose that domestic firms produce a homogeneous good which is imperfect

substitute from the goods produced by foreign firms. Labor is the only variable input used in production and domestic firms had monopoly power so that they can fix the prices of their goods in the level higher than unit labor cost. Then prices are fixed by a mark-up over unit direct costs of production, but mark-up depends on the level of real exchange rate: the more depreciated is the level of real exchange rate, higher will be the mark-up that domestic firms can set without losing market-share for foreign firms.

Then domestic prices are determined by equation (14):

$$(14) \quad p_t = \sigma_0 E_t^\theta \frac{w_t}{q_t}$$

Where:  $1 + \tau = \sigma_0 E_t^\theta > 1$ ,  $\tau$  is the mark-up rate and  $\theta > 0$  is the sensitivity of the mark-up rate to real exchange rate.

From (14) we can define the real product wage as:

$$(14a) \quad \frac{w_t}{p_t} = \frac{q_t}{\sigma_0 E_t^\theta}$$

Households spend their wage income both in domestic and foreign consumption goods. So, the workers effective real wage rate will be given by:

$$(15) \quad V_t = \frac{w_t}{(p_t)^\varepsilon (e_t p_t^*)^{1-\varepsilon}}$$

Where:  $0 < \varepsilon < 1$  is the weight of domestic goods in the consumption basket of households.

From (14a) and (15) we get:

$$(16) \quad V_t = \frac{q_t}{\sigma_0 E_t^{\sigma_1}}$$

Where:  $\sigma_1 \equiv 1 + \theta - \varepsilon > 0$

Equation (16) shows that workers effective real wage rate is a positive function of labor productivity and a negative function of the level of real exchange rate.

Regarding the labor market, we will consider an economy where workers are organized by means of Unions so that wages are set by a collective bargaining between labor unions and firms. Labor unions defines a target real wage ( $\bar{V}_t$ ) which will be used as the reference for nominal wage bargaining with firms. We will suppose that the target real wage depends on the rate of employment ( $l_t$ ), due to the fact that a tight labor market increases the bargaining power of unions. So, we get:

$$(17) \quad \bar{V}_t = v_0 + v_1 l_t$$

Where:  $l_t = \frac{L_t}{N_t}$  is the rate of employment,  $N_t$  is the labor force and  $L_t$  is the number of workers that are employed.

It can be easily show that the rate of employment is given by:

$$(17a) \quad l_t = \frac{u_t k_t}{q_t}$$

Where:  $k_t = \frac{K_t}{N_t}$  is the stock of capital per-worker.

The growth rate of nominal wages is thus determined by:

$$(18) \quad \hat{w}_t = \hat{p}^T + \varpi(\bar{V}_t - V_t)$$

In which ( $\varpi$ ) is a positive coefficient that measures the bargaining power of workers<sup>2</sup>.

Taking logs in (14) and time derivative of the resulting expression we get:

$$(19) \quad \hat{p}_t = \left[ \frac{\theta}{1 + \theta} \right] (\hat{e}_t + \hat{p}_t^*) + \left[ \frac{1}{1 + \theta} \right] (\hat{w}_t - \hat{q}_t)$$

Equation (19) shows that domestic inflation is a weighted average of the growth rate of prices of foreign goods (which is the sum of the rate of nominal exchange rate devaluation and international inflation) and the growth rate of unit labor cost (which is the difference between wage inflation and productivity growth).

The growth rate of labor productivity is given by:

$$(20) \quad \hat{q}_t = \alpha_0 + \alpha_1 h_{t-1} \hat{y}_{t-1} + \alpha_2 l_{t-1}$$

In equation (20) the labor productivity growth rate ( $\hat{q}_t$ ) depends on an autonomous component ( $\alpha_0$ ) and on the growth rate of output in the previous period multiplied by the manufacturing share in output and a positive coefficient ( $\alpha_1$ ). The influence of the previous period growth rate of output over productivity growth is due to the existence of dynamic economies of scale in manufacturing industry, which is the basis of the so-called “Kaldor-Verdoorn Law”. The magnitude of such an effect depends on the size of the manufacturing sector relative to the rest of the economy (Oreiro et al. 2020; Botta, 2009, Gabriel et al. 2016). The third element on equation (20) captures the Marxist idea that a tight labor market may induce capitalists to introduce labor-saving innovations in order to preserve the rate of profit in an environment of increasing real wages.

Using (16), (17) and (17a) in (18) we get:

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<sup>2</sup>This coefficient depends on several structural variables of the labor market as the level of centralization of wage bargains, the share of workers that are unionized, the level of unemployment insurance, among other variables. See Amadeo (1994) for a detailed discussion regarding this issue.

$$(18a) \quad \hat{w}_t = (\varpi v_0 + \hat{p}^T) + \varpi \left( \frac{v_1 \sigma_0 k_t u_t E_t^{\sigma_1} - q_t^2}{\sigma_0 q_t E_t^{\sigma_1}} \right)$$

Using (5b), (18a), (20) in (19) we get:

$$(21) \quad \hat{p}_t = \left[ \frac{1}{1 + \theta(1 + f\kappa\beta_1)} \right] \left[ (1 + f\kappa\theta\beta_1)\hat{p}^T + \theta\hat{p}_t^* + (\varpi v_0 - \alpha_0) - \frac{\alpha_2 u_{t-1} k_{t-1}}{q_{t-1}} - \frac{\varpi q_t}{\sigma_0 E_t^{\sigma_1}} - \alpha_1 h_{t-1} \hat{y}_{t-1} + f\kappa\theta\beta_2 u^T + \left( \frac{\varpi v_1 k_t - f\kappa\theta\beta_2 q_t}{q_t} \right) u_t \right]$$

From equation (21) we get:

$$\frac{\partial \hat{p}_t}{\partial \hat{p}^T} = \left[ \frac{(1 + f\kappa\theta\beta_1)}{1 + \theta(1 + f\kappa\beta_1)} \right] > 0 \quad (21a)$$

$$\frac{\partial \hat{p}_t}{\partial \hat{p}_t^*} = \left[ \frac{\theta}{1 + \theta(1 + f\kappa\beta_1)} \right] > 0 \quad (21b)$$

$$\frac{\partial \hat{p}_t}{\partial u_t} = \left[ \frac{\varpi v_1 k_t - f\kappa\theta\beta_2 q_t}{(1 + \theta(1 + f\kappa\beta_1) q_t)} \right] \quad (21c)$$

In equation (21c)  $\frac{\partial \hat{p}_t}{\partial u_t} > 0$  if and only if  $(\varpi v_1 k_t - f\kappa\theta\beta_2 q_t) > 0$ .

### 3. Short Run Equilibrium

In the short run the real exchange rate, the profit-share, the growth rate of world income, the international inflation rate, the target capacity utilization, the stock of capital per-worker and the manufacturing share are constants, as well as all variables that had pre-determined values, i.e., determined from previous periods. The model is thus composed by the following equations:

$$(1) \quad \hat{x}_t = \chi_0(\hat{p}_t^* - p_t + \hat{e}_t) + \chi_1 \hat{z}_t$$

$$(2) \quad \hat{m}_t = \mu_0(\hat{p}_t - \hat{p}_t^* - \hat{e}_t) + \mu_1 \hat{y}_t$$

$$(3) \quad \hat{e}_t + \hat{p}_t^* + \hat{m}_t = \theta_1(\hat{p}_t + \hat{x}_t) - \theta_2(\hat{p}_t + \hat{d}_t) + \theta_3(\hat{p}_t + \hat{f}_t)$$

$$(4a) \quad \hat{f}_t = f[\beta_1(\hat{p}_t - \hat{p}^T) + \beta_2(u_t - u^T)]$$

$$(5b) \quad \hat{e}_t = -\kappa f[\beta_1(\hat{p}_t - \hat{p}^T) + \beta_2(u_t - u^T)]$$

$$(13a) \quad u_t = \left( \frac{1}{\pi_t + \eta_1 + \varphi_1 \beta_1} \right) [(\varphi_0 + \eta_0 - \varphi_1 \beta_0) + \varphi_1 \beta_2 u^T + \eta_2 E_{t-1} + \varphi_1((1 - \beta_1)\hat{p}^T - \beta_1 \hat{p}_t)]$$

$$(21) \quad \hat{p}_t$$

$$= \left[ \frac{1}{1 + \theta(1 + f\kappa\beta_1)} \right] \left[ (1 + f\kappa\theta\beta_1)\hat{p}^T + \theta\hat{p}_t^* + (\varpi v_0 - \alpha_0) - \frac{\alpha_2 u_{t-1} k_{t-1}}{q_{t-1}} - \frac{\varpi q_t}{\sigma_0 E_t^{\sigma_1}} - \alpha_1 h_{t-1} \hat{y}_{t-1} + f\kappa\theta\beta_2 u^T + \left( \frac{\varpi v_1 k_t - f\kappa\theta\beta_2 q_t}{q_t} \right) u_t \right]$$

Then we have a system of seven independent equations with seven unknowns:  $\hat{x}_t$ ,  $\hat{m}_t, \hat{y}_t, \hat{f}_t, \hat{e}_t, u_t$  and  $\hat{p}_t$ . This means that we had a determinate system. Some observations are required to make about this system of equations. First of all, it allowed us to calculate the level of employment and capacity utilization as well as the growth rate of real output. This is an important advance in respect with balance of payments constrained growth models where only the growth rate of real output can be determined, but not the general level of resources utilization. Second, the interdependence of the equations of the model is not general: the model is *block-recursive* (Sargent, 1987). Indeed, the short-run equilibrium values of capacity utilization and inflation rate could be entirely determined by equations (13a) and (21) alone. Once these values are determined, equations (4a) and (5b) determine the growth rate of foreign capital flows and the rate of exchange rate depreciation. Once these values are determined, equations (1)-(3) determine the growth rate of imports, the growth rate of exports and the growth rate of real output that is compatible with the balance of payments equilibrium.

Equations (13a) and (21) are, respectively, the aggregate demand and aggregate supply equations of the model. The determination of the short-run equilibrium values for the inflation rate and capacity utilization can be visualized by figure 1.

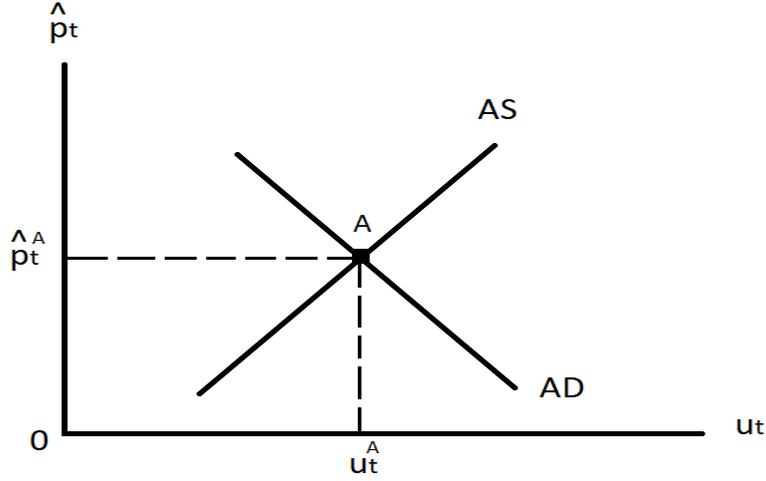


Figure 1: Aggregate Demand and Aggregate Supply Curves and Short-Run Equilibrium

Solving the model for equations (13a) and (21) we get the short-run equilibrium value for the domestic rate of inflation:

$$(22) \hat{p}_t^A = \rho_0 + \rho_2 \hat{p}_t^* + (\rho_1 + \rho_3) \hat{p}^T + \rho_4 u^T + \rho_5 E_{t-1} - \frac{\rho_6 u_{t-1} k_{t-1}}{q_{t-1}} - \rho_7 h_{t-1} \hat{y}_{t-1}$$

Where:

$$\rho_0 = \frac{(\varpi v_0 - \alpha_0) (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} (\varphi_0 + \eta_0 - \varphi_1 \beta_0)}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_1 = \frac{(\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_2 = \frac{\theta (\pi_t + \eta_1 + \varphi_1 \beta_2)}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_3 = \frac{f \kappa \beta_1 (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_4 = \frac{f \kappa \theta \beta_2 (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_2}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_5 = \frac{(\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \eta_2}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_6 = \frac{\alpha_2 (\pi_t + \eta_1 + \varphi_1 \beta_2)}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

$$\rho_7 = \frac{\alpha_1 (\pi_t + \eta_1 + \varphi_1 \beta_2)}{[1 + \theta(1 + f \kappa \beta_1)] (\pi_t + \eta_1 + \varphi_1 \beta_2) + (\varpi v_1 k_t - f \kappa \beta_2 q_t) q_t^{-1} \varphi_1 \beta_1} > 0$$

Substituting (22) in (13a) we get the short-run equilibrium value for the capacity utilization:

$$(23) \quad u_t^A = \frac{\gamma_0 + \gamma_1 u^T + \gamma_2 E_{t-1} + (\gamma_3 + \gamma_4) \hat{p}^T - \gamma_5 \hat{p}_t^* + \gamma_6 u_{t-1} k_{t-1} q_{t-1}^{-1} + \gamma_7 h_{t-1} \hat{y}_{t-1}}{\pi_t + \eta_1 + \varphi_1 \beta_2}$$

Where:

$$\gamma_0 = (\varphi_0 + \eta_0 - \varphi_1 \beta_0) - \varphi_1 \beta_1 \rho_0 > 0 \leftrightarrow (\varphi_0 + \eta_0) > \varphi_1 (\beta_0 + \beta_1 \rho_0)$$

$$\gamma_1 = \varphi_1 (\beta_2 - \beta_1 \rho_4) \leftrightarrow \beta_2 > \beta_1 \rho_4^3$$

$$\gamma_2 = (\eta_2 - \varphi_1 \beta_1 \rho_5) > 0 \leftrightarrow \eta_2 > \varphi_1 \beta_1 \rho_5^4$$

$$\gamma_3 = \varphi_1 (1 - \beta_1 \rho_1) > 0 \leftrightarrow 1 > \beta_1 \rho_1$$

$$\gamma_4 = \varphi_1 \beta_1 (1 - \rho_3) > 0 \leftrightarrow 1 > \rho_3$$

$$\gamma_5 = \varphi_1 \beta_1 \rho_2 > 0$$

$$\gamma_6 = \varphi_1 \beta_1 \rho_6 > 0$$

$$\gamma_7 = \varphi_1 \beta_1 \rho_7 > 0$$

From equation (23) we get:

$$\frac{\partial u_t^A}{\partial \gamma_0} = \frac{1}{\pi_t + \eta_1 + \varphi_1 \beta_2} > 0 \quad (23a)$$

$$\frac{\partial u_t^A}{\partial E_{t-1}} = \frac{\gamma_2}{\pi_t + \eta_1 + \varphi_1 \beta_2} > 0 \quad (23b)$$

$$\frac{\partial u_t^A}{\partial \hat{p}^T} = \frac{(\gamma_3 + \gamma_4)}{\pi_t + \eta_1 + \varphi_1 \beta_2} > 0 \quad (23c)$$

$$\frac{\partial u_t^A}{\partial \hat{p}_t^*} = - \frac{\gamma_5}{\pi_t + \eta_1 + \varphi_1 \beta_2} < 0 \quad (23d)$$

$$\frac{\partial u_t^A}{\partial \pi_t} = - \frac{u_t^A}{\pi_t + \eta_1 + \varphi_1 \beta_2} < 0 \quad (23e)$$

Two results are noteworthy to be commented. The first one is equation (23b). A real exchange devaluation in period t-1 will increase the actual level of capacity utilization. This

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<sup>3</sup>This requires that Monetary Authorities give more weight to deviations of actual capacity utilization from the target level than deviations of inflation from the target level. This could occur in a double mandate regime of monetary policy.

<sup>4</sup>This condition requires a high price elasticity of net exports to real exchange rate variation, which means that the economy under consideration is mainly an exporter of manufacturing goods.

means that exchange rate devaluation had an expansionary effect over aggregate demand only after a time lag. The second one is equation (23e) which shows that an increase in the profit share in period  $t$  – due, for example, a real exchange rate devaluation in period  $t$  – will decrease the actual level of capacity utilization, which means that demand regime is wageled in the short run, and a devaluation of real exchange rate had a short-run negative impact over the level of economic activity.

Using equations (23) and (22) in the system (1)-(3), (4a) and (5b) we arrive at the value of the growth rate of real output that is compatible with balance of payments equilibrium:

$$(24) \quad \hat{y}_t = \phi_0 + \phi_2(u_t^A - u^T) + \phi_3\hat{p}_t^* - (\phi_1 + \phi_4)\hat{p}^T$$

Where<sup>5</sup>:

$$\begin{aligned} \phi_0 &= (\phi\beta_0 - \theta_1\chi_1\hat{z}_t + \theta_2\hat{d}) \mu_1^{-1} > 0 \\ \phi_1 &= (\phi f\beta_1 + \theta_1\chi_0 + \mu_0) \mu_1^{-1} > 0 \\ \phi_2 &= \phi f\beta_2\mu_1^{-1} > 0 \\ \phi_3 &= (\theta_1\chi_0 + \mu_0) \mu_1^{-1} = \left(\theta_1\left(\frac{\chi_0}{\mu_0}\right) + 1\right)\mu_1^{-1} > 0 \\ \phi_4 &= \phi f\beta_1\mu_1^{-1} > 0 \\ \phi &= (1 + \mu_0 - \theta_1\chi_1)(\kappa\mu_1 + \theta_3) > 0 \end{aligned}$$

From equation (24) we get:

$$\frac{\partial \hat{y}_t}{\partial u_t^A} = \phi_2 > 0 \quad (24a)$$

Expression (24a) shows that an increase in the short-run equilibrium level of capacity utilization produces an increase in the growth rate of real output compatible with balance of payments equilibrium. This occurs because an increase in the level of capacity utilization will increase the rate of employment and hence on the domestic rate of inflation. For a given rate of change of nominal exchange rate and international inflation, this will allow an increase in the *terms of trade* and hence on the growth rate of imports (and growth rate of real output) compatible with balance of payments equilibrium. Furthermore, the increase in the domestic inflation will induce Central Bank to increase the level of domestic interest rate, increasing interest rate differential and hence increasing the growth rate of capital flows.

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<sup>5</sup>The sufficient conditions for these parameters be positive are:  $[\beta_0(1 + \mu_0)] > \theta_1\chi_1$  and  $(\phi\beta_0 + \theta_2\hat{d}) > \theta_1\chi_1\hat{z}_t$ .

The balance of payments constraint is thus relaxed, allowing a faster growth rate of real output in the short-run.

From expressions (23e) and (24a) we can conclude that the short-run effect of a exchange rate devaluation will be (i) a decrease in the actual level of capacity utilization and, hence, in the rate of employment and (ii) a decrease in the growth rate of real output that is compatible with the balance of payments equilibrium.

#### 4. Balanced Growth without Structural Change

In the steady-steady balanced growth path it is required that the following conditions hold<sup>6</sup>:

$$\hat{p}_{t-1} = \hat{p}_t = \hat{p} \quad (25a)$$

$$\hat{p}^e = \hat{p}^T \quad (25b)$$

$$u_{t-1} = u_t = u^T = u \quad (25c)$$

$$\hat{y}_{t-1} = \hat{y}_t = \hat{y} \quad (25d)$$

$$h_{t-1} = h_t = h \quad (25e)$$

$$E_{t-1} = E_t = E \quad (25f)$$

Substituting (25a)-(25f) in equations (23) and (24) we get:

$$(26) \quad u^* = \frac{\gamma_0 + \gamma_1 u^* + \gamma_2 E + (\gamma_3 + \gamma_4) \hat{p}^T - \gamma_5 \hat{p}^* + \gamma_6 l^* + \gamma_7 h \hat{y}^*}{\pi + \eta_1 + \varphi_1 \beta_2}$$

$$(27) \quad \hat{y}^* = \phi_0 + \phi_3 \hat{p}_t^* - (\phi_1 + \phi_4) \hat{p}^T$$

Solving (26) for  $u^*$  we get:

$$(26a) \quad u^* = \frac{\gamma_0 + \gamma_2 E + (\gamma_3 + \gamma_4) \hat{p}^T - \gamma_5 \hat{p}^* + \gamma_6 l^* + \gamma_7 h \hat{y}^*}{(\pi + \eta_1 + \varphi_1 \beta_2)(1 - \gamma_1)}$$

In equation (26a) we can see that the steady-state value of capacity utilization is a function of the steady-state rate of employment. The issue is how the steady-state level of employment is determined?

In order to answer this question, we have to notice that in a balanced growth path the actual growth rate of output must be equal to the natural growth rate, which is the sum of the

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<sup>6</sup>Throughout this section we will consider both the real exchange rate and manufacturing share as exogenous variables.

growth rate of the labor force and the growth rate of labor productivity. Let us assume that the growth rate of labor force is exogenous and equal to  $n$ . Then, the natural growth rate is given by:

$$g_N = n + \alpha_0 + \alpha_1 h \hat{y} + \alpha_2 l \quad (28)$$

For a balanced growth path to exist is required that  $g_N = \hat{y}$ , so we get:

$$(29) \quad l^* = \frac{\hat{y}(1 - \alpha_1 h) - (n + \alpha_0)}{\alpha_2}$$

Equation (29) presents the rate of employment in the balanced growth path as a positive function of the growth rate of real output and a negative function of the manufacturing share<sup>7</sup>. The rate of employment is the variable that adjusts the natural growth rate, which is an endogenous variable, to the actual growth rate of real output, through variations in the rate of productivity growth. **It is important to notice that there is no reason to believe that  $l^*$  corresponds to the full employment of the labor force.**

Notice that an increase in the manufacturing share will have a negative impact over the steady-state level of employment (see equation 28a). This occurs because an increase in the employment rate will increase the growth rate of productivity for a given growth rate of real output, thereby reducing the demand for labor in the economy.

$$\frac{\partial l^*}{\partial h} = -\frac{\alpha_1}{\alpha_2} \hat{y}^* < 0 \quad (29a)$$

The domestic rate of inflation in the balanced growth path is given by:

$$(30) \quad \hat{p}^{**} = \rho_0 + \rho_2 \hat{p}^* + (\rho_1 + \rho_3) \hat{p}^T + \rho_4 u^* + \rho_5 E - \rho_6 l^* - \rho_7 h \hat{y}^*$$

In equation (30) we can see that in steady-state, for a given level of real exchange rate, the value for the domestic inflation only by chance will be equal to the target inflation<sup>8</sup>.

For a balanced growth path to exist it is necessary that the growth rate of real output is equal to the growth rate of capital stock in order for capacity utilization to be constant over time. This condition requires that:

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<sup>7</sup>The approach used here follows Ros (2013, chapter 10).

<sup>8</sup> If actual inflation does not converge in the long-run for the target inflation one could ask why monetary policy is taken to be credible by economic agents, making them to set their inflation expectations equal to the target set by Monetary Authorities? The most obvious reason is that in most countries that adopt an Inflation Targeting Regime, the target inflation is not a number but a band with floor and ceiling for the inflation rate. This is, for example, the case of Brazil. This means that the inflation rate is within this band there is no reason to distrust the commitment of the Central Bank with the target inflation.

$$(31) \quad \phi_0 + \phi_3 \hat{p}_t^* - (\phi_1 + \phi_4) \hat{p}^T = [\phi_0 - \phi_1(i^* + r - \hat{p}^T)]$$

In equation (31) all variables are parameters, which means that only by chance this condition will be satisfied. We came back to Harrod's first problem. In order to solve this problem, we will assume that in the long-run the autonomous component of investment is no longer an exogenous variable but adjusts over time according to the following equation (See Lavoie, 2016):

$$(32) \quad \frac{d\phi_0}{dt} = \phi(\hat{y}^* - g) = \phi_0 + \phi_3 \hat{p}_t^* - (\phi_1 + \phi_4) \hat{p}^T - [\phi_0 - \phi_1(i^* + r - \hat{p}^T)]$$

In steady-state, we have  $\frac{d\phi_0}{dt} = 0$ , so we get:

$$(32a) \quad \phi_0^* = \phi_0 + \phi_3 \hat{p}_t^* - (\phi_1 + \phi_4 + \phi_1) \hat{p}^T + \phi_1(i^* + r)$$

Summing-up the steady-state balanced growth path solution of the model is given by:

$$(26a) \quad u^* = \frac{\gamma_0 + \gamma_2 E + (\gamma_3 + \gamma_4) \hat{p}^T - \gamma_5 \hat{p}^* + \gamma_6 l^* + \gamma_7 h \hat{y}^*}{(\pi + \eta_1 + \phi_1 \beta_2)(1 - \gamma_1)}$$

$$(27) \quad \hat{y}^* = \phi_0 + \phi_3 \hat{p}_t^* - (\phi_1 + \phi_4) \hat{p}^T$$

$$(29) \quad l^* = \frac{\hat{y}^*(1 - \alpha_1 h) - (n + \alpha_0)}{\alpha_2}$$

$$(30) \quad \hat{p}^{**} = \rho_0 + \rho_2 \hat{p}^* + (\rho_1 + \rho_3) \hat{p}^T + \rho_4 u^* + \rho_5 E - \rho_6 l^* - \rho_7 h \hat{y}^*$$

$$(32) \quad \phi_0^* = \phi_0 + \phi_3 \hat{p}_t^* - (\phi_1 + \phi_4 + \phi_1) \hat{p}^T + \phi_1(i^* + r)$$

Regarding the steady-state solution of the model, it should be noticed the non-neutrality of the monetary policy. In fact, if the Monetary Authority changes the target inflation, then this will affect the growth rate of real output, the capacity utilization and the rate of employment. From equation (27) we get:

$$(27a) \quad \frac{\partial \hat{y}^*}{\partial \hat{p}^T} = -(\phi_1 + \phi_4) < 0$$

In expression (27a) we can see that an increase in the target inflation will produce a reduction in the long-run growth rate of real output.

From (26a) and (27a) we get:

$$(26b) \quad \frac{\partial u^*}{\partial \hat{p}^T} = \left[ \frac{(\gamma_3 + \gamma_4)}{(\pi + \eta_1 + \varphi_1 \beta_2)(1 - \gamma_1)} \right] - \left[ \frac{\gamma_7 h(\phi_1 + \phi_4)}{(\pi + \eta_1 + \varphi_1 \beta_2)(1 - \gamma_1)} \right]$$

From (29) and (27a) follow:

$$(29b) \quad \frac{\partial l^*}{\partial \hat{p}^T} = - \frac{(1 - \alpha_1 h)(\phi_1 + \phi_4)}{\alpha_2} < 0$$

In expression (28b) the effect of an increase in the target inflation is clearly negative over the long-run value of employment rate. **This means that the long-run Phillips curve is positive sloped.** The effect of an increase in target inflation over long-run capacity utilization is ambiguous in equation (26b), but the second term in the brackets is clearly an increasing function of the manufacturing share. This means that for highly industrialized small open economies, an increase in the target inflation is likely to be associated with a reduction in the long-run level of capacity utilization.

## 5. Monetary Policy, Real Exchange Rate and Structural Change

Until now, we have maintained manufacturing's share and the real exchange rate constant throughout the analysis. Thus, the present section has the purpose of relaxing the hypothesis of constant of real exchange rate and manufacturing share.

The dynamics of manufacturing share over time are influenced by the *price competitiveness* as well as *non price competitiveness*. With regards to the *price competitiveness*, an overvalued exchange rate; i.e. a real exchange rate below some long-run equilibrium value, may lead to a progressive reduction of the share of manufacturing industry in GDP, since such a situation induces an increased transfer of productive activities to other countries. We will call this level of the real exchange rate of “industrial equilibrium level”.<sup>9</sup> Thus, an overvalued RER is associated with a negative structural change on the economy, which we may call premature deindustrialization (Palma, 2005). An undervalued exchange rate, that is, above its industrial equilibrium level would have the opposite effect,

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<sup>9</sup>See Bresser-Pereira and Gala (2010) and Bresser-Pereira, Oreiro and Marconi (2015) about the exchange rate at the industrial equilibrium level. Industrial equilibrium exchange rate is defined in these works as the level of real exchange rate that makes firms which operate with the state of art technology competitive both in domestic and international markets. The problem with this concept is that, for developing countries, firms in general operate behind the technological frontier. For overcome this conceptual problem, we will redefine industrial equilibrium exchange rate as the level of real exchange rate that, for a given level of technological gap, makes the share of manufacturing industry on real output constant over time.

to induce a transfer of productive activities to the domestic economy, thereby increasing the share of the manufacturing industry in the GDP.

A fundamental feature of developing economies is that these economies are far from the technological frontier and therefore their firms cannot operate with the state-of-art technology. This technological gap negatively affects the *non price competitiveness* of manufacturing firms in developing economies, which produce manufactured goods that are of inferior quality and/or lower technological intensity than the manufactured goods produced in the developed economies (Verspagen, 1993). It follows that the existence of the technological gap is an aspect that acts to reduce the competitiveness of developing countries industries, thus contributing to a reduction in its share of the manufactured industry on real output.

From above discussion, we will assume that the dynamics of the share of manufacturing industry in real output is given by the following differential equation:

$$(33) \quad \hat{h} = h_0 + h_1 E - h_2 G$$

Where:  $\hat{h}$  is the growth rate of the share of manufacturing industry in real output ;  $E$  is the level of the real exchange rate;  $G = \frac{T_f}{T_d}$  is the *technological gap*, defined as the ratio between the level of scientific and technological knowledge at the technological frontier ( $T_f$ ) and level of scientific and technological knowledge at domestic economy ( $T_d$ );  $h_1 > 0$  is a parameter that represents the discretionary policies that directly address the industrial development such as trade tariffs;  $h_2 > 0$  is a coefficient that captures the sensitivity of the productive structure to the technological gap and  $h_0 < 0$  is a parameter that captures the effect of “mature deindustrialization” due to the effects of the rising levels of per-capita income over the demand for manufacturing goods (Rowthorn and Ramaswamy, 1999).

The industrial equilibrium exchange rate will be defined as the level of real exchange rate for which the manufacturing share is constant over time (Oreiro, 2020). From (33), making  $\hat{h} = 0$ , we get:

$$E^i = \frac{h_2}{h_1} G - \frac{h_0}{h_1} \quad (34)$$

Where:  $E^i$  is the industrial equilibrium level of real exchange rate.

In equation (34) we can see that the industrial equilibrium level of real exchange rate is an increasing function of the technological gap, which means that higher is the distance of

a developing country to the technological frontier, higher will be the real exchange rate required to hold manufacturing share constant over time. We can also see that industrial equilibrium exchange rate is a negative function of the level of trade tariffs, captured by the coefficient  $h_1$ .

It can be easily shown that the dynamics of manufacturing share will be dependent on the level of real exchange rate overvaluation compared to the industrial equilibrium level. For doing so, let us make some algebraic manipulation in equation (33) as shown below.

$$(35) \quad \hat{h} = h_0 + h_1 E^i + h_1 (E - E^i) - h_2 G = h_0 + h_2 G - h_0 + h_1 (q - q^i) - h_2 G$$

Then we get:

$$(36) \quad \hat{h} = h_1 (E - E^i)$$

In steady-state  $\hat{h} = 0$ , so we get:

$$(36a) \quad E^* = E^i = \left[ \frac{h_2}{h_1} G - \frac{h_0}{h_1} \right]$$

The dynamics of real exchange rate is given by:

$$(36b) \quad \hat{E}_t = \hat{e}_t + \hat{p}_t^* - \hat{p}_t$$

From equation (5b) that in steady-state we have:

$$(37) \quad \hat{e}_t = -\kappa f [\beta_1 (\hat{p}_t - \hat{p}^T)]$$

Substituting (37) in (36b) we get:

$$(38) \quad \hat{E}_t = -\kappa f [\beta_1 (\hat{p}_t - \hat{p}^T)] + \hat{p}_t^* - \hat{p}_t = \hat{p}_t^* + \kappa f \beta_1 \hat{p}^T - (1 + \kappa f \beta_1) \hat{p}_t$$

Substituting (30) in (38) we get:

$$(39) \quad \hat{E}_t = \{[1 - (1 + \kappa f \beta_1)] \rho_2\} \hat{p}^* - (1 + \kappa f \beta_1) \rho_0 + [\kappa f \beta_1 - (1 + \kappa f \beta_1) (\rho_1 + \rho_3)] \hat{p}^T \\ - (1 + \kappa f \beta_1) \rho_4 u^* - (1 + \kappa f \beta_1) \rho_5 E + (1 + \kappa f \beta_1) \rho_6 l^* \\ + (1 + \kappa f \beta_1) \rho_7 h \hat{y}^*$$

In steady-state we have  $\hat{E}_t = 0$ , so we get:

$$(40) \quad E = \frac{1}{(1 + \kappa f \beta_1) \rho_5} \{ [1 - (1 + \kappa f \beta_1)] \rho_2 \} \hat{p}^* - (1 + \kappa f \beta_1) \rho_0 \\ + [\kappa f \beta_1 - (1 + \kappa f \beta_1) (\rho_1 + \rho_3)] \hat{p}^T - (1 + \kappa f \beta_1) \rho_4 u^* + (1 + \kappa f \beta_1) \rho_6 l^* \\ + (1 + \kappa f \beta_1) \rho_7 h \hat{y}^* \}$$

Equation (40) defines a locus of combinations between the level of real exchange rate and the manufacturing share for which real exchange rate is constant over time. Taking the derivative of (40) in respect to  $E$  and  $h$ , we get:

$$(41) \quad \left[ \frac{\partial E}{\partial h} \right]_{\hat{E}=0} = -\frac{\rho_4}{\rho_5} \frac{\partial u^*}{\partial h} + \frac{\rho_6}{\rho_5} \frac{\partial l^*}{\partial h} + \frac{\rho_7}{\rho_5} \hat{y}^*$$

We know that:

$$\frac{\partial l^*}{\partial h} = -\frac{\alpha_1}{\alpha_2} \hat{y}^* < 0 \quad (29a)$$

Taking the derivative of  $u$  in respect to  $h$  in (26a) we get:

$$(42) \quad \frac{\partial u^*}{\partial h} = \left[ -\frac{\gamma_6}{\pi + \eta_1 + \varphi_1 \beta_2} \left( \frac{\alpha_1}{\alpha_2} \right) + \frac{\gamma_7}{\pi + \eta_1 + \varphi_1 \beta_2} \right] \hat{y}^*$$

Substituting (42) and (28a) in (41) we get:

$$(43) \quad \left[ \frac{\partial E}{\partial h} \right]_{\hat{E}=0} \\ = \left\{ \left[ \frac{\gamma_6}{\pi + \eta_1 + \varphi_1 \beta_2} \left( \frac{\alpha_1}{\alpha_2} \right) - \frac{\gamma_7}{\pi + \eta_1 + \varphi_1 \beta_2} \right] \left( \frac{\rho_4}{\rho_5} \right) - \left[ \left( \frac{\rho_6}{\rho_5} \right) \left( \frac{\alpha_1}{\alpha_2} \right) \right] + \left( \frac{\rho_7}{\rho_5} \right) \right\} \hat{y}^*$$

In expression (43) the third component is positive  $\left( \frac{\rho_7}{\rho_5} \right)$ , the second component is negative  $\left\{ - \left[ \left( \frac{\rho_6}{\rho_5} \right) \left( \frac{\alpha_1}{\alpha_2} \right) \right] \right\}$  and the first component can be positive or negative  $\left\{ \left[ \frac{\gamma_6}{\pi + \eta_1 + \varphi_1 \beta_2} \left( \frac{\alpha_1}{\alpha_2} \right) - \frac{\gamma_7}{\pi + \eta_1 + \varphi_1 \beta_2} \right] \left( \frac{\rho_4}{\rho_5} \right) \right\}$ . So, the locus  $\hat{E} = 0$  can be either positive or negative sloped.

The stability of the system composed by equations (36) and (38) can be analyzed once the system is linearized around its long-run equilibrium position and presented in the matrix form:

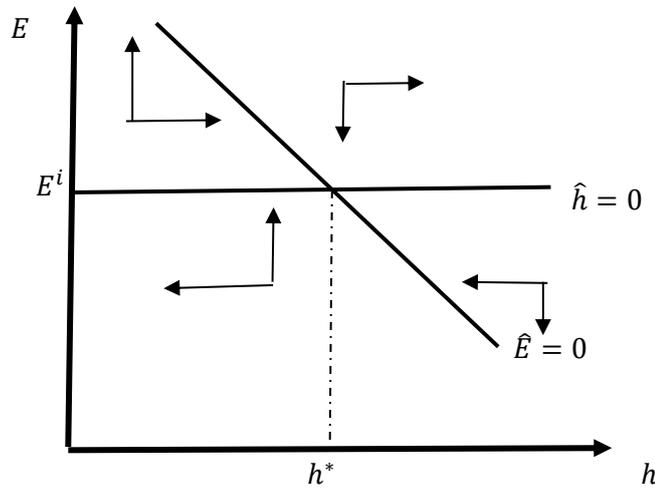
$$(44) \quad \begin{bmatrix} \hat{h} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} \frac{\partial \hat{h}}{\partial h} & \frac{\partial \hat{h}}{\partial E} \\ \frac{\partial \hat{E}}{\partial h} & \frac{\partial \hat{E}}{\partial E} \end{bmatrix} \begin{bmatrix} h - h^* \\ E - E^* \end{bmatrix}$$

The Jacobian Matrix is given by:  $\begin{bmatrix} \frac{\partial \hat{h}}{\partial h} & \frac{\partial \hat{h}}{\partial E} \\ \frac{\partial \hat{E}}{\partial h} & \frac{\partial \hat{E}}{\partial E} \end{bmatrix}$ . According to Olech's Theorem for the

system to be asymptotically stable in the large the Trace of the Jacobian Matrix must be negative, and the Determinant must be positive (Gandolfo, 1997, pp. 354-355).

The trace is given by:  $TR J = \left( \frac{\partial \hat{h}}{\partial h} + \frac{\partial \hat{E}}{\partial E} \right)$ . From equation (36) we know that  $\frac{\partial \hat{h}}{\partial h} = 0$  and from equation (43) we know that  $\frac{\partial \hat{E}}{\partial E} = -(1 + \kappa f \beta_1) \rho_5 < 0$  which is negative. The determinant is given by:  $DET J = \frac{\partial \hat{h}}{\partial h} \frac{\partial \hat{E}}{\partial E} - \frac{\partial \hat{E}}{\partial h} \frac{\partial \hat{h}}{\partial E}$ . The first term of the determinant is equal to zero because  $\frac{\partial \hat{h}}{\partial h} = 0$ . From equation (36) we get  $\frac{\partial \hat{h}}{\partial E} = h_1 > 0$ . So, the sign of the determinant depends on the sign of  $\frac{\partial \hat{E}}{\partial h}$ . If  $\frac{\partial \hat{E}}{\partial h} < 0$  then the determinant will be positive, and the system will be stable; otherwise, the determinant will be negative, and the system will have a saddle-path.

Assuming  $\frac{\partial \hat{E}}{\partial h} < 0$  we can show the long-run equilibrium position by means of figure 2.

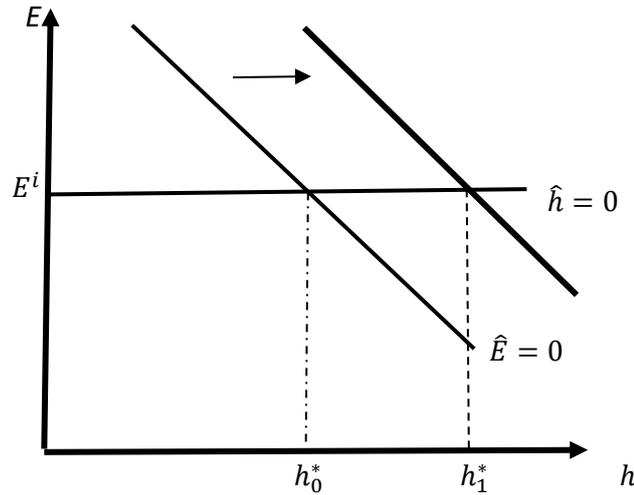


**Figure 2: Equilibrium with structural change**

Let us analyze the effect of a reduction of target inflation over the long-run equilibrium value of manufacturing share and real exchange rate. According to equation (36a) the long run equilibrium value of real exchange rate depends only on the level of technological gap, so it is independent of monetary policy. However, the position of the locus  $\hat{E} = 0$  depends on target inflation as we can see in equation (40). Taking the derivative of this equation to  $E$  and  $\hat{p}^T$ , we get:

$$(45) \quad \frac{\partial E}{\partial \hat{p}^T} = \left\{ \frac{[\kappa f \beta_1 - (1 + \kappa f \beta_1)(\rho_1 + \rho_3)]}{(1 + \kappa f \beta_1)\rho_5} \right\} - \frac{\rho_4}{\rho_5} \left( \frac{\partial u^*}{\partial \hat{p}^T} \right) + \frac{\rho_6}{\rho_5} \left( \frac{\partial l^*}{\partial \hat{p}^T} \right) + \frac{\rho_7}{\rho_5} \left( \frac{\partial \hat{y}^*}{\partial \hat{p}^T} \right)$$

From equations (27a) and (28b) we know that  $\left(\frac{\partial l^*}{\partial \hat{p}^T}\right) < 0$  and  $\left(\frac{\partial \hat{y}^*}{\partial \hat{p}^T}\right) < 0$ , so the last two terms in the R.H.S of equation (45) are clearly negative. From (26b) we know that the sign of  $\left(\frac{\partial u^*}{\partial \hat{p}^T}\right)$  is ambiguous, but a decreasing function of  $h$ . For low levels of manufacturing share  $\left(\frac{\partial u^*}{\partial \hat{p}^T}\right) > 0$ , so the term  $-\frac{\rho_4}{\rho_5} \left(\frac{\partial u^*}{\partial \hat{p}^T}\right)$  will be negative. Finally, it seems reasonable to assume that  $\kappa f \beta_1 - (1 + \kappa f \beta_1)(\rho_1 + \rho_3) < 0$ , so the term  $\left\{ \frac{[\kappa f \beta_1 - (1 + \kappa f \beta_1)(\rho_1 + \rho_3)]}{(1 + \kappa f \beta_1)\rho_5} \right\} < 0$ . This means that:  $\frac{\partial E}{\partial \hat{p}^T} < 0$ , and a decrease in target inflation will shift the  $\hat{E} = 0$  to the right, as it is show in figure 3, producing an increase in the steady-state level of the manufacturing share.



**Figure 3: Decrease in target inflation and equilibrium with structural change**

The effect of changes in target inflation over the growth rate of real output, capacity utilization, employment rate and manufacturing share are summarized in Table I below.

**Table I: Effects of changes in target inflation over the growth rate of real output, capacity utilization, employment rate and manufacturing share**

	$\hat{y}^*$	$l^*$	$u^*$	$h^*$
$\hat{p}^T$	-	-	-	-

## 6. Concluding Remarks

The main objective of this paper was to show that monetary policy, specifically the target inflation can have a long-term influence on real output growth, capacity utilization, employment rate and the productive structure of a small open economy with imperfect capital mobility, flexible inflation targeting and hysteresis. With this goal, a Kaldorian model of balance of payments constrained growth was developed in which monetary policy, price and wage setting, productivity growth and structural change had a central role in the interaction between inflation and economic growth.

Although the model was developed for a small open economy, it is designed for a *mature economy* in the sense of Lewis (1954): labor supply is inelastic and real output growth had to be equal to the natural growth rate in the long-run. The natural growth rate is, however, endogenous because productivity growth depends both on output growth and employment rate. The natural growth rate adjusts itself to the actual growth rate of real output, determined by the balance of payments constraint, due to changes in the level of employment. In order for a balanced growth path to exist, it was also necessary to make the autonomous component of investment demand an endogenous variable in the long-run, as done by Lavoie (2016).

The main result of the model is that a reduction in the target inflation will result in a increase in the long run growth rate of real output, rate of employment, capacity utilization and manufacturing share. This means that monetary policy is not neutral, even in the long-run and has the capacity to permanently change the productive structure of the economy, increasing the manufacturing share. As the manufacturing share is decisive for the process of technological progress and economic development; it is clear that monetary policy is capable of affecting the development path of an economy. This means that is not possible to separate macroeconomics from economic development. This conclusion is in accordance with the so-called structuralist development macroeconomics, which is the theoretical basis of the Brazilian New-Developmentalism school<sup>10</sup>.

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<sup>10</sup>See [www.sdmg.com.br](http://www.sdmg.com.br).

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## Annex - I - Stability Analysis of the Dynamic Model in the Case with No Structural Change

The stability analysis of the two-dimensional linear systems of short and long term was based on SHONE, R. (1997).

From equations (22) and (23) and considering (7), we have:

$$(22. a) \quad \hat{p}_t = \rho_0 + \rho_2 \hat{p}_t^* + (\rho_1 + \rho_3) \hat{p}^T + \rho_4 [u^{T-1} + \tilde{\theta}(u_{t-1} - u^{T-1})] + \rho_5 E_{t-1} \\ - \rho_6 \frac{k_{t-1}}{q_{t-1}} u_{t-1} - \rho_7 h_{t-1} \hat{y}_{t-1}$$

$$(23. a) \quad \hat{u}_t = \left( \frac{1}{\pi_t + \eta_1 + \varphi_1 \beta_2} \right) \left[ \gamma_0 + \gamma_1 [u^{T-1} + \tilde{\theta}(u_{t-1} - u^{T-1})] + \gamma_2 E_{t-1} \right. \\ \left. + (\gamma_3 + \gamma_4) \hat{p}^T - \gamma_5 \hat{p}^* + \gamma_6 \frac{k_{t-1}}{q_{t-1}} u_{t-1} + \gamma_7 h_{t-1} \hat{y}_{t-1} \right]$$

So, the matrix form of the system (22.a) and (23.a) has the following arrangement:

$$\begin{bmatrix} \hat{p}_t \\ \hat{u}_t \end{bmatrix} = \begin{bmatrix} 0 & \rho_4 \tilde{\theta} - \rho_6 \frac{k_{t-1} q_{t-1}^{-1}}{\pi_t + \eta_1 + \varphi_1 \beta_2} \\ 0 & \frac{\gamma_1 \tilde{\theta} + \gamma_6 k_{t-1} q_{t-1}^{-1}}{\pi_t + \eta_1 + \varphi_1 \beta_2} \end{bmatrix} \begin{bmatrix} \hat{p}_{t-1} \\ u_{t-1} \end{bmatrix}$$

The determinant of matrix A is clearly equal to zero. In effect, the eigenvalues are real and equal. In this case, if the matrix A feature is negative, the system will be a stable node. For this to occur, the following condition must be met:

$$\left| \frac{\gamma_1 \tilde{\theta} + \gamma_6 k_{t-1} q_{t-1}^{-1}}{\pi_t + \eta_1 + \varphi_1 \beta_2} \right| < 1$$

Or to be more specific:

$$\gamma_1 \tilde{\theta} + \gamma_6 k_{t-1} q_{t-1}^{-1} < \pi_t + \eta_1 + \varphi_1 \beta_2$$

Condition that is most likely to be satisfied if:  $\beta_2 < \beta_1 \rho_4$ .

## Annex - II - Stability Analysis of the Dynamic Model in the Case with Structural Change

Equations (36) and (39) make up the system of the model of section 5.

$$(36) \quad \hat{h} = h_1(E - E^i)$$

$$(39) \quad \hat{E}_t = (1 - \tilde{\epsilon})\rho_2\hat{p}^* - \tilde{\epsilon}\rho_0 + [(\tilde{\epsilon} - 1) - \tilde{\epsilon}(\rho_1 + \rho_3)]\hat{p}^T - \tilde{\epsilon}\rho_4u^* - \tilde{\epsilon}\rho_5E + \tilde{\epsilon}\rho_6l^* \\ + \tilde{\epsilon}\rho_7h\hat{y}^*$$

Where  $\tilde{\epsilon} \equiv (1 + \kappa f \beta_1) > 1$

So, the matrix from of the system (36) and (39) has the following arrangement:

$$(A1) \quad \begin{bmatrix} \hat{h} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 0 & h_1 \\ J_{21} < 0 & J_{22} < 0 \end{bmatrix} \begin{bmatrix} h - h^* \\ E - E^* \end{bmatrix}$$

Where

$$(A2) \quad J_{21} \equiv \frac{\partial \hat{E}}{\partial h} = -\tilde{\epsilon}\rho_4 \frac{\partial u^*}{\partial h} - \tilde{\epsilon}\rho_6 \frac{\alpha_1}{\alpha_2} \hat{y}^* < 0$$

$$(A3) \quad J_{22} \equiv \frac{\partial \hat{E}}{\partial E} = -\tilde{\epsilon}\rho_5 < 0$$

The partial derivative  $\frac{\partial u^*}{\partial h}$  will be positive as long as  $\alpha_2 > \alpha_1$ . That is, as long as the rate of growth in labor productivity is more sensitive to the rate of employment than the growth rate of output.

So, we have to  $Tr = J_{22} < 0$  and  $Det = -h_1 J_{21} > 0$ . Therefore, the system has a stable equilibrium. *Quod erat demonstrandum.*

### Annex - III - Calibration of model parameters and initial conditions

The *model calibration* was performed using *Matlab/Simulink R12*. As the model is hybrid, that is, it presents discrete dynamics in the short term and continuous dynamics in the long term (when there is structural change), the 50 years calibrated in the long term was based on 50,000 short term interpolations.

The calibration procedure consisted of determining the values of the short-term parameters that would generate a *plausible equilibrium* in economic terms. These values were used to *calibrate* the parameters in the presence of structural change (long term). Thus, the long-term behavior is, at each instant, consistent with the short-term equilibrium.

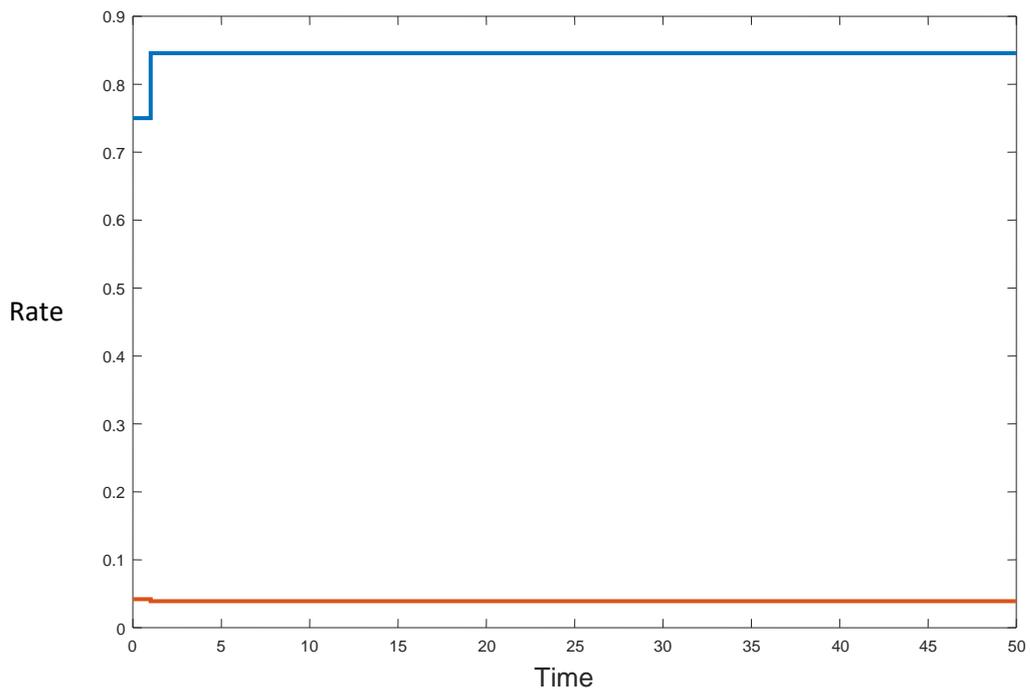
**Table A1 - Model calibrated initial values of the model**

Initial Variables	Description	Calibrated values
$y_{t-1}$	Domestic income growth rate in the period $t-1$	1
$h_{t-1}$	Industry share in GDP	0.25
$E_{t-1}$	Real exchange rate in period t-1	1
$u_{t-1}$	Degree of capacity utilization in period t-1	0.75
$E_t$	Real exchange rate in period t	1
$u^{T-1}$	Target Capacity utilization in period t-1	0.75
$\hat{p}^T$	Inflation Target	0.05
$\hat{p}_t^*$	International inflation rate	0.02
$i_t^*$	International interest rate	0.02
R	Country risk	0.1
$K_t$	Capital stock	1
$k_t$	Capital stock per worker	1
$N_t$	Workforce	1
$q_t$	Labor productivity	1
$\pi_t$	Profit share in income	0.25
$\hat{z}_t$	International GDP growth rate in period t	0.05
$\theta_1$	Ratio between the initial value of exports and the initial value of imports	1
$\theta_2$	Ratio between the initial value of external liability services and the initial value of imports	0.24
$\theta_3$	Restriction $\theta_3 \equiv (1 - \theta_1 + \theta_2)$	0.24
G	Technological gap	0.5

**Table A2- Calibrated model parameter values**

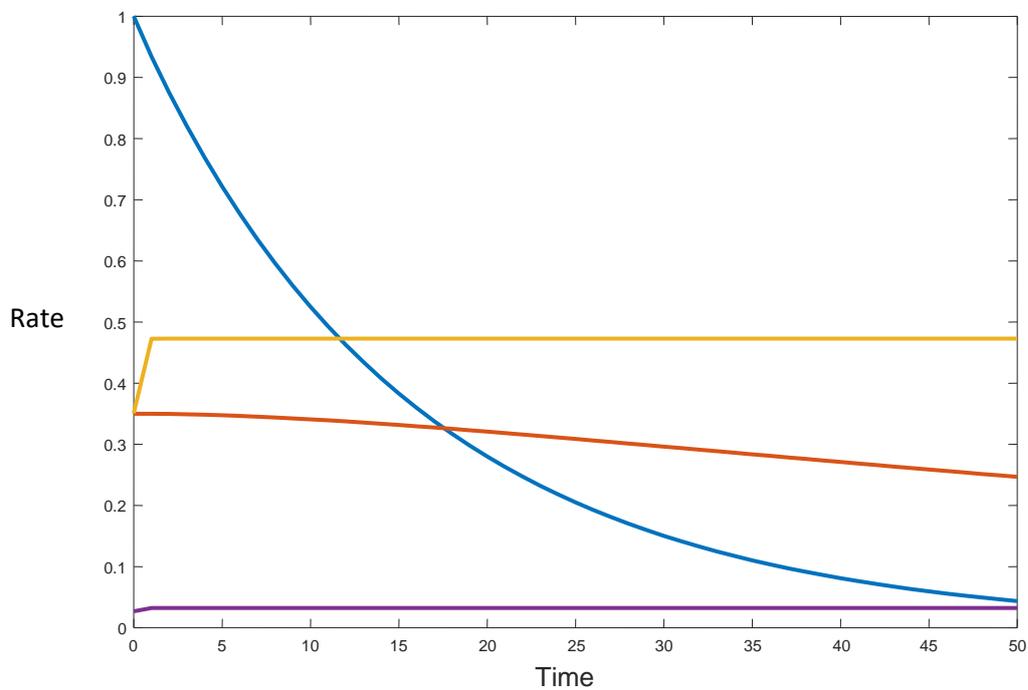
Parameter	Description	Calibrated values
$\chi_0$	Price elasticity of exports	0.4
$\chi_1$	Income elasticity of exports	0.25
$\mu_0$	Price elasticity of exports	0.01
$\mu_1$	Income elasticity of imports	0.25
$\kappa$	Coefficient of sensibility of the rate of change of nominal exchange rate in relation to the growth rate of external capital flows	0.25
$f$	Sensibility of the growth of external capital flows to the interest differential	0.25
$\beta_0$	Sum of the international interest rate and the country risk premium	0.12
$\beta_1$	Coefficient of sensibility of the interest rate to the inflation gap	0.25
$\beta_2$	Coefficient of sensibility of the interest rate to the output gap	0.25
$\tilde{\theta}$	Coefficient of sensibility of the target level of capacity utilization to the capacity utilization gap	0.25
$\varphi_0$	Autonomous component of investment	0.06
$\varphi_1$	Sensitivity coefficient (positive) that captures the influence of the expected real interest rate on investment decisions	0.25
$\eta_0$	Autonomous component of net exports	0.12
$\eta_1$	Sensitivity of net exports the degree of capacity utilization	0,8
$\eta_2$	Marshall-Lerner's condition	0.41
$\varpi$	Coefficient that measures the bargaining power of workers	0.12
$\alpha_0$	Autonomous component	0.01
$\alpha_1$	Coefficient of Kaldor-Verdoorn	0.12
$\alpha_2$	Sensibility of labor productivity growth rate to the employment rate	0.02
$h_0$	Autonomous share of the manufacturing industry in GDP	0.5
$h_1$	Sensibility of structural change to the real exchange rate	0.01
$h_2$	Sensibility of structural change to the technological gap	1
$n$	Population growth rate	0.01
$\theta$	Sensitivity of the mark-up rate to real exchange rate	0,5

**Figure A1: Dynamics of the level of capacity utilization and the rate of inflation**



Source: Authors' own elaboration. Note: blue - capacity utilization; red - inflation rate.

**Figure A2: Dynamics with structural change, growth, and employment**



Source: Authors' own elaboration. Note: red – manufacturing share in GDP, yellow - employment rate and purple: GDP growth rate.