Inconsistency and Over-determination in neo-Kaleckian growth models: a note

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Abstract

This short note argues that the canonical neo-Kaleckian growth model does not yield a balanced growth path due to the absence of an inbuilt mechanism by which desired and actual rates of capital accumulation are equalized. Introducing non-generating capacity autonomous demand does not solve such inconsistency. Contrary to what Lavoie (2016a) claims, we show that the latter is also unable to bring capacity utilization to its normal level. In light of recent contributions (e.g. Nikiforos, 2013; 2016), we suggest that making normal capacity utilization an endogenous variable is an alternative better suited to deal with the issue.

1. Introduction

Neo-Kaleckian models of growth and distribution have been highly popular among non-Neoclassical economists in the last thirty years. However, from a theoretical point of view, a substantial literature has stressed their failure to ensure the equalization between actual and normal capacity utilization rates in the long-run. Indeed, early contributions such as Rowthorn (1981), Taylor (1983) or Dutt (1984), did not even refer to such a notion of a normal rate of capacity utilization.

Recently, some authors have began to explore the idea (probably originating in a different 1940s context, with Hicks and Goodwin) of introducing non-capacity creating autonomous demand as the driver of long-run economic growth. Lavoie (2016a) proposed the inclusion of such component in the *canonical* neo-Kaleckian framework. This would provide a mechanism to bring about such an equalization, while at the same time controlling Harrodian dynamic instability, and preserving the core “Keynesian message”. Such proposition has initiated a still ongoing debate (e.g. with Skott, 2016a, 2016b, Lavoie, 2016b).

The present note has two main objectives. First, we argue that the canonical neo-Kaleckian growth model is inconsistent with a balanced growth path (being over-determined) because there is no mechanism by which desired and actual rates of capital accumulation can be equalized. The introduction of a non-generating capacity autonomous demand would therefore not do with solving such inconsistency. Second, contrary to what is stated in Lavoie (2016a), we show that autonomous consumption is unable to bring capacity utilization to its normal level. It is only the adoption of a very particular representation of Harrodian dynamics that does the job. Non-capacity creating autonomous demand can at most be considered a mechanism *softening* one type of Harrodian instability.

On the other hand, recent developments on production theory have showed that cost minimizing firms increase the utilization rate of their capital if the rate of returns to scale decreases as production increases, i.e. the normal rate of utilization is endogenous to variations in demand (Nikiforos 2013; 2016). In the light of such recent contributions, this note also suggests that endogeneizing normal capacity utilization is a more appealing alternative. In fact, this was the original route pursued by Lavoie himself and others (e.g. Lavoie 1995; 1996; Dutt, 1997).

2. Discussion

2.1 Inconsistency over the long run

Consider a closed economy without public sector, and let investment and saving be determined by:

$$g^i = \frac{I}{K} = \gamma + \gamma_u(u - u_n) \tag{1}$$

$$g^s = \frac{S}{K} = \frac{s_p \pi u}{v} - z \tag{2}$$

Notation follows that in Lavoie (2016a): g^i and g^s denote desired and actual rates of capital accumulation; I denotes investment, S stands for savings, K is capital, v is the inverse of capital productivity, and u is the level of capacity utilization; u_n stands for the normal or desired level of capacity utilization and is determined by technology; the profit share π would be taken to be exogenously given;[1] γ_u and s_p are positive parameters satisfying the Keynesian stability condition, that is, $s_p\pi > \gamma_u v$; $z = \frac{Z}{K}$ is the ratio between autonomous (capitalist) consumption and capital stock; finally γ can be understood as the average expectation of the secular rate of growth, as perceived by the managers of firms.

Short-run solutions for utilization and accumulation are given by:

$$u^* = \frac{(\gamma - \gamma_u u_n + z)v}{s_p\pi - \gamma_u v} \tag{3}$$

$$g^* = \frac{\gamma_u v z + s_p \pi(\gamma - \gamma_u u_n)}{s_p\pi - \gamma_u v} \tag{4}$$

Set $z = 0$ so as to recover the traditional short-run equilibrium solutions.[2]

Defining the long-run as a state in which the level of capacity utilization is equal to *normal* rate, that is:

$$u = u_n \tag{5}$$

From equation (1) we have:

$$g^i = \gamma \tag{6}$$

while from equation (2):

$$g^s = \frac{s_p \pi u_n}{v} \tag{7}$$

The equilibrium in goods market requires:

$$g^i = g^s \tag{8}$$

The system formed by equations (5), (6), (7) and (8) has only *three* endogenous variables (u, g^i and g^s) but *four* independent equations, which means that it is over-determined.

2.2 Non-capacity generating autonomous demand and the inconsistency problem

[1] In the neo-Kaleckian framework income distribution is determined at a microeconomic level by pricing decisions of firms.

[2] In the canonical neo-Kaleckian growth model for a closed economy without governmental activities there is no such a thing as non-capacity generating autononous demand. See, for example, Skott (2010). The only autononous component of demand is located in the investment equation.

Set now $z \neq 0$. Lavoie (2016a) explicitly states, "Some Sraffian economists have long been arguing that **the presence of non-capacity autonomous expenditures provides a mechanism that brings back the model to normal rates of capacity utilization**, while safeguarding the main Keynesian message and without going back to classic conclusions. **The present article provides a very simple proof of this** […]" (p. 1, emphasis added).

Suppose for a moment that *it is true* that autonomous consumption brings capacity utilization to its normal level. Therefore, it immediately follows that equations (5) and (6) are satisfied. The actual rate of capital accumulation now is given by:

$$g^s = \frac{s_p \pi u_n}{v} - z \tag{9}$$

Autonomous consumption is supposed to grow at a constant rate, g_z. By definition we have:

$$\frac{\dot{z}}{z} = \frac{\dot{Z}}{Z} - \frac{\dot{K}}{K} = g_z - g^* \tag{10}$$

In a balanced growth path $\frac{\dot{z}}{z} = 0$, so recalling equation (4):

$$g_z = \frac{\gamma_u v z + s_p \pi(\gamma - \gamma_u u_n)}{s_p \pi - \gamma_u v} \tag{11}$$

Our system is now constituted by equations (5), (6), (8), (9), (11), which means that we have 5 independent equations; however, there are still only 4 endogenous variables (u, g^i, g^s, z). The inconsistency problem remains.

If *it is not true* that autonomous consumption brings capacity utilization to its normal level (as in fact it is the case), then substituting equation (4) in (10) gets us a one-dimensional differential equation for z. Lavoie (2016a) and Skott (2016b) extensively discuss dynamic properties. As long as the Keynesian stability condition is satisfied, that is $s_p \pi > \gamma_u v$, and g_z is high enough, the share of autonomous consumption converges to a non-zero equilibrium. This in turn implies that the equilibrium level of capacity utilization becomes:

$$u^{**} = u_n + \left(\frac{g_z - \gamma}{\gamma_u}\right) \tag{12}$$

The simple introduction of *non-capacity generating autonomous demand* is no sufficient condition to solve the inconsistency problem nor to bring capacity utilization to its normal level.[3]

The solution requires that one exogenous variable of the canonical model to be converted into an endogenous one. That is precisely what is done by Lavoie (2016a) when he makes the *autonomous* component of investment demand to be a function of the difference between actual and desired rate of capital accumulation.[4] This means that γ should assume the required value for closing the model in the long-run, which is given by:

$$\gamma^{**} = g_z \tag{13}$$

Non-capacity creating autonomous demand can at most be considered a mechanism that attenuates(?) a very particular representation of Harrodian instability. Moreover, according to equation (13), in the balanced growth path of our modified neo-Kaleckian growth model the rate of capital accumulation must be determined by g_z. This means that Keynesian uncertainty and

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[3] Later on Lavoie labels the equilibrium obtained in equation (12) as concerning the medium-run, somehow in contradiction with the previous statement. This mistake does not appear in Allain (2015).

[4] We shall mention that it is quite puzzling to model Harrodian instability while keeping the Keynesian short-run stability condition.

entrepreneurs´ animal spirits have no role in long-run growth. It is hard to see how this solution preserves the Keynesian message!

2.3 An alternative solution

A simple alternative that ensures that actual and normal utilization coincide while overcoming the inconsistency (or over-determination) problem (without eliminating Keynesian uncertainty and animal spirits from the model!) is by making the normal level of capacity utilization an endogenous variable. Indeed, this route was originally pursued by Lavoie himself and others (e.g. Lavoie 1995; 1996; Dutt, 1997). As pointed out by Skott (2012), the Dutt-Lavoie argument is correct from a logical perspective but lacks economic rationale.

Nikiforos (2013) has recently demonstrated that the choice of the system of production is related to demand and higher demand can lead to an increase in the desired utilization, conditional on the behavior of the economies of scale. The basic idea is that the firm will tend to utilize its capital more as the output grows, if there are increasing returns to scale and the rate of the returns to scale decreases as demand expands. As result, in the face of the increase in demand, the firm will increase its production increasing the normal level of capacity utilization. Therefore, from a technological point of view, it is not correct to take the normal level of capacity as exogenous as constant in the long-run.[5] The importance of Nikiforos results is in the fact that endogeneity of normal capacity derives from technology properties.

Just to show a very simple example, consider:

$$\dot{u}_n = \lambda(g^{i*} - g^{s*}) \qquad (14)$$

where $g^{i*} = \gamma$ and $g^{s*} = \frac{s_p \pi u_n}{v}$ are desired and actual rates of capital accumulation when normal and actual levels of capacity utilization are equal. The expression above indicates that the normal level of capacity utilization adjusts to the difference between desired and actual capital accumulation. The logic towards equation (14) follows the intuition of Nikiforos results.

Suppose $g^{i*} > g^{s*}$. This means that the average expectation of the secular rate of demand growth, as perceived by the managers of firms, is above actual long-run capital accumulation. Once managers realize this, they increase production trying to fill the gap. That is, since for a given normal rate of utilization productive capacity is growing less than long-run expected demand, firms increase production in order to satisfy such demand. This increase will tend to materialized through an increase in u_n. On the other hand, for $g^{s*} > g^{i*}$, expectations over secular growth are bellow actual long-run capital accumulation. Managers respond reducing production trying to match expected demand and u_n will decrease.

Manipulating equation (14) it follows that:

$$\dot{u}_n = \alpha(u^* - u_n) \qquad (15)$$

where $\alpha = \frac{\lambda(s_p \pi - \gamma_u v)}{v} > 0$ is an adjustment parameter.[6] The complete derivation of the expression above can be found in the appendix.

[5] Nikiforos (2016) provides some empirical evidence of the endogeneity of the normal utilization for the United States.

[6] One could also follow Skott (1989) and considering average costs of production which opens the door for non-trivial multiple equilibria solutions (e.g. Oreiro, 2004). For instance, Nikiforos (2016) himself proposes to make $\dot{u}_n = \lambda(g^* - g_0)$ where g_0 is the expected rate of accumulation and from which he claims it is possible to obtain an expression similar to (15).

Making normal capacity utilization an endogenous variable in the system of equations (5), (6), (7) and (8) eliminates the over-determination problem since we now have 4 endogenous variables and 4 independent equations. The normal level of capacity utilization is given by:

$$u_n = \frac{\gamma v}{s_p \pi} \tag{16}$$

The long-run rate of growth of real output and capital stock is determined by the autonomous element of investment decision. Contrary to what happens in Lavoie´s model, uncertainty and animal spirits now do have a role in long-run growth, by inducing exogenous changes in the desired growth rate of capital stock.

3. Conclusion

As pointed out by Skott (2010), from a theoretical perspective, the problems with the neo-Kaleckian growth model arise from the combination of an exogenous markup with the extension to the long run of a standard Keynesian short-run stability condition. In the context of the present discussion, it is quite puzzling how (and why) Kaleckians have tried to introduce Harrodian instability while keeping the relative insensitivity of investment to aggregate demand, given the constraint $s_p \pi > \gamma_u v$.

In any case, this note has shown that extending the canonical neo-Kaleckian model to the long-run makes it inconsistent because the is no mechanism by which desired and actual rates of capital accumulation can be equalized. The simple introduction of a non-capacity generating autonomous demand would not be sufficient to overcome such inconsistency problem nor to bring capacity utilization to its normal rate. This problem may be solved by making autonomous investment an endogenous variable, but this would eliminate all role for uncertainty and animal spirits in long run growth. It is hard to see how this solution would preserve the "Keynesian message" of the neo-Kaleckian growth model.

If we are to update and extend the Kaleckian approach, we argue, treating as endogenous normal capacity utilization appears to be a better promising alternative.

Appendix

We propose that the normal level of capacity utilization adjusts to the difference between desired and actual long-run capital accumulation:

$$\dot{u}_n = \lambda(g^{i*} - g^{s*}) \tag{A1}$$

With $g^{i*} = \gamma$ and $g^{s*} = \frac{s_p \pi u_n}{v}$. Substituting those values in (A1) we have:

$$\dot{u}_n = \lambda\left(\gamma - \frac{s_p \pi u_n}{v}\right) \tag{A2}$$

This is equivalent to:

$$\dot{u}_n = \frac{\lambda}{v}\left(\gamma v - s_p \pi u_n\right)\frac{s_p\pi - \gamma_u v}{s_p \pi - \gamma_u v} \tag{A3}$$

Which in turn is not different from:

$$\dot{u}_n = \frac{\lambda}{v}\left(\gamma v - s_p \pi u_n + \gamma_u u_n v - \gamma_u u_n v\right)\frac{s_p\pi - \gamma_u v}{s_p \pi - \gamma_u v} \tag{A4}$$

Factoring the expression above:

$$\dot{u}_n = \frac{\lambda}{v}\left[(\gamma - \gamma_u u_n)v - (s_p\pi - \gamma_u v)u_n\right]\frac{s_p\pi - \gamma_u v}{s_p \pi - \gamma_u v} \tag{A5}$$

we obtain:

$$\dot{u}_n = \frac{\lambda(s_p\pi - \gamma_u v)}{v}\left[\frac{(\gamma - \gamma_u u_n)v}{s_p\pi - \gamma_u v} - u_n\right] \quad \text{(A6)}$$

At this point recall from equation (3) that setting $z = 0$ the current level of capacity utilization is given by $u^* = \frac{(\gamma - \gamma_u u_n)v}{s_p\pi - \gamma_u v}$. Substituting this expression in (A6) we arrive to:

$$\dot{u}_n = \alpha(u^* - u_n) \quad \text{(A7)}$$

Where $\alpha = \frac{\lambda(s_p\pi - \gamma_u v)}{v}$ is an adjustment parameter that results from the combination of behavioural, technological and distributive parameters.

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