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VERDOORN'S LAW IN RETROSPECT: A COMMENT

1. It was gratifying to find Mr Rowthorn's note[1] devoted exclusively to a critical discussion of the theoretical appendix of my 1949 article in *L'Industria*, whereas most authors simply discard it when discussing my so-called 'law'. This gives me the opportunity for some comments on this note as well as on my own conclusions thirty years ago.

2. The model in the appendix aimed to explain the alleged invariance of the long-term elasticity between the growth rates of productivity and output in manufacturing. For this purpose I used Tinbergen's (1942) neoclassical model.[2] Apart from this, the appendix is based on what, at that time, I called the 'normal assumptions of long-period analysis' (section 3 of the main text), namely that the growth-rates of the variables concerned are approximately constant, and hence that their period-averages are roughly representative of what is currently called the steady-state of the system.

To simplify the formulae, the initial values (at $t = 0$) of the variables, also that of capital (b) and output (y), have been put at unity. The investment function $b_t = \gamma y_t$ then leads to $\dot{b}_0/b_0 = \gamma y_0/b_0 = \gamma$. Mr Rowthorn rightly points at this formula being an anomaly in the context of the appendix. In itself, however, it is quite all right if one remembers that by this choice of units γ represents not the ratio of net investments to output but $\gamma = \gamma'/\kappa_0'$ (γ' being the true value of this ratio and κ_0' that of the capital–output ratio at $t = 0$).

It is relevant, however, that following this approach the rate of capital accumulation is constant.[3, 4] At the same time I overlooked that the steady-state requires the model to be solved for the asymptotic growth rates, at $t \to \infty$, of capital and output as did, for example, Domar (1946). As a consequence, my final formula for the productivity–output elasticity is burdened by quite a few terms that vanish in the asymptotic case.

3. A correct and at the same time more elegant and readily interpretable form for the elasticity is obtained, as the present author did (1959, with a follow-up in the 1960 report by a group of experts of the European Community), following Domar and Solow by studying the asymptotic growth rates, and replacing the labour demand and supply equations by one single relation for effective labour demand. Denoting labour by a, the system then reduces to:

$$y_t = a_t^{\lambda} b_t^{\mu} e^{\nu t}, \tag{1}$$

[1] See the March 1979 issue of this JOURNAL, pp. 131–3.

[2] This is, I think, the first example of a model of this kind. Written as it was in German, it has remained little known. An English translation, however, is to be found in his *Selected Papers* (1959).

[3] Since γ' is assumed constant, γ is uniquely determined by the value of κ_t at $t = 0$.

[4] Tinbergen avoided this pitfall and solved the system for $b(t)$, anticipating one of Solow's results (1956, p. 76) by 14 years. Primarily interested in differences in the time-shape of the trends, unlike Domar and Solow, Tinbergen did not analyse the asymptotic case. One wonders whether this foreshadows the misgivings voiced by some later authors as to the *actual* significance of the steady-state.

$$\dot{b}_t = \gamma y_t, \tag{2}$$

$$a_t = a_0 e^{\pi t}, \tag{3}$$

where the production function (1) allows for both economies of scale,[1]

$$\xi = \lambda + \mu > 1,$$

and technical progress.

The time-path of $\dot{y}_t/y_t$ then takes the form

$$\dot{y}_t/y_t = \frac{(\pi\lambda + \nu)}{1 - \mu} A_t, \tag{4}$$

where A_t is given by:

$$A_t = 1 + \mu \frac{\gamma(1-\mu) - \kappa_0(\pi\lambda + \nu)}{\gamma(1-\mu)\left[e^{(\pi\lambda+\nu)t} - 1\right] + \kappa_0(\pi\lambda + \nu)}, \tag{5}$$

κ_0 being the initial capital–output ratio.[2]

As is easily verified, $A_t \to 1$ if $t \to \infty$, yielding as the steady-state values of the system:

$$\lim_{t\to\infty} \frac{\dot{y}}{y} = \lim_{t\to\infty} \frac{\dot{b}}{b} = \frac{\pi\lambda + \nu}{1 - \mu}, \tag{6}$$

$$\lim_{t\to\infty} \kappa = \frac{\gamma(1-\mu)}{\pi\lambda + \nu}, \tag{7}$$

whereas the growth-rates of productivity and the productivity-output elasticity, η, are:

$$\lim_{t\to\infty} d\ln(y/a) = \frac{\pi(\xi - 1) + \nu}{1 - \mu}, \tag{8}$$

$$\lim_{t\to\infty} \frac{d\ln(y/a)}{d\ln y} = \lim_{t\to\infty} \eta = \frac{\pi(\xi - 1) + \nu}{\pi\lambda + \nu}. \tag{9}$$

One inference from (7) and (8) is that in the steady-state the neoclassical model degenerates into quasi-complementarity, a fact hinted at by Mr Rowthorn when he states that in my 1949 model the growth-rates of productivity and output are constant (his equation 16).

4. As stated in the footnote on the first page of the 1949 article, the latter is not more than a progress report. With the hindsight of my 1959 publication its main shortcoming, from the operational point of view, was that it insufficiently emphasised that rigid constancy *over time* of the productivity-output elasticity is only to be expected in the steady-state. As a matter of fact, considerable changes in $d\ln(y/a)$ and η_t are to be expected if in the case of disequilibrium growth A_t in (4) differs appreciably from unity. To demonstrate this: let in the case of an

[1] See, for a macro-economic justification of economies of scale and their long-run impact on inflation, Verdoorn (1973).

[2] Equations (4) and (5) are obtained by solving the system for b_t in terms of $\dot{b}_t$ and the initial values of the variables. Upon integration and substitution of the resulting expression for b_t in the production function and logarithmic differentiation of the latter (4) and (5) follow.

originally steady-state development, a sudden and permanent change of the investment–output ratio, from γ_0 to γ_1, set in. The value of A_t then becomes

$$A_t = 1 + \mu \frac{\gamma_1 - \gamma_0}{\gamma_1[e^{(\pi\lambda+\nu)t} - 1] + \gamma_0}, \tag{10}$$

and

$$\eta_t = 1 - \frac{\pi(1-\mu)}{(\pi\lambda+\nu)A_t}, \quad t = 0, \ldots, \infty. \tag{11}$$

Table 1. *Effects of Doubling of Net Investment–Output ratio*

($\gamma_1 = 2\gamma_0$; $\xi = 1{\cdot}1$; $\lambda = 0{\cdot}734$; $\mu = 0{\cdot}366$; $\pi = 0{\cdot}0174$; $\nu = 0{\cdot}00785$)

	Ratios to asymptotic values		
t	$d \ln (y/a)$	$d \ln y$	η_t
0	1·776	1·361	1·305
1	1·761	1·345	1·301
5	1·647	1·301	1·266
10	1·540	1·251	1·231
15	1·456	1·212	1·201
20	1·357	1·166	1·164
30	1·290	1·135	1·137
40	1·220	1·102	1·107
50	1·170	1·079	1·085
60	1·134	1·062	1·067
80	1·084	1·039	1·043
100	1·054	1·025	1·028
∞	1·000	1·000	1·000
Corresponding absolute steady-state values:			
∞	0·0151	0·0325	0·465

Quantitatively, the effects of a doubling of the investment–output ratio are shown in Table 1 for an arbitrary case. Analogous effects occur if the initial value of the capital–output ratio is too low, i.e. if

$$\kappa_0 < \frac{\gamma(1-\mu)}{\pi\lambda+\nu}. \tag{12}$$

It is probable that the superposition of these two effects goes far in explaining the boisterous post-war growth in Western continental European countries (an initially too low value of κ_0 followed by a large increase in the rate of saving).

5. Conversely, it should be clear that η's derived from periods subject to disequilibrium growth cannot but yield unreliable values if used for extrapolation under conditions that differ appreciably from those of the period of observation. Nor can they be taken as representative for the steady-state, if A_t – taken as a period average – differs non-negligibly from unity. As suggested by Mr de Vries of Erasmus University a simple test on the existence of the steady-state in the period observed is the equality of $\dot{b}/b$ and $\dot{y}/y$, as required by (6). The condition implied is a necessary but not sufficient one: compensating gradual changes of π and of γ or one or more of the technical coefficients are possible.

A moot point, finally, that also affects *inter-country comparisons*, is that even given steady-state and identical values for the technical coefficients λ, μ and ν, differences in π may lead to quite different values of η. As is seen at a first glance from (9):

$$\lim_{\pi \to 0} \eta = 1 \quad (\nu \neq 0), \tag{13}$$

a situation in manufacturing we have gradually learned to live with.

6. The 'law' that has been given my name appears therefore to be much less generally valid than I was led to believe in 1949.

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