4

The Essay in Dynamic Theory

Abstract "The Essay in Dynamic Theory" (1939) developed further the principle of instability and the idea of endogeneity and inevitability of the trade cycle. However, the conception of dynamics and instability in the Essay differs from that of the *Trade Cycle* (1936) in two important aspects. Instability is no longer the result of the interaction between the multiplier and the accelerator, but of assuming the independency of the rate of growth, the propensity to save and the incremental capital-output ratio. Also imperfect competition is no longer seen as a microfoundation of the trade cycle. In the Essay, Harrod articulated his dynamics and its central core, the instability principle, around the fundamental equation. He defined it in terms of the warranted rate of growth: that rate of growth that validates the capital accumulation decisions of entrepreneurs. Along with the warranted rate of growth, Harrod introduced the natural and actual rates of growth. The draft Essay (1938) received comments for J. Marschak and J. M. Keynes who pointed out that the model required additional hypotheses that were not made explicit. Marschak also provided the first mathematical formulation of Harrod's dynamics and raised several important points including the lack of clarity in the definition of the warranted rate of growth. Keynes's comments led Harrod to compress

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his discussion of the application of the dynamic determinants to the cycle that give rise to instability of the moving equilibrium and the non-linearities of the trade cycle. This facilitated the interpretation of the fundamental equation as a constant coefficient model.

In "The Essay in Dynamic Theory" (1939) Harrod developed further the principle of instability and the idea of endogeneity and inevitability of the trade cycle. However, the conception of dynamics and instability in the Essay differs from that of the *Trade Cycle* (1936) in two important respects. Instability is no longer the result of the interaction between the multiplier and the accelerator but of assuming the independency of the rate of growth, the propensity to save and the incremental capital-output ratio. Also imperfect competition does not play a role in defining the microfoundations framework for the trade cycle.

Harrod articulated his dynamics around the fundamental equation which defined the warranted rate of growth, the rate of growth that validates the capital accumulation decisions of entrepreneurs. Some of the influences that led him to the discovery of the fundamental equation included, E. Lundberg, D. H. Robertson, F. Hayek, G. Haberler and most importantly Keynes.

In the Essay, Harrod summarized his dynamic theory in two propositions: "(i) A unique warranted line of growth is determined jointly by the propensity to save (s) and the quantity of capital required by technological and other considerations per unit increment of total output (C)...Only if producers keep to this line will they find that on balance their production in each period has been neither excessive nor deficient. (s and C do not remain constant and change in the cycle, particularly as a result of growth itself) (ii) On either side of this is a 'field' in which centrifugal forces operate, the magnitude of which varies directly as the distance of any point in it from the warranted line. The moving equilibrium of advance is thus a highly unstable one" (CIPC, Vol. III, p. 1196). The instability of the moving equilibrium constitutes an epistemic principle rather than a result of Harrod's model as the later interpretations would assert (Besomi 2002).

The draft Essay was submitted for publication to the *Economic Journal* in August 1938 and underwent important modifications as

a result of the exchanges with J. Marschak (1899–1977) and Keynes (Besomi 2003, CIPC, Vol. III, pp. 1205–1206, note 1).

Marschak provided the first mathematical formulation of Harrod's dynamics, and he raised several important points that would be part of the future debate on Harrod's dynamics. Marschak pointed to the lack of clarity in the definition of the required capital-output ratio (C_r). He also remarked that Harrod's model did not yield non-linearities and was in fact explosive. Finally, he argued, as other commentators would do later on (i.e., Alexander 1950) that the model required additional hypotheses that were not made explicit by Harrod. Keynes was also of the opinion that Harrod's model presumed a hypothesis that the latter never made explicit ($C_r < s$). However, the exchange was blurred by misunderstanding, miscommunication, and misrepresentation.

Overall Harrod did not pay much attention to Marschak or Keynes's comments. But he did downplay the role played by the non-linearities of the parameters thus facilitating the interpretation of his dynamics as a constant coefficient model.

The Essay was published in a final form in March 1939 in *The Economic Journal*. The Essay received only one review (Hawtrey 1939) and was overshadowed by the concerns of World War II. The interest in Harrod's dynamics resumed almost a decade later associated with Domar's growth model, Samuelson's multiplier–accelerator formulation and the publication of Harrod's *Towards Economic Dynamics* in 1948.

4.1 The Fundamental Equation

4.1.1 The Fundamental Equation and Its Main Implications

The Essay was written between July and August 1938. Although Harrod was grappling with the formulation of his fundamental equation, since at least 1937, he stated in his later book *Dynamic Economics* (1973, p. 41) that he arrived at his fundamental equation in July 1938 in a moment's flash. As he puts it:

> In that book (The Trade Cycle) there are to be found many of the ideas that have been developed in my subsequent writings on dynamic economics. But I did not when writing it, have the advantage of having in my mind my fundamental growth equation, which also came to me in a flash on a particular day. I recall a kindly Russian enquiring by what process of research and reflection I had evolved my growth equation. Not so. 'Reflection' is, I suppose, relevant. In the course of my 'reflections' I suddenly saw it in a split second. That was not in my Oxford study, but in John Betjeman's cottage in Berkshire, which he lent me in July 1938.

Harrod's fundamental equation determined the warranted rate of growth (G_w). The warranted rate of growth constituted the marriage between the multiplier and the accelerator. He defined it as (Harrod 1939, p. 16): "...that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount."[1] The warranted rate of growth described an equilibrium at a particular point in time in the sense that staying on it will lead to a repetition of the same growth rate.

Harrod explained that the warranted rate was highly unstable and that its determinants were constantly changing. Yet as a starting point, as in his previous writings on the subject, he applied the procedure followed by the theory of static equilibrium in order the identify and analyze the determinants and the conditions for continuity.

A way to derive the warranted rate of growth is to start with a given marginal capital-output ratio (C).[2] With a known C_r the stock of capital goods that are required to produce Y units of output is C_rY, and the increment of capital required when output (Y) is expanding at a given rate of growth (G_w) is G_wC_rY. Within the same time period,

[1]Actually as McCord Wright (1949, p. 326) pointed out Harrod provided in the Essay six different definitions of the warranted rate of growth: the rate which "(i) leaves each entrepreneur satisfied with what he has done;...(2) allows for some individual disappointments but keeps entrepreneurs as a group, on balance, satisfied;...(3) keeps them doing the same thing;... (4) equates ex ante S and... (5) concerns only the part of investment directly linked to consumption;...(6) somehow differs from the 'proper' warranted rate which would obtain in conditions of full employment."

[2]This derivation is partly based on Shackle (1965, pp. 98–108).

the addition to the capital stock is made possible by adding the unconsumed part of output (Y), that is savings (S), to the stock of capital goods. By postulating that savings (S) is a given proportion of output ($S = sY$),

$$G_w C_r Y = S \Leftrightarrow G_w C_r Y = sY \Leftrightarrow G_w C_r = s \Leftrightarrow G_w = \frac{s}{C_r} \quad (4.1)$$

where

G_w = warranted rate of growth

C_r = increment in the stock of capital divided by the increment in total output "required by technological and other conditions (including the state of confidence, the rate of interest, etc.)."[3]

According to Harrod, the fundamental equation traced a line of output that is a moving equilibrium, in the sense that it leaves all producers satisfied by their decisions. It is "the equation specifying the determinants of growth in a regime of *laissez-faire* capitalism" (Harrod 1973, p. 28).

Later on, Harrod (1973, pp. 16–17) also distinguished between the actual and the desired fraction of income saved (s and s_d) so that the fundamental equation is defined in terms of desired magnitudes in both the numerator and the denominator,

$$G_w = \frac{s_d}{C_r} \quad (4.2)$$

On some occasions, Harrod (1973, p. 27) referred to another "variant" of the fundamental equation which expressed the natural rate of growth (G_n), the rate of growth consistent with the potential of the economy

[3]This is the definition of C_r provided in the Essay. In *Towards Dynamic Economics* (1948, p. 82), he defined it as a "...an equilibrium term expressing requirements for new capital." In his Second Essay (1960, p. 278), it is the "optimum amount of capital required, per unit of time, given current technology, for the output of an extra unit...." In the Essay, he used the term C_p and C for the incremental capital-output ratios corresponding to the actual and warranted rates of growth. In *Towards Dynamic Economics* (1948) and thereafter he uses the terms C and C_r (Harrod 1948, pp. 77, 81; 1960, p. 279; 1973, pp. 16–17; 1969, pp. 199–200).

("the current rate for growth determined by the working population and the current potential for technical progress," Harrod 1969, p. 196) as a function of the optimum rate of savings (s_o) and the required incremental capital-output ratio (C_r),

$$G_w = \frac{s_o}{C_r} \tag{4.3}$$

Actually, in Eq. (4.3) the dependent variable should be the optimum rate of savings (s_o) expressed as a function of the natural rate of growth (G_n) and the required incremental capital-output ratio (C_r) ($s_o = G_n C_r$).

Harrod's dynamics centered on the interaction between the actual, warranted, and natural rates of growth (G, G_w, G_n respectively). Harrod defined the actual rate of growth $\left(G = \frac{s}{C}\right)$ as a truism, a necessary truth, expressing the equality between ex-post savings and ex-post investment.[4] The variables refer to actual values so that, for example, G is the rate of growth which actually occurs. The same reasoning applies to C and s. Since C is the addition to capital it includes all goods and need not include exclusively capital goods (Harrod 1939, p. 18; 1948, pp. 80, 85).[5] It is a dynamic equation because it contains G which is a rate of growth at a point in time.[6]

For Harrod, the deviations of the actual from the warranted rate ($G > or < G_w$) were cumulative upward or downward. While the warranted rate of growth could exceed the natural rate of growth, the actual rate of growth could not and this generated the cycle.

The warranted rate of growth could maintain a steady advance only under certain specific and unlikely conditions such as the constancy in s_d and C_r (1938, CIPC, Vol. III, p. 1195; 1948, p. 84).

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[4]As Harrod explained (1948, note 1, p. 80): $GC = \left(\frac{\Delta Y}{Y}\right)\left(\frac{I}{\Delta Y}\right)$ and $s = \frac{S}{Y}$.

[5]As Harrod (p. 85) explained: "This capital…covers both equipment and stock-in-trade.…C consists in part of consumer goods, including non-durable consumer goods. In an advancing community goods in the pipeline, shops, warehouses, transit and producers' stores, have to increase in proportion to turnover. All these goods are part of capital."

[6]Also G and C is independent of the unit period chosen. Finally, s is not a constant but its changes must be small relative to those of G.

In fact, the warranted rate changed over the course of the business cycle. As put by Harrod (1973, p. 36): "...the warranted growth rate itself... changes under the influence of boom or slump" due to the change in the parameters.[7] This gave rise to the distinction between the normal warranted rate ("the warranted rate pertaining to a steady advance") and the special warranted rates ("the warranted rates that do not correspond to a steady advance"). The desired propensity to save (s_d) changed in accordance with changes in income. Similarly C_r can also changed due to changes in "the proportion of existing equipment that needs to be replaced," the capital intensity of innovations or "the overall capital intensity of the goods people want to buy" (ibid., pp. 38–39). The values of s and C_r are independent of the value of G, and thus the moving equilibrium is thus inherently unstable. In an open economy, fluctuations in the balance of payments also affected the warranted rate (CIPC, Vol. I, p. 1024; Harrod 1939, p. 25).

The interaction between G, G_w, G_n generates Harrod's non-linear trade cycle. This non-linear cycle is the result of three sets of conditions: (i) changes in G_w as a result of changes in the parameters; (ii) the cumulative divergence between G and G_w; and (iii) G cannot exceed G_n but G_w can be above G_n for some time.

4.1.2 Influences on Harrod's Fundamental Equation: Keynes and Lundberg

J. M. Keynes was a decisive influence in Harrod's development of the fundamental equation. Erik Lundberg, albeit a less important influence than Keynes, is also credited with having contributed to Harrod's formulation of the warranted growth.[8]

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[7]See also Harrod (1948, p. 89): "G_w itself fluctuates in the trade cycle. Even if savings...is fairly steady in the long run, it is not likely to be so in the short run".

[8]Young (1989) also mentions other possible influences such as G. Cassel, O. Lange, M. Kalecki, and J. Tinbergen. Young also argues that Tinbergen's influence was decisive in the formulation of Harrod's dynamic theory. Goodwin (1982, p. vii) refers to the influence of Tinbergen on Harrod's Essay: "I also told him that Tinbergen reviewing the book in a German periodical, had pointed out that the accelerator plus multiplier gave a first order differential equation and hence

J. M. Keynes

In the development of his dynamic theory, Harrod was influenced by both the *Treatise on Money* (TM) and the *General Theory* (GT) (Keynes 1964 [1936], 1981) and also by his exchange with Keynes on the Trade Cycle (Harrod 1936; Keynes 1973, pp. 151–179). The TM provided an important influence on Harrod's fundamental equations and on his views on dynamics. The TM placed the focus of its analysis on the trade cycle rather than on the "equilibrium system as a whole" as in the GT (Harrod 1933, 1970, p. 618). The TM was also the inspiration for Harrod's instability principle, as Harrod explained in an exchange with Haberler in a discussion on Harrod's "Expansion of Credit in an Advancing Community" (1934):

> The puzzle of the cycle is that when a departure from equilibrium occurs, the system tends to move further from and not back to the equilibrium position. This movement seems contrary to the principles of supply and demand. Now if Keynes shows that these principles don't operate in the case of interest, which clearly lies at the heart of the system, it would seem that he is supplying just the very kind of explanation that is required. (Letter from Harrod to G. Haberler, 19 October 1934, CIPC, Vol. I, p. 304. See also the letters 5th and 10th November 1934, ibid., pp. 333, 339)

This had important implications for the development of Harrod's thought. Instability meant that there is no such thing as an equilibrium rate of interest or an equilibrium volume of output even if there were no rigidities (CIPC, Vol. I, pp. 339–340, 346). The TM also provided the basis for the distinction between the actual and required incremental capital-output ratio (C and C_r) which are a basis for the definition for the actual and warranted rates of growth (G and G_w) and thus of Harrod's dynamics (Harrod 1969, p. 165).

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could only produce exponential growth. Harrod seems to have been convinced by this, for two years later he produced his famous article on growth." The evidence regarding Cassel, Lange, and Kalecki is scant and counterfactual. The existing correspondence between Harrod and Tinbergen does not provide evidence for the claim that Tinbergen influenced Harrod. Jolink (1995, p. 435) argues that the influence of Tinbergen on Harrod was "on formal, mathematical grounds" but did not influence Harrod's methological views. See the discussion on Tinbergen on pages 195–196 and in Chapter 3 (The Trade Cycle). Besomi also contends that Harrod's instability principle derives from "Hayek's rendition of A. Lowe" in his *Monetary Theory and the Trade Cycle* (1933) but is careful to point out that the "evidence is no conclusive" (Besomi 2002, p. 48).

Harrod also recognized the influence of Keynes's GT. He explained to Keynes in August 1938 after having arrived at his fundamental equation: "I have just finished writing my re-statement of the 'dynamic' theory...I have been throwing out hints in a number of places of the possibility of formulating a simple law of growth and I want to substantiate the claim. It is largely based on the ideas of the general theory of employment" (Keynes 1973, CW, Vol. XIV, p. 301).

Harrod's exchanges with Keynes on the Trade Cycle in the period running from March to April 1937 led Keynes to formulate a "fundamental equation" that was according to Besomi (1999, p. 134): "...one of the major sources influencing the elaboration of his (Harrod's) 1938–1939 'fundamental equation', to which it also bears a strict formal analogy." Keynes's fundamental equation (Keynes 1973, CW, Vol. XIV, p. 171) expressed the rate of growth of capital (y) as an inverse function of the product of the relation (R) (i.e., accelerator) and the multiplier (M),

$$y = \frac{100}{MR - 1} \tag{4.4}$$

Harrod found Keynes's algebraic formulation extremely useful and adopted in the Essay an analogous formulation.[9] As put by Besomi (ibid., p. 139), the adoption of the fundamental equation led to a change in the epistemic status of the multiplier and the accelerator: "On the one hand, in the 'Essay' the three dynamic determinants of the Trade Cycle (Harrod 1936) (whose interaction explain the fluctuation of the trade cycle) were substituted by the variables s and C (or M and R in Keynes's formulation); the qualitative discussion of the resultant of the dynamic forces in *The Trade Cycle* thus gave place to a quantitative link. On the other hand, the Relation and multiplier were no longer considered as the mechanisms of two interacting processes, but as independent coefficients codetermining the growth rate. The coefficients s and C were entirely new concepts, thus needing interpretation."

[9]Keynes's dependent variable is the rate of growth of capital not of output as in Harrod. Keynes pointed out that steady growth in capital and output are the same when the relation is constant (Keynes, ibid.).

Also Keynes's discussion with Harrod clarified some of the conditions required for steady growth (or steady advance). First, Keynes argued that if expectations are not taken into account steady growth requires a falling rate of interest. Steady growth can be obtained if the product of the multiplier and the Relation (MR) is constant. If M falls as the level of income increases, as Keynes assumed in the GT, steady growth demands a sufficient fall in the rate of interest to induce the rise in the Relation to offset the decline in the multiplier.[10]

Second, Keynes argued that the system had several degrees of freedom and that steady growth was not equivalent to full employment and vice versa and thus that the warranted rate of growth was not uniquely determined. Third, he recognized that Harrod showed that steady growth was unlikely under a regime of *laissez-faire* ("What you have shown is that, if conditions of steady growth exist, the smallest fluctuation in the value of *MR* will cause those conditions to break down," ibid., p. 173).

E. Lundberg

Another credited influence on Harrod's development of the fundamental equation is Erik Lundberg's (1907–1987), Studies in the *Theory of Economic Expansion*, written in December 1936 and published in 1937. Lundberg's book was written (as he put it in the foreword to the second edition of the book) as a reaction to Keynes's use of the equilibrium method to explain underemployment equilibrium in the GT. Its main contribution was to analyze dynamic models (which he called "model sequences") on the basis of the methodology of Wicksell and the Stockholm School (Lundberg 1964 [1937], p. ii).

According to Lundberg Keynes's theory was valid in "determining the degree of employment which is possible under given conditions" but was incomplete. Equilibrium positions also imply some knowledge of the actual tendencies toward these since equilibrium positions are

[10]As Keynes (1964 [1936], p. 120) explained: "The marginal propensity to consume is not constant for all levels of employment, and it is probable that there will be, as a rule, a tendency for it to diminish as employment increases; when real income increases...the community will wish to consume a gradually diminishing proportion of it."

established over time. This process of adjustment in turn affected the business cycle. As he put it: "Keynes does not further investigate how these adjustments take place, nor how and why they lead to the equilibrium...this cannot be done when using as a starting point the formulation of the problem given by the postulated equilibrium system and Keynes adheres to this approach almost throughout" (Lundberg, ibid., p. 35).[11] According to Lundberg, overcoming the limitations of Keynes's analysis required the construction of dynamics based on sequence analysis and, in this regard, Wicksell was considered a pioneer because he combined "states of disequilibrium over successive periods" (ibid., p. 45 and note 1, p. 45). Sequence analysis involved the introduction of time through the use of time lags in the analysis. As Lundberg explains (ibid., p. 50):

> The primary question is in what way we may introduce time elements in order to carry the static theory further. In reality, of course, there are an infinite number of time-lags that could be considered. Every cause is followed by its effect after a certain time. If it does not seem satisfactory to qualify the results of static analysis by general statements about the time-consuming nature of the tendencies found, an infinite number of possibilities opens up for dynamizing the static relationships.

In his review of Lundberg's book (August 1937), Harrod contrasted sequence analysis with dynamic analysis. Harrod recognized that Lundberg's analysis of the relationship between the investment and

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[11]Note that Joseph Alois Schumpeter (1883–1950). Schumpeter also argued that the wave motion is a key characteristic of capitalist development, and that the cycle is the normal way through which capitalism develops and grows (Elliott 1997). As Schumpeter (1997 [1934], pp. 214–215) put it: "The point of view according to which the wave-like fluctuation in business...appears to be the fundamental thing to be explained....We agree in this conception... that the alternating situation are the form economic development takes in an era of capitalism." Schumpeter published in 1946 an article also arguing that Keynes's GT was anchored in static analysis. As he put it (1946, p. 511), "The exact skeleton of Keynes's system belongs, to use the terms proposed by Ragnar Frisch, to macro statics, not to macro dynamics..." and (ibid., p. 512): "All the phenomena incident to the creation and change in this apparatus, that is to say, the phenomena that dominate the capitalist processes, are thus excluded from consideration." Years later Hyman Minsky (1919–1996), a former student of Schumpeter, made a similar criticism of Keynes (Minsky 1975).

the consumption goods industries (ibid., e.g., pp. 181–210) took into account the consequences of the acceleration principle. But the consequences of this dynamic principle were mixed with those of the sequential assumption according to which entrepreneur's disbursements in any given period depended only on the receipts received in the previous period. By assuming that entrepreneurs' decisions depended only on the past without consideration for present conditions, sequence analysis postulated that change, say due to an unforeseen event, depended on ignorance. For Harrod ignorance had little to do with expansion or growth: "Changes may occur in a system that is not expanding... I suggest that we shall be making a methodological error of the first order if we do not rigidly distinguish between matters related to ignorance and matters related to growth" (Harrod 1937, p. 497). The point can be made that in Lundberg's analysis, insofar as dynamics are entangled with ignorance or, which is the same thing, forecasting errors, his analysis bears a close resemblance to Pigou's trade cycle theory founded upon errors, which Harrod criticized (see Chapter 2).

For Harrod, the problems related to growth and dynamics should be solved by a system of equations of growth rates relating to a given point in time. In developing his critique of Lundberg's book, Harrod had the chance to develop two central concepts that would be part of his fundamental equation, the warranted rate of growth and the notion of the length of the time period which Harrod refers to as "reaction time" (Young 1989). The following quote from the review illustrates the first explicit formulation of Harrod's warranted growth rate:

> I suggest that it may be possible to construct a method of dynamic analysis more closely analogous to the dynamics of mechanics. In place of the static question, what rate of production of eggs per day will be consistent with the maximization of the advantage of egg producers, we ask, what rate of increase in the production of eggs per day will lead to this result? This is a natural extension of the static theory, appropriate to an expanding economy. At what rates of increase (or decrease) must all members of the system pursue their operations so that, when they all do this, no one shall find it to his advantage to do otherwise than continuing expanding at this rate? A system of equations in which these rates of increase (or decrease) are unknowns should be elaborated, on lines similar to those of

traditional static systems, with sufficient unknowns to determine the values of the additional unknowns also. (Harrod 1937, p. 495)[12]

4.2 The Fundamental Equation in the Draft Essay (1938)

4.2.1 The Fundamental Equation and the Principle of Instability

"The Essay in Dynamic Theory" was submitted for publication in the *Economic Journal* in August 1938. This version is referred to as the draft Essay to distinguish it from the final published version in March 1939.

As in his other later works on dynamics (1948, 1960, 1973), Harrod began the draft Essay by distinguishing between statics and dynamics and qualifying both terms. He referred to "a tentative and preliminary attempt to give an outline of dynamic theory," and "to a framework of concepts" (CIPC, Vol. III, p. 1888). He also made the contrast between static and dynamics and explained that dynamics are the analogue to static analysis when "certain forces are operating to increase or decrease certain magnitudes in the system."

Then Harrod stated his fundamental equation in the most simple form as:

$$G_w = \frac{s}{C} \tag{4.5}$$

where

G_w = warranted rate of growth.

s = total savings of individuals and firms divided by output (average propensity to save).

C_r = the value of the capital goods required for the production of a unit increment of output.

[12]Young (op. cit.) also argues that Harrod developed the idea of instability in his critique of Lundberg.

The propensity to save, s, changed with the level of income, its distribution, the phase of the business cycle as well as a result of institutional changes. Later on in the Second Essay on Dynamic Theory, Harrod would emphasize the influence of subjective factors on the propensity to save. These included: "the private convenience of many individuals, on the desires of many firms to finance themselves for their own ploughed-back profits, etc." (Harrod 1960, p. 279). In *Economic Dynamics* (1973, pp. 20–21), he also included as determinants of the propensity to save institutional arrangements (social provision and pension arrangements and the development of capital markets).

For its part, the variable C_r depended on the state of technology and the composition of capital, variations in income, the phases of the business cycle (as in the case of the propensity to save) and "somewhat dependent on the rate of interest" (CIPC, Vol. III, p. 1191).

Once the propensity to save (s) and the value of the capital goods required for the production of a unit increment of output (C_r) were known, the warranted rate of growth, G_w (the unknown in the fundamental equation) could be determined. Harrod then provided a "derivation proof" for his fundamental equation. This can be derived by assuming that planned savings are equal to planned investment. That is,

$$I = S \tag{4.6}$$

Assuming two periods (0 and 1), a short interval between both that investment was equal to the increment of capital (K) between both periods, $I = K_1 - K_2$, and that total savings were undertaken out of income in the first period ($S = sY_0$),

$$I = S \Leftrightarrow K_1 - K_0 = sY_0 \tag{4.7}$$

Dividing both sides of Eq. (4.7) by the increment in output between period 0 and 1,

$$\frac{K_1 - K_0}{Y_1 - Y_0} = \frac{sY_0}{Y_1 - Y_0} \tag{4.8}$$

Rearranging terms,

$$\frac{Y_1 - Y_0}{Y_0} = \frac{Y_1 - Y_0}{K_1 - K_0} * s \tag{4.9}$$

Equation (4.9) states that the rate of growth of output $\left(\frac{Y_1-Y_0}{Y_0}\right)$ equaled to the increment of output divided by the increment of capital $\left(\frac{Y_1-Y_0}{K_1-K_0}\right)$, multiplied by the propensity to save (s). Or,

$$G = \frac{s}{C}, \tag{4.10}$$

where G is equal to the rate of growth of output $\left(G = \frac{Y_1-Y_0}{Y_0}\right)$ and $\left(C = \frac{K_1-K_0}{Y_1-Y_0}\right)$.

Equating C to C_r, that is, the actual value of the increment of capital per unit of increment of output to the amount of capital per unit of the increment of output, which is required by the determinants of C_r, led to the equality between the actual and the warranted rate of growth ($G = G_w$).

This set the stage for the characterization of Harrod's dynamics.

Any disparity between the actual (G) and the warranted rate of growth of output (G_w) set up centrifugal forces causing a cumulative upwards or downward movement amplifying the distance between both variables. This can be shown below for the cases where $G > G_w$ and $G < G_w$

$$G > G_w \Leftrightarrow \frac{s}{C} > \frac{s}{C_r} \Leftrightarrow C_r > C \Rightarrow G > G_w \tag{4.11}$$

$$G < G_w \Leftrightarrow \frac{s}{C} < \frac{s}{C_r} \Leftrightarrow C_r < C \Rightarrow G < G_w$$

When the actual rate of growth exceeded the warranted rate, $G > G_w$ (when the actual rate of growth is below the warranted rate, $G < G_w$), the actual value of the increment of capital per unit of increment of output would be below the required amount of capital per unit of output, $C_r > C$ (above $C_r < C$). This called forth increased (less) capital investment pushing the actual rate further above (below) its warranted level. The same exercise could be carried out by assuming different propensities to save, although at this stage, Harrod did not distinguish between the actual and the desired propensity to save.[13]

[13]He made the distinction in *Economic Dynamics* (1973, pp. 16–17).

Harrod's summarized his dynamic theory in two propositions: "(i) A unique warranted line of growth is determined jointly by the propensity to save (s) and the quantity of capital required by technological and other considerations per unit increment of total output (C_r)...; (ii) On either side of this is a 'field' in which centrifugal forces operate, the magnitude of which varies directly as the distance of any point in it from the warranted line. The moving equilibrium of advance is thus a highly unstable one" (CIPC, Vol. III, p. 1196).

4.2.2 Autonomous Capital Expenditures and the External Sector

Harrod modified the fundamental equation to include capital expenditures that are not related to the level of income and/or to its variations and to the external sector. The inclusion of the former implies defining investment (I) as the sum of the additions to the capital stock that depended on income and those that were independent of income (denoted by the term $\overline{K}$) and equating investment to savings. This can be shown as follows. Starting from the definition of $C = \frac{K_1 - K_0}{Y_1 - Y_0}$, investment can be defined as being equal to $C(Y_1 - Y_0)$. That is,

$$C = \frac{K_1 - K_0}{Y_1 - Y_0} \Leftrightarrow C(Y_1 - Y_0) = K_1 - K_0 \Leftrightarrow C(Y_1 - Y_0) = I \tag{4.12}$$

The term $\overline{K}$ is then included in the definition of investment, so that investment is equal to the additions to the capital stock that depend on income and those that are independent on income $\left(C(Y_1 - Y_0) \text{ and } \overline{K}\right)$. Investment is then equated with savings

$$C(Y_1 - Y_0) + \overline{K} = S \Leftrightarrow C(Y_1 - Y_0) + \overline{K} = S \tag{4.13}$$

Rearranging and manipulating the terms in Eq. (4.13) yield a modified actual rate of growth (G). That is,

$$G = \frac{Y_1 - Y_0}{Y_0} = \frac{s - \frac{\overline{K}}{Y_0}}{C} \tag{4.14}$$

When the increase in the stock of capital divided by the increase in total output which actually occurs was equal to that which was justified by the current conditions ($C = C_r$), the actual rate of growth equals to the warranted rate of growth ($G = G_w$), and the modified warranted rate of growth could be defined as,

$$G_w = \frac{s - \frac{\overline{K}}{Y_0}}{C_r} \tag{4.15}$$

In addition Harrod also incorporated the foreign sector by postulating that total savings (S) equaled domestic and foreign savings $(S_d + S_f)$

$$S = S_d + S_f \tag{4.16}$$

Foreign savings were defined as the difference between imports (M) and exports (X),

$$S_f = M - X \tag{4.17}$$

Substituting Eq. (4.17) into Eq. (4.13) yielded the equality between investment and savings for an open economy. That is,

$$\begin{aligned} C(Y_1 - Y_0) + \overline{K} &= S_d + (M - X) \\ &\Leftrightarrow C(Y_1 - Y_0) + \overline{K} + X = S_d + M \end{aligned} \tag{4.18}$$

Assuming that both domestic savings and imports depended on the level of income in the period 0 led to the following result,

$$C(Y_1 - Y_0) + \overline{K} + X = sY_0 + mY_0, \tag{4.19}$$

where m = average propensity to import.

Dividing Eq. (4.19) by Y_0 and rearranging terms, the modified actual rate of growth (with investment that depended and did not depend on income and the foreign sector) was obtained,

$$\frac{C(Y_1 - Y_0)}{Y_0} + \frac{\overline{K}}{Y_0} + \frac{X}{Y_0} = s + m \Leftrightarrow \tag{4.20}$$

$$G = \frac{(Y_1 - Y_0)}{Y_0} = \frac{s + m - \frac{\overline{K}}{Y_0} - \frac{X}{Y_0}}{C}$$

Finally, corresponding modified warranted rate of growth could be derived by equating the actual value of the increment of capital per unit of increment of output (C) with the amount of capital per unit of the increment of output which is required by the determinants of C_r, that is,

$$G_w = \frac{s + m - \frac{\overline{K}}{Y_0} - \frac{X}{Y_0}}{C_r} \tag{4.21}$$

According to Eq. (4.21), the warranted rate of growth (G_w) varied positively with the propensity to save and the propensity to import (s and m) and negatively with exports (X), and capital outlays ($\overline{K}$) that are independent of income. The warranted rate of growth varied inversely with the capital requirements per unit increase in output (C_r). Yet due to the instability principle, the same factors that affected the warranted rate of growth in one direction (including changes in the parameters) have the opposite effect on the actual rate of growth. It is worth to quote Harrod at length on this central point of his dynamics:

> It is essential...to grasp the point that a change in the fundamental conditions tends to have the opposite effect on the actual rate to that which it has on the warranted rate. An increasing propensity to save, a decreasing capital coefficient...and a declining active balance or growing passive balance on the international current account, all tend to increase the warranted rate; but they all tend to have a depressing effect on the actual rate. If the actual rate is equal to the warranted rate, by raising the warranted rate above it, they precipitate a downward movement. If the actual rate is above the warranted rate, by raising the warranted rate, they reduce the gap and so diminish the drive to expansion. If the actual rate is below the warranted rate they increase the gap and so increase the forces of depression. These propositions are all connected with the instability of the moving equilibrium. In the 'field' on either side of the warranted line of growth, there are centrifugal forces, which increase with the distance of the actual output from the warranted output. (CIPC, Vol. III, p. 1203)

4.2.3 The Natural and Normal Warranted Growth Rates

In order to complete his dynamic theory and provide a full explanation of the cycle, Harrod introduced two additional growth rates: the natural growth rate (G_n) and the normal value of the warranted growth rate (G_{wn}).

He defined the former as the maximum rate of growth, which is allowed by population, technology, and preferences assuming full employment.[14] It provided a ceiling to the expansion of the actual growth rate above the warranted rate ($G > G_w$). Under this scenario as the actual rate of growth increasingly diverged from the warranted rate, it eventually reached the natural rate, associated with full employment, above which it could not increase. However, the warranted rate could exceed the natural rate ($G_w > G_n$).

The normal value of the warranted rate (G_{wn}) is defined as the value of the warranted rate under a steady rate of growth of the economic system, or what Harrod terms a state of steady advance ("The special value of G_w....which would obtain if the actual rate of growth had coincided with the warranted rate for some time," CIPC, Vol. III, p. 1998). In this state, the propensity to save represented "the normal habits of individuals and companies" and that C_r is determined by technology and "the regular trend in of consumer's choice" (ibid.). The normal value of the warranted rate (G_{wn}) was assumed to be independent of the phases of the cycle and captured the idea of stationary conditions, that is, of the idea that contractions are offset in an exact manner by expansions.

4.3 A Sketch of the Trade Cycle

Having established, defined, and characterized the above elements (the four rates of growth: the actual, warranted, and normal warranted, and natural rates of growth; and the instability principle) Harrod sketched

[14]It is curious to note that in his correspondence with Marschak on the draft Essay, Harrod explicitly stated that the natural rate of growth did not represent the ceiling of full employment (CIPC, Vol. II, p. 859).

his theory of the trade cycle. It consisted of two parts: The first is related to the cumulative deviations of the actual from the warranted rate of growth and the second is referred to the endogeneity of the warranted over the course of the cycle.

4.3.1 The Cumulative Divergence Between the Actual and Warranted Rates of Growth

The first case illustrated "how the forces driving growth away from the warranted rate are checked by the productive forces of the system" in the boom phase of the cycle.

In that phase of the cycle both the actual (G) and the warranted rates (G_w) increased. Following the principle of instability, the former rose at a faster and diverging pace from the latter as long as the boom lasted. At every point in time ($G_t > G_{wt}$), G moved away from G_w as the actual increase of capital goods (C) exceeded that which required or desired (C_r) by the current conditions.

However, G cannot exceed the maximum rate of growth, which is allowed by population, technology, and preferences assuming full employment, i.e., the natural rate of growth (G_n). Given this point the "expansion hit a ceiling". If it is assumed that G_w catches up with G and eventually exceeds G_n, a reversal would occurs and a recession becomes inevitable. Harrod referred to this case as the theory of recession presented in the Trade Cycle (1936).

4.3.2 The Endogeneity of the Warranted Rate of Growth

The second part is concerned with the case of a displacement of the warranted rate from its "normal value," which occurs when the actual rate growth has been displaced from the warranted rate for some time. In this case, the warranted rate of growth "chased after the actual rate of growth upwards or downwards."

In the downward phase of the cycle, the actual rate of growth was driven below the warranted rate ($G < G_w$) and as a result income

fell. If the situation persisted, the fall in income led to a decline in the warranted rate of growth. Harrod identified three forces that tend to depress the warranted rate as income falls. First $\frac{\overline{K}}{Y_0}$ and $\frac{X}{Y_0}$ increased directly as income fell. Second, the propensity to save (s) also declined in line with the fall in income. Third, insofar as Harrod defines C_r, in a recession, as the "amount of capital that can be most conveniently dispensed with" (CIPC, Vol. III, p. 1199). C_r would tend to increase.[15]

He did not consider what happens to the propensity to import (m). It can be conjectured, however, that it can move in tandem with the cycle phases, so that in slump (boom) m decreased (increased) providing a further impulse to the fall (increase) in the warranted rate (G_w).

The fall in the warranted rate of growth may be reinforced as a result of a negative actual rate of growth ($G < 0$) due, in turn to a negative value of C. As a result $C < C_r$ and entrepreneurs would be de-accumulating capital at a rate below that which is required and thus expanding instead of contracting supply. Harrod further postulated that in a contraction the required de-accumulation is "abnormally depressed" due to the fact that a great part of fixed capital cannot be liquidated. If only circulating capital can be liquidated, a moving equilibrium would be attained when the actual de-accumulation equals to the required de-accumulation of circulating capital ($C = C_r$). The recovery would start when $C > C_r$.

In the boom phase as income and output increases, the propensity to save (s) increased as corporations earned more profits and decided to increase their retained earnings to finance more investment; $\frac{\overline{K}}{Y_0}$ falls since $\overline{K}$ is independent of output. For its part $\frac{X}{Y_0}$ declined only insofar as the expansion of internal demand offset that of the rest of the world. Harrod assumed that at the end of the contractionary phase of the cycle and beginning of the boom phase C_r had a lower value at the start of the boom and remained at that level due to the existence of surplus capacity.

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[15]Harrod (ibid.) provided a different definition for C_r in the boom ("...the amount of extra capital required per unit increment of output") and in the slump ("...the amount of capital that can most conveniently be dispensed with"). This adds to the confusion regarding the different definitions of C_r. See footnote 1. Harrod also pointed out that a quicker turnover of equipment during the slump would increase the value of C_r (CIPC, Vol. III, p. 1201).

According to Harrod, the development and fulfillment of the recovery faced three challenges. First, if there is significant excess capacity and, at the same time, the warranted rate of growth is "abnormally" high due to a low value of C_r, the warranted rate of growth would rise above the actual rate putting a brake on the recovery and bringing about a recession. Second, a similar situation could occur if the normal warranted rate was much higher that the natural rate of growth. In this case, the warranted rate of growth would also exceed the actual rate of growth before nearing full or quasi-full employment.

The third challenge referred to a situation where the normal warranted rate of growth is below the natural rate. The actual and warranted rates of growth departed upwards from the normal warranted rate. Eventually, the warranted rate expanded above the natural rate. If the actual rate of growth maintained a level compatible with full employment, the resulting inflation led to an increase in the propensity to save as a result of an increase in the share of profits.[16] In turn the rise in the propensity to save translated in an upward movement in the warranted rate of growth, and if it increased above the actual rate of growth, a recession would result.

In his sketch of the trade cycle, Harrod presumed that the specification of rates of growth at a point in time, which Harrod defined as being the essence of dynamic analysis (see his criticism to Lundberg's sequence analysis above), had to give way to the analysis of rates of growth over time. In other words, Harrod was forced to include time lags in his analysis: "The objection must be admitted; a lag is implied. It is only in the formulation of the fundamental equation in its different forms for determining the warranted rate of growth that considerations of lags is rigidly excluded. This is dynamic because it embodies a rate of growth as an unknown variable; and the whole of the argument which follows depends upon it" (CIPC, Vol. III, p. 1203).

The Draft Essay received detailed comments from Marschak and especially from Keynes.

[16]This is the "shift-to-profits" determinant that Harrod introduced in the *Trade Cycle* (1936). See Chapter 3.

4.4 The Reactions to the Draft Essay: The Correspondence with J. Marschak

4.4.1 Marschak's Comments of the Use of Lags and Non-linearities

Marschak agreed with Harrod that dynamic theory involved rates of change at a point in time and is not equivalent to expressing variables in terms of lags at different points in time. But he pointed out that concepts such as acceleration and velocity (second derivatives) necessarily involved the comparison between two points in time and hence the use of lags[17] (CIPC, Vol. II, p. 848).

The use of second derivatives is implicit in Harrod's equation and in his description of the trade cycle. As pointed out earlier, Harrod's characterization of the trade cycle involves analyzing the oscillatory movement of rates of growth. This necessarily implies comparing second derivatives (the change in the rates of growth) over time. Nonetheless, he persistently struggled throughout his writings to distinguish his view of dynamics from those that are more associated with the use of lags.[18]

Marschak also understood, commented, and valued the endogenous mechanism of the trade cycle theory which led to the turning points and the oscillatory movement. As he stated: "it can be postulated that G_w continues to rise after intersecting the ceiling (some sort of 'inertia'), the turning of G does follow from the postulate of cumulative process. This is, I think, one of the most interesting points in the Essay." The same logic of cumulative movement applied in the downward phase of the cycle when G_w fell at a greater pace than G also led to the bottom of the contraction and eventually the recovery. Thus, the endogeneity mechanism worked in both phases of the cycle and guarantees the oscillatory movement of the cycle.

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[17]Marschak's comment was made in relation to Harrod's statement that "those who define dynamics as having a cross reference to two points in time will not regard this equation (the fundamental equation) as dynamic" (CIPC, Vol. II, p. 1191).

[18]In this regard, Besomi (2003, CIPC, Vol. III, p. 1206, n. 5) argues that Harrod "pigeon-holed numerous trade cycle theories under the heading of 'time.lag' theories of the trade cycle." He includes here Robertson's period analysis, Lundberg's analysis in his *Theory of Economic Expansion* (1937), Tinbergen's work (1935, 1938) and Hicks's *Value and Capital* (1939). See also Besomi (1998b).

4.4.2 Marschak's Graphical Representation of Harrod's Dynamics

In addition, Marschak introduced a graphical method of representation depicting the possible trajectories of the normal and warranted rates of growth with a ceiling and a floor (Fig. 4.1). Marschak's figure can be interpreted to reveal four cycle stages. In the first stage, both G and G_w increased with the latter increasing at a greater rate than the former and reaching the natural rate of growth (G_n). In a second stage, G_w surpassed both G and G_n which signaled the end of the expansion and set the scenario for the turning point from an expansion to a recession. In a third stage, both G and G_w decreased. In addition, G_w decreased at a faster pace than G. In the final and fourth stage, both G and G_w have negative growth rates and G_w intersects G setting the basis for the recovery.

Harrod (CIPC, Vol. II, pp. 859–861) remarked that the graphical method was not suited for the analysis of dynamics and pointed to several misrepresentations by Marschak. Marschak confused levels and rates

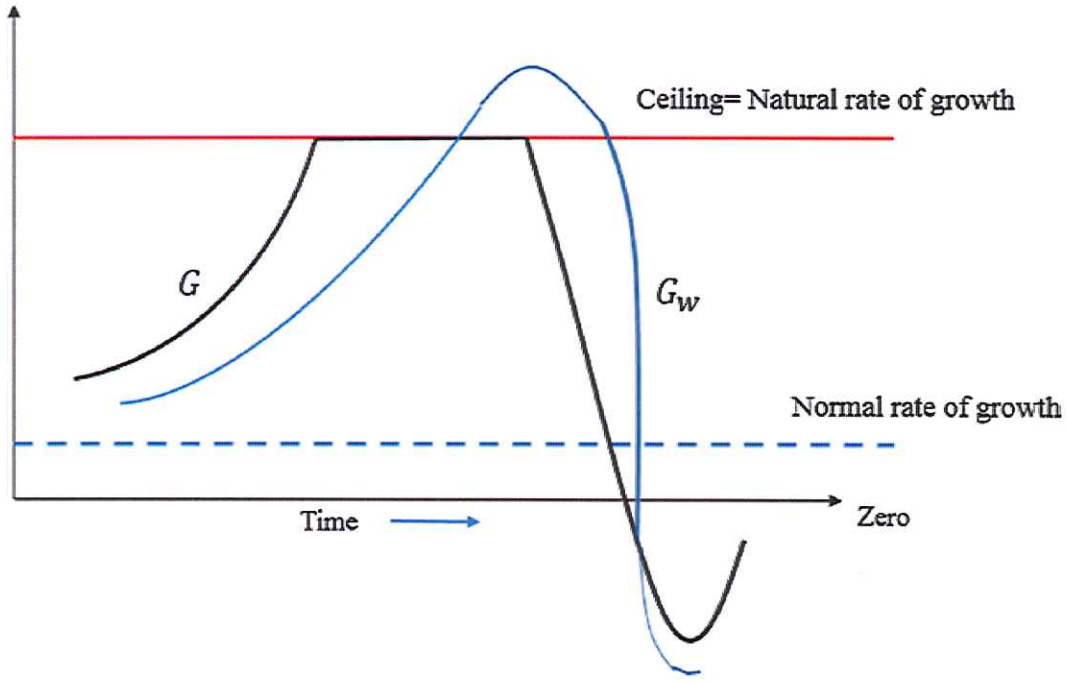

Fig. 4.1 Marschak's depiction of the Harrodian trade cycle (*Source* On the basis of Besomi, CIPC, Vol. II, pp. 845, 859)

of growth by equating full employment with the natural rate of growth. As a result, the actual rate of growth could expand above the natural rate, and only once it reached the level corresponding to full employment would it decline toward the natural rate (see footnote 219). The velocity with which it reached the ceiling depended on the "magnitude of the acceleration." This implied that G could follow several possible paths in the upturn and downturn of the economic cycle and that Marschak's figure portrayed only a particular rather than the general case.

Harrod also explained the difficulty in plotting the warranted rate since it could have several fluctuations within a cycle and thus not necessarily just two as in Marschak's figure. In addition, he explained that the warranted rate had to cut the actual rate at its turning point. Moreover, depending on the circumstances, the normal warranted rate could be above and not below (as in Marschak's figure) the natural rate, the actual rate could expand far above or not very far above the normal warranted rate, and the warranted rate could also rise far above or very slightly above the normal warranted rate.

Harrod also considered the possibility that when the normal warranted rate was far above the natural rate, the warranted rate could not even reach the normal rate. Harrod's explanation points to the fact that a coherent framework to analyze the relations between the natural, actual, and warranted rates of growth were through case analysis rather than through the use of a figure. This foreshadows the type of case presentation he provided in *Economic Dynamics* (1973, Chapter 7: Problems and Conflicts, p. 104) for seven different cases depicting different relationships among the actual, warranted, and natural rates of growth.

4.4.3 Marschak's Formalization of Harrod's Dynamics

In his letter to Harrod, Marschak's not only provided graphical representations of Harrod's dynamics but also its first mathematical formalization. Marschak derived the fundamental equation using Harrod's axiomatic basis of his theory, something that Harrod had not done (CIPC, Vol. 3, p. 1188): (i) The supply of savings is determined by income; (ii) the demand for savings depends on the rate of increase of income; and (iii) demand is equal to supply.

Marschak's mathematical formulation was the first of a series of models developed over time that would try to capture the instability and non-linearity of Harrod's dynamics (see Annex for a selected sample of the models from 1939 to 1962).

The fact that Harrod did not realize that his multiplier-accelerator model could not account for non-linearities and yielded explosive solutions, as well as his indifference to the formal aspects of his model in his reply to Marschak can perhaps be attributed to an insufficient knowledge of mathematics. In his review of the *Trade Cycle*, Tinbergen (1937) made similar remarks and Harrod took no account of it. A year prior to the writing of the Draft Essay, Harrod wrote in relation to Bowley's *Mathematical Framework of Economics*, that he was not fond of mathematical foundations and ill educated in mathematics (CIPC, Vol. II, p. 600). In a letter to Tinbergen (1 July 1937), Harrod also mentioned the rudimentary state of his mathematics (CIPC, Vol. II, p. 705).

However, this is somewhat contradictory with the facts that he took a seminar on Einstein's theory of relativity in 1922 and seems to have known the theory well as indicated in his book *The Prof.* (1959).[19] Also he became a close friend of Joseph Lindeman, a physicist by training, and he befriended Einstein during one of his visits at Oxford in the 1920s.

A second explanation is related to a difference in method between Harrod and Marschak. In his "Scope and Method of Economics" (1938), Harrod had already outlined his approach to dynamics in three stages which he followed in the Essay: "...the determination of the dynamic equilibrium in one instant and the discussion of its stability properties (Sections 1–11...Second the succession of events occurring in the trade cycle (Sections 12–21) is studied...variations in the fundamental conditions...(are) allowed and played an important part in the argument. Third economic policies are discussed (Sections 22–25)... according to the relative positions of natural and normal...rates of growth" (Besomi 1996, p. 288; see also Sember 2009).

[19]While it is true that the Special Theory of Relativity (1905) requires only elemental mathematics, *the General Theory of Relativity* (1915) requires more sophisticated mathematics, including non-Euclidean geometry and non-linear differential equations.

Marschak (1941) had a different approach. He was of the view that the trade cycle theory consisted of two necessary steps: to ascertain the existence of regularities in the behavior of economic variables and to explain these on the basis of an elementary hypothesis (p. 446). He did not see a difference of method between static and dynamic regularities. For Marschak a dynamic model consisted in a "comprehensive system of elementary postulates." He exemplified this approach with the case of Tinbergen's *Statistical Testing of Business Cycles* (1939), where the latter applied a general linear difference equation to derive cycles and compare these with existing time series. The equation could be solved in the abstract and use historical data to fill in the values of the variables under consideration (Marschak 1942, pp. 191–192 in Marschak 1974).

A third explanation is that the type of mathematical formulation provided by Marschak simply does not capture adequately Harrodian dynamics. By postulating that Harrod's model provides a periodic solution Marschak was assuming regularity in behavior and more specifically in entrepreneurial behavior. Regularity implies predictability and unless errors are invoked, this ultimately negates the very basis of instability.[20] In fact, Harrod explicitly stated that there was nothing in his theory that suggested that the phases of the cycle "must follow each other with any regularity" (Harrod 1957, p. 3).[21] The type of formulation provided by Tinbergen requires giving numerical values to the parameters but may not be useful to explain different cycles

[20]As Besomi (1996, p. 292) points out Marschak's remarks on the non-constancy of C suggests that at the time "the econometricians were aware of the limitations of linear analysis" and speculates that Le Corbeiller's article (1933) of the application of the van der Pol oscillator to problems of political economy including crises and price movements did not go unnoticed. But once again the van de Pol oscillator provides periodic solution and thus suffers from the same type of weakness as Tinbergen's linear difference equation. Some of the current literature on Harrod's dynamics (i.e. Sportelli 2000) also provides a periodic solution and thus assumes regularity and predictability on the part of entrepreneurs and thus, in spite of the use of modern mathematical techniques, suffers from the same weakness in the conceptual interpretation of Harrod's dynamics and the instability principle.

[21]He made a similar comment earlier on in the *Trade Cycle* (1936, p. 31): "In the actual course of the trade cycle the movement is not found to be uniform. This is largely connected with the relation between capital and consumable goods, a matter reserved for discussion at a later stage. The lack of uniformity may be connected with the fact that the stabilizing principles or their inverse operate with different force in different occupations."

(Harrod to Keynes, 3 August 1938; Keynes, CW, Vol. XIV, p. 301) or different phases of one cycle. Harrod made a similar point in a letter to Tinbergen (1 July 1937, CIPC, Vol. II, pp. 705–706): "...any single-handed attempt to give a rigid mathematical formulation to my theory would not be successful...I am strongly of the opinion that there is a stray element of discontinuity in the slump and of asymmetry as between slump and boom, and that conditions remain for a while in the slump more decidedly out of short period equilibrium even than they do in the upward swing."

4.5 The Reactions to the Draft Essay: The Debate with Keynes

Keynes's exchanges with Harrod on the draft Essay which went on for more than a month (17 August–24 September 1938) were marked by misunderstandings, reciprocal misinterpretations, errors in some of the formulas and formulation of their arguments.[22] Keynes made a critique of the presentation and exposition of the ideas presented in the Essay and a substantive critique.

With respect to the former, Keynes complained about Harrod's notation and ("muddled") symbolism which simply made the argument confusing and difficult to grasp and understand. As he stated (Keynes, CW, Vol. XIV, p. 339): "I feel even more strongly that, from the reader's point of view, you have not done justice to them, and I doubt whether the ideas will have the full impact on the reader which they deserve. I still find the exposition half-baked and prolix, and I have found it practically impossible to work things out for myself in terms of your symbolism which is so contrived as to lose sight of the dimensions of your quantities which makes it very difficult to handle." Throughout the exchange, Keynes tried to use numbers and a different notation to clarify the arguments but he himself made errors in his presentations increasing the difficulty of communication.

[22]See Besomi (1995, p. 325) and Kregel (1980).

His substantive criticism focused on the definition of the concept of a warranted rate of growth and the instability of the warranted rate.[23] Keynes rejected the concept of a warranted rate of growth (ibid., pp. 345–346). Regarding instabilbility Keynes argued that it required a premise that Harrod never made explicit. That is, the propensity to save must be less than the required incremental capital-output ratio ($s < C_r$). According to Keynes, Harrod's troubles began when the latter illustrated the instability principle with the case where the actual growth rate exceeded the warranted rate ($G > G_w$), the actual increase in capital goods (C) fell below its desired level (C_r). This set in motion a divergence of the actual growth rate from the warranted rate (ibid., p. 340). Keynes asserted that the premise underlying Harrod's argument which he never proved, $s < C_r$, "is what seems to me to be the fundamental presupposition of the whole story, without which it would fall to the ground…it is the fundamental fact in the world of experience which lends significance to your whole story" (pp. 339–340).

Harrod could not understand Keynes's critique. He replied that "in any warranted position $s < C_r$ since $C_r(\Delta Y) = s(Y + \Delta Y)$" (ibid., p. 335).[24] And as he affirmed in his last letter to Keynes on the subject: "You seem to have some substantial objection to my argument which I am quite unable to see…I cannot comprehend the force of your criticism" (ibid., p. 342).

Keynes accepted that he became partly confused in trying to disentangle Harrod's ideas and ultimately argued that the condition $s < C_r$ was a requirement for the existence and uniqueness of the warranted rate of growth. Nonetheless, he questioned the practical feasibility of Harrod's model. As he put it (ibid., pp. 348–349): "unless C_r is much greater than s, the warranted rate will have to be so great as not be practically feasible, since the risks of acting on a sufficiently drastic scale will be too great for the entrepreneurs. For example if C_r is ten times s,

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[23]See Kregel (1980), pp. 102–113.

[24]Keynes explained Harrod's model in the following terms. $\Delta S = sY$, where s is the proportion of income saved in any period, ΔS = increment of savings per unit of time; $\Delta I = C_r \Delta Y$ = 'increment of capital, corresponding to increment of income at a rate ΔY per unit of time required by "normal" technological considerations'; and $\frac{\Delta Y_W}{Y} = \frac{s}{C_r}$ = warranted rate of growth during the time period (ibid., p. 333). From here the equation $C_r \Delta Y_w = sY$ can be obtained (ibid., p. 333). As Besomi (1995, p. 328) indicates Keynes forgot that $\Delta S = sY + s\Delta Y$, so that $C_r \Delta Y = sY + s\Delta Y$ as Harrod correctly stated. Thus $C_r \Delta Y = sY + s\Delta Y \Leftrightarrow \Delta Y(C_r - s) = sY$ and $\Delta Y > 0$ implies that $C_r > s$.

the warranted rate of growth will have to be 11 per cent per annum... If you imagine a prospect of this persisting year after year with a stationary population, the change in the direction of new demand and of technique will be so great as to baffle the entrepreneurs...an all-round continuing expansion at the rate of 11 per cent is simply not practicable." Similarly, he pointed out that a feasible warranted rate of growth of 2.5% annually would mean that ΔC_r would have to be forty times $s\Delta Y$ which was unlikely.

4.6 The 1939 Essay on Dynamic Economics

The published version of the Essay included mainly the comments made by Keynes. A comparison of the draft and published version of the Essay shows six explicit instances where Harrod introduced changes due to Marschak's comments (Besomi 1996).

The most relevant one is the elimination of the concept of normal warranted rate of growth which also responded to one of Keynes's comments. Both Marschak and Keynes found the concept confusing.

In the draft Essay, Harrod had introduced the concept in direct relation with the fact that the warranted rate is not constant and varies in the course of the cycle. When the warranted rate is at its normal value, the system advances "steadily along its warranted rate." When the actual rate of growth has diverged for some time from the warranted rate, then the warranted rate becomes, in turn, displaced from its own value. This ultimately leads to the cycle movement with the warranted rate always chasing the actual rate in the upward and downward phase of the cycle.

In the final version of the Essay, Harrod abandoned the notion of the normal value of the warranted rate. As soon as there is a divergence of the actual from the warranted rate, the warranted rate moved and "may chase the actual rate in either direction" (Harrod 1939, p. 29). And also: "...there is no unique warranted rate; the value of the warranted rate depends upon the phase of the trade cycle and the level of activity" (ibid., p. 30). The separation between the actual and the warranted rate perhaps responded also to Marschak's criticism that Harrod needed to

communicate more clearly that G_w and G were two distinct variables and that G_w was not a particular value of G (CIPC, Vol. III, p. 848).

Harrod seems to have substituted the normal warranted rate with the proper warranted rate. However, both are different concepts. The proper warranted rate referred to "that warranted rate which would obtain under conditions of full employment" (Harrod 1939, p. 30). Contrarily the normal warranted rate did not entail full employment.[25]

Harrod made several changes to the draft version as a result of Keynes's comments. Perhaps the most significant one is the reduction of the application of his dynamics to the theory of the cycle. In the draft Essay, Harrod provided a detailed sketch of the phases of the cycle and of the workings of the principle of instability. The sketch made obvious the dependency of the warranted rate on the different phases of the cycle and the instability of the moving equilibrium, the non-constancy of the parameters in the fundamental equation, and the need for non-linearities to be able to account for the different phases and turning points of the cycle (CIPC, Vol. I, pp. 1198–1203).

In the published version of the Essay, Harrod stated: "The fundamental dynamic equation has been used to demonstrate the inherent tendency of the system to instability. Space forbids an application of this method of analysis to the successive phases of the cycle" (Harrod 1939, p. 28). As a result, the discussion of the trade cycle was compressed significantly, and this may have contributed to downplay the originality of Harrod's contributions and the misinterpretation of his dynamics as a fixed coefficient model.

Another relevant change resulting from Keynes comments is the explicit inclusion of the rate of interest and the state of confidence as a determinant of C_r and thus of G_w to distinguish their influence from that of technology. In the draft Essay, Harrod stated that: "If the value of the increment of the stock of capital per unit increment of output which actually occurs (C) is equal to the amount of capital required by technological and other conditions…" (CIPC, Vol. I, p. 1191). In the

[25]It is curious to note that in his discussion on policy proposals and more specifically public works he does refer in one instance to the normal warranted rate (ibid., p. 32).

Essay itself, he replaced the last part of the sentence with "required by technological and other conditions…(including the state of confidence, the rate of interest, etc.)…"

However, he was careful to point out that C_r is independent of rates of growth at a point in time and, as a result, it is independent of G. This contradicted Marschak's suggestion of making C_r an endogenous variable to the actual rate of growth.

In fact, according to Harrod, the independence of C_r from G (as well as the independence of s from G) was crucial to the instability of the "warranted line of advance." He emphasized the fact that the system did not provide any self-correcting mechanism when the rate of growth was situated on either side of the warranted rate. He illustrated this point with respect to the concepts of over and underproduction. Overproduction (underproduction) occurred when the actual rate of growth was below (above) its warranted level $G < G_w (G > G_w)$. Overproduction (underproduction) would continue as long as G remained below (above) G_w or to put more precisely as long as the condition determining the warranted rate remained unchanged. It is useful to quote Harrod on this point due to the clarity of exposition:

> It must be noted that a rate of growth lying on either side of the warranted rate is regarded here as unwarranted. If the actual rate exceeds the warranted rate, producers on balance will not feel that they have produced or ordered too much; on the contrary, they will be running short of stocks and/or equipment. Thus they will not feel that they have produced the warranted amount plus something; on the contrary, they will feel that everything which they have produced has been warranted, and that they might warrantably have produced something more. None the less, we define their production as unwarrantably large, meaning by that that they have produced in excess of the unique amount which would leave them on balance satisfied with what they had done and prepared to go forward in the next period on similar lines.[26] (Harrod 1939, p. 24)

[26]Following the introduction of capital that is independent of income and the external sector, Harrod does not describe the effects of the downward and upward phase of the cycle on the warranted rate due to lack of space (Harrod, ibid., p. 28).

4.7 The Policy Proposals

Both the draft Essay and the final published versions end on a policy note. Leaving aside the substitution of the normal warranted rate with the proper warranted rate, no significant changes exist between both versions.

The focus is placed on avoiding a slump which ensues when the actual growth rate reached the ceiling and the warranted rate expands beyond the natural rate ($G = G_n$ and $G_w > G_n$). Harrod mentioned that when the proper warranted rate was above the natural rate the economic system will display a chronic tendency to depression. It is in this sense that Harrod stated: "...the anti-cyclical policy has to be converted into a permanent policy for keeping down the warranted rate" (ibid., p. 32). The main tools for this task are permanent public works and low long-term rates of interest. This view was in line with Keynes's recommendations to maintain full employment (Harrod 1956, p. 482) as well as that of several other economists at the time.[27]

Harrod always thought that fiscal policy was the adequate tool to avoid runaway movements of G_w from G_n (1969, p. 197; 1973, p. 173). But he grew more cautious about public works and eventually rejected them as a policy tool. In *Towards a Dynamic Economics* still found use for "that old fashioned remedy of public works" (1948, p. 88) with the

[27]From late 1924 on, Keynes consistently recommended the undertaking of public works as a way to overcome recessions. In his earlier articles, "Does Unemployment Need a Drastic Remedy?" and a "Drastic Remedy for Unemployment" (1924) Keynes analyzed unemployment in terms of structural causes, wage rigidity, and demand considerations. While he thought structural programs, reconversion, and reorganization of depressed sectors, and wage flexibility could improve competitiveness, it was de facto an impractical solution. He thus favored increasing aggregate demand through a program of public work expansion. Kahn's multiplier provided, eventually, the basis for its rationale. Increasing demand could transfer workers from less to more productive sectors. Keynes's case for demand and output expansion through fiscal policy, in the guise of public works, was the basis for his recommendations to the Macmillan Committee, for the views presented in "Can Lloyd George Do It?," and for his earlier views on the "Great Slump." In the *Treatise on Money* (1930), Keynes made the case for public works for an open economy under conditions of wage rigidity and a fixed exchange rate regime. As a group, Chicago economists advocated on more than one occasion an increase in aggregate demand to revamp a stagnant economy. In January 1932, twenty-four economists participating at a conference at the University of Chicago urged President Hoover to pursue more aggressively open market operations and to continue the government's public works program. Kahn (1984, pp. 128–129) identifies only one passage in the GT (Chapter 10) where Keynes refers to "loan expenditure 'by

proviso that these focused on productive investments and not be used simply as a means to create employment ("We certainly do not want public works which are thought up as a means to spend money," p. 136; "We do not want to dig holes in the ground," p. 176). Later on, in *Economic Dynamics* (1973, p. 106) he argued: "Public works should be taken out of the list of the available tools…for regulating the business cycle or maintaining optimum growth."[28]

This permanent policy had to be complemented by temporary measures (i.e., variations in the short-term rate of interest) that offset changes in the warranted rate due to the "the varying incidence of inventions and fluctuations in the foreign account." [29]

In this regard, Harrod's Eq. (4.19) raises an important point regarding the sustainability of trade deficits. Rising trade deficits (i.e., increasing capital flows) increase the warranted rate and thus open the possibility that by expanding above the natural rate these lead to a recession. In this sense, avoiding the possibility of a recession implies that capital inflows must bear a relation with productivity. Framing this idea in more modern terms, capital flows that do not correspond to productivity are necessarily of a more speculative nature and thus have the potential to cause instability. As a result, it follows that besides public works and the short- and long-run rate of interest another instrument to avoid a slump could well be capital controls (which in fact Keynes supported in his Clearing Union proposal). Harrod, however, tended to oppose capital controls (see Chapter 7).

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public authorities'": "'wasteful' loan expenditure may […] enrich the community on balance. Pyramid building, earthquakes…may serve to increase wealth." Public works were also advocated by Robertson as exemplified in his article in the Listener ("Is Another Slump Coming?" 28 July 1937) which led Keynes to remark. "I doubt if there is a sentence from which I disagreed" (Keynes, CW, Vol. XIV, p. 250). Pigou also supported public works as attested by the evidence he gave to the Macmillan Committee (Howson and Winch 1977, p. 66). *The Twenty Second Report of Committee on Economic Information* (February 1937) also advocated public works (ibid., pp. 346–350). In 1938, Keynes focused on policy measures (monetary, fiscal, and also related to trade) to restrain the boom phase of the cycle (Moggridge and Howson 1974, pp. 240–242).

[28]One of the drawbacks mentioned by Harrod was the difficulty in implementing public works. This is an aspect also mentioned by Galbraith (1940, pp. 107–111).

[29]In the draft Essay, Harrod explicitly associated the use of the short-term rate of interest in connection to the control of inflation (CIPC, Vol. III, p. 1025), whereas in the Essay it is associated simply with mitigating cycle fluctuations.

4.8 The Fate of the Essay in Dynamic Theory

The Essay in Dynamic Theory did not receive the attention it deserved. It got only one review written by Hawtrey (1939), and its contents were overshadowed by the economic concerns brought about by World War II. Interest in Harrod's fundamental equation and, more generally his dynamics, revived in the late 1940s in connection with Evsey Domar's work on growth theory (Schelling 1947), Samuelson's multiplier–accelerator model (Baumol 1948, 1949) and especially with the publication of Towards a Dynamic Economics in 1948.[30] The literature on the subject has grown considerably overtime specially that devoted to the stability/instability properties of Harrod's model with different variants, modifications, and additions. A review of the Harrodian stability (selected) results is presented in Table 4.1 from different studies spanning the period 1939–1962. Chapter 9 shows a similar table for the period 1964–2018.

Harrod himself was never able to provide a fully articulated model of his dynamics, to adequately clarify the concepts he used or respond satisfactorily to the queries, questions and doubts raised by his equations and his dynamic framework. The core idea of his model, the "cumulative deviations from an unstable moving equilibrium",[31] faded into the background and his dynamics came to be identified with growth theory, more particularly with balanced growth, and his contribution subsumed under the Harrod–Domar growth formula.[32] Harrod is also to blame for this since initially he did not put up a strong opposition to the categorization of his dynamics as a steady-state growth model. Only much later in the mid-1960s, he began to complain about being grossly misrepresented by most commentators. By then he also felt that there had been limited progress in the development of economic dynamics including in the efforts to formalize dynamics.

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[30]Domar (1946, 1957) and Samuelson (1939a, b).

[31]Besomi (2002, p. 42).

[32]See Pugno (1998, pp. 89–90) in Rampa et al. (1998) for a summary of the Harrod–Domar textbook interpretation. See also Besomi (1998a).

Table 4.1 A review of Harrodian stability results 1939–1962 on the basis of selected articles

Year	Title	Author	Journal	Stable/unstable	Mechanism	Notes
1939	Interaction between the multiplier analysis and the principle of acceleration	P. Samuelson	*The Review of Economics and Statistics* 21(2)	Unstable (for some parameter configurations)	Multiplier accelerator	Seminal contribution to multiplier–accelerator models. Two equations: Consumption depended on lagged incomes and investment depends on changes in lagged consumption. If the marginal propensity to consume and the accelerator coefficient are small, the model is globally stable. This would serve as a benchmark to analyze Harrodian instability in the following years
1949	Formalization of Mr. Harrod's model	W. Baumol	*The Economic Journal* 59(236), 625–629	Unstable	Multiplier accelerator	Warranted and actual incomes are defined with different difference equations. After solving the system, it is shown that Harrodian instability exists

(continued)

Table 4.1 (continued)

Year	Title	Author	Journal	Stable/unstable	Mechanism	Notes
1950	A contribution to the theory of the trade cycle	J. Hicks	Book. Clarendon Press; Oxford	Locally unstable; Globally stable	Non-linear multiplier accelerator	Another seminal contribution to multiplier–accelerator model. A non-linear model is broken down in several linear pieces, with an exogenous "floor" and "ceiling" bounding instability
1956	The stability of growth models	D. Bodenhorn	*American Economic Review* 46(4), 607–631	Stable	Expectations	Introducing adaptive expectations on a standard Keynesian investment function and on the consumption function makes the model globally stable. The model is of the multiplier–accelerator type
1960	On stability in the sense of Harrod	D. Jorgensen	*Economica* 27(107), 243–248	Stable	Excess demand function	Formulates a standard multiplier–accelerator model, where the warranted growth rate can differ from the actual growth rate. An excess demand function is added, which achieves global stability of the model

(continued)

Table 4.1 (continued)

Year	Title	Author	Journal		Stable/unstable	Mechanism	Notes
1962	Dynamic equilibrium and instability in the sense of Harrod	J. Green	*Economica* 29(113), 53–57		Unstable (for some parameter configurations)	Change in equilibrium concept	The author proposes that v, the average capital-output ratio should be at an exogenous required rate_. This creates a non-linear differential equation which has two equilibriums, and one of them is unstable
1962	The mathematical formulation of Harrod's growth model	J. W. Nevile	*The Economic Journal* 72(236), 367–370	Keynesian	Unstable	Expectations and investment function	The author takes a standard multiplier–accelerator model and makes two modifications: The Investment function has a standard accelerator term and a term representing capital shortages, D. Expected output is a function of past output. The resulting difference equation is unstable

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