An IS-LM Model for a Closed Economy in a Stock and Flow Consistent Framework[♣]

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Abstract: **T**he objective of the present article is to build a *Stock-and-Flow Consistent* version of the IS-LM model for a closed economy with endogenous money supply in order to analyze the dynamic properties of the model. We show that once the capacity effect of investment is taken into account, the equilibrium of the model is no longer characterized by a stationary level of real output; but by *a constant growth rate of real output*. The level of activity is represented in the model by the variable capacity utilization, which is constant and lower than one over the equilibrium path. This means that the SFC version of the IS-LM model reproduces the traditional Keynesian result of *underemployment equilibrium*. Moreover, in steady-state disposable income, capital stock and private wealth all grow at a same constant rate, meaning that a *balanced growth path* exists for the model at hand. It is also shown that along balanced growth path, the economy is *dynamically efficient* and firms had a *hedge financial posture*, ruling out financial instability in the sense of Minsky. Regarding the comparative dynamics of the model, we had performed some numerical simulations about the dynamic effects of shocks over the time path of endogenous variables. We showed that some traditional results of Keynesian Theory as the "paradox of thrift" and a wage-led regime of accumulation still hold in a SFC framework.

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1. Introduction

Since the seminal paper "Mr. Keynes and the Classics: a suggested interpretation" written by John Hicks in 1937, IS-LM model has become the main theoretical framework for the exposition and spreading of Keynesian ideas all over the world. Despite the strong criticism of some Post-Keynesian economists as Luigi Pasinetti, who claimed that IS-LM model deformed the central message of Keynes´s General Theory, neutralizing its revolutionary spirit (Pasinetti and Mariutti, 2008, p.59); the IS-LM model has been perceived ever since as a true piece of Keynesian economics.

One of the major weaknesses of the IS-LM apparatus is the clear inconsistency between flows and stocks. Indeed, not only the capacity effect of investment is not taken into account, but also the effect of private and government savings over the stock of public bonds is completely neglected. In words of Godley and Lavoie (2007) the IS-LM model has a huge "black hole" inside it, since nothing is said about what families do with their savings or how government finance the excess of public expenditures over taxes. Some important flows of income (as savings) or expenditure (as investment) have no consequence over the magnitude of the existing stocks. They simply disappear from the model, as if they had fallen in a "black hole".

As stressed by Tobin (1982), the first inconsistency in IS/LM model is the solution, since it "cannot generally be a stationary equilibrium. The values that the solution gives to the flow of variables in the model usually imply that stocks are increasing or decreasing. Thus net investment may be positive, so that the capital stock is increasing. Savings may be positive, so that household net worth is increasing. Yes, the government deficit may be positive, so that public debt, in some form monetary or non-monetary, is increasing. These stocks changes matter because the stocks are, or should be, arguments in the functions determining the flows: for example, capital in the investment and production functions, wealth in the saving function. As a result of these internal dynamics, the IS/LM is generally changing as time passes, even though no exogenous shocks are occurring. The only stationary solutions, if any exist, are those which imply stationary stocks – or balanced growth equivalent, stocks all growing at a common proportional rate" (Ibid, pp. 74-75).

Even if we consider the IS/LM model a slice in a continuous-time dynamic model and then the model has a new solution to each microsecond, we still need to ask how momentary solution be different if, for a given values of stocks and other state variables, government expenditure or taxes or transfer payments are different. The stocks could no change by finite amounts during an infinitesimally small interval of time. As for monetary policy, it is possible to ask how the momentary solution will be different if the central bank, by finite open market transactions with the public which take zero time, alters the historically determined supplies of its monetary and non-monetary liabilities. The central bank is not free to engineer instantaneously, via open market operations, any finite change it desires in the state variables representing outstanding stocks of its liabilities. Thus, as professor Tobin said, some writers prefer to imagine variation of monetary stock by sudden helicopter drop a newly minted currency, rather than by central bank transactions in the market.

The theoretical problem posed by this inconsistency is that the results of comparative statics of the IS-LM model – for instance, the partial derivatives of equilibrium level of output and interest rates relative to changes in government expenditures and money supply – could be completely different if the relation between flows and stocks are taken into account. This is a very weak basis for Keynesian ideas.

The objective of this article is precisely to correct this deficiency of the IS-LM apparatus. We will build a *Stock-and-Flow Consistent* version of the IS-LM model for a closed economy with endogenous money supply in order to analyze the dynamic properties of the model. Although the traditional specification of IS-LM model considers a constant stock of money supply, recent developments in both theory and practice of monetary policy emphasizes the role of short-term nominal interest rate as the basic policy instrument at the hands of monetary authority. This means that nominal interest rate, instead of money supply, must be taken as a policy parameter in the structure of the IS-LM model, as it is done in Romer (2000).

Redefining the standard macroeconomic models in terms of a SFC framework is a not novel task. Indeed, Godley and Shaik (2012) had already shown that the *Classical Macroeconomic Model* had an important inconsistency at the heart of its logical structure. As is well-known the classical model "exhibits a block recursive structure beginning from the equilibrium in the labor market and moving to real output demand and its components, including the real demand for money, and ending finally in nominal wages and prices" (Ibid, p.66). This structure assure that "doubling the money supply must double prices so as to keep the real money supply equal do an unchanged real money demand" (Ibid, p.66), so that classical dichotomy holds. The problem is that this result follows from the assumption that all real net income of business sector is somehow distributed to households. Since the only instrument available for firms to distribute profits to households is interest payments on the bonds they have issued, then household income has to be redefined as wage and interest income, instead of wage and profits income. Once household income is correctly specified, the classical dichotomy disappears, because a "change in the price level (due, say, to a change in the money supply) changes the real value of the bonds outstanding, and hence changes the level of real interest flows. Since real interest flows enter into the household income, this affects real consumption demand, real investment demand (…) and the interest rate" (Ibid, p.67). This means that "real variables such as consumption, investment, the interest rate and the real money demand became intrinsically linked to nominal variables such as price level and the money supply" (Ibid, p.77).

The novelty to be presented in this paper is to combine the theoretical structure of the standard IS-LM model – that is the equilibrium conditions for goods, money and bond markets – with an exogenous interest rate-endogenous money supply approach as the one developed by Romer (2000) in a SFC framework. This differs from Godley and Shaikh (2002) in two important aspects. First of all, the level of output is demand-determined in the model to be presented in the following sections. Second, the money supply is now an endogenous variable, not being subject to the control of monetary authorities.

Regarding the specification of behavior equations of the model, our approach rests on the work of Lavoie and Godley (2012) for investment demand. More precisely, investment demand is defined in terms of a desired rate of capital accumulation, as in Kaleckian models of growth such as the ones as developed by Rowthorn (1981), Dutt (1990) and Lavoie (1995). This specification for investment function allows that, once the capacity effect of investment is taken into account, the equilibrium of the model is no longer characterized by a stationary level of real output; but by *a constant growth rate of real output*. The level of activity is represented in the model by the variable capacity utilization, which is constant and lower than one over the equilibrium path. This means that the SFC version of the IS-LM model reproduces the traditional Keynesian result of *underemployment equilibrium*. Moreover, in steady-state disposable income, capital stock and private wealth all grow at a same constant rate, so a *balanced*

growth path exists. We also show that distribution of wealth between money and bonds is also constant in steady-state growth. Finally, it is also shown that along *balanced growth path*, the economy is *dynamically efficient* and firms had a *hedge financial posture*. This means that financial fragility in the sense of Minsky (1986) is ruled out of the model.

Turning to the comparative dynamics of the model, we had performed some numerical simulations about the dynamic effects of shocks over the time path of endogenous variables. We have tested changes in the autonomous rate of capital accumulation, the propensity to consume out of disposable income, the coefficient of profit distribution, the tax rate, the nominal interest rate and the wage share. One important but expected result is that the qualitative effects of changes in the parameters of investment and consumption functions are very similar. Indeed, an increase in the propensity to consume out of disposable income generated an increase in the level of capacity utilization and an increase in the growth rate of capital stock and private wealth; an increase in the autonomous rate of capital accumulation generated just the same qualitative effects.

Another interesting result is about the old "paradox of thrift". An increase in the marginal propensity to consume (a reduction in the marginal propensity to save) resulted in an increase in the level of capacity utilization, and also an increase in the growth rate of capital stock and private wealth[1].

Regarding the effects of changes in fiscal policy over the dynamic behavior of the economy, the model showed that an increase in the tax rate – in a possible attempt of the government to reduce fiscal deficit and the ratio of public debt to GDP – has also no effect over the time path of the public debt to GDP ratio. This can be due to the fact that being growth rate of disposable income higher than interest rate along the balanced growth path then even if government runs a primary deficit than the ratio of public debt to GDP will be decreasing over time, making "fiscal adjustment" unnecessary.

Finally, the model showed a clear *wage-led* regime of accumulation[2], since an increase in the wage share resulted in an increase in the growth rate of both capital stock and private wealth and also an increase in the level of capacity utilization.

The article is organized in six sections, including the present introduction. In section two we presented the accounting structure and the theoretical assumptions of the SFC version of IS-LM model. In section three we will present the behavior equations of the model, that is, its formal theoretical structure. Section four is dedicated to the calibration of the model and the performing of the basic numerical simulation. In section five we perform the comparative dynamic exercises, evaluating the effects of exogenous shocks in some behavior and policy parameters over the dynamic path of the endogenous variables. In section six, we do some final remarks.

2. Accounting Structure and Theoretical Assumptions

We will consider a closed economy (there is no import and export of goods and services and no capital flows) with four sectors: Households, Firms, Government and Central Bank. The balance sheet of these sectors is summarized in table 1 below.

[1] This result replicates the "paradox of thrift" in the growth and distribution model of Joan Robinson. See Harcourt (2006, p. 29).

[2] This is not a surprising result since we are supposing the existence of a strong accelerator effect in the investment function. If investment spending was sensitive to changes in the profit share, as in Bhaduri and Marglin (1990), then this result could be reversed to a *profit-led* regime.

Table 1: Balance Sheet of Model IS-LM

	Households	Firms	Government	Central Bank	∑
Fixed Capital		$+K_f$			$+K_f$
Money	$+H_h$			$-H$	
Bills / Corporate Notes	$+B_h$	$-B_f$	$-B_g$	$+B_{cb}$	
Balance (net worth)	$-V_h$	$-V_f$	$+V_g$		$-K_f$
∑	0	0	0	0	0

Note: Positive variables are assets, while negative ones are liabilities.

Looking at balance sheet, you should notice that the only asset owned by Firms is the fixed capital (tangible goods). Thus all their funds are used to finance the purchase of new fixed capital equipment. We don't consider commercial banks in the composition of monetary system. However, we consider the issuance of corporate notes. We will suppose that corporate notes are *perfect substitutes* of government bills. The basic idea was to follow the original structure of IS-LM model and deal with just one interest rate for different assets. All funds used to finance firms come from retained profits plus the new corporate bonds issued. Households accumulate financial wealth, which can be allocated in the form of money or buying bills issued by the government or corporate notes issued by firms. The Central Bank is considered as an institution in its own right. The central bank purchases bills from government and also corporate notes from firms, thereby adding to its stock of assets. On its liability side, the central bank provides money to households. This money can take the form of either cash or deposits at the central bank. It is assumed that central bank has zero net worth. The value of bills insured by government is the public debt. As usual all rows and columns must sum zero. The exception is the fixed capital row.

Table 2 shows the transactions-flow matrix of our model. Once again, all columns and all rows must sum zero to ensure that all transactions are taken into account. Thus we avoid *black holes* in the system. The government pays interest arising from government debt both to households and central bank. Interest payments each period are generated by stocks of assets in existence at the end of previous period. Because of this time lag, the rate of interest on bills relevant in period t is the rate of interest that was set at the end of previous period, at time t_{-1}.

Table 2: Transactions-flow matrix of Model IS-LM

		Firms			Central Bank		
	Households	Current	Capital	Government	Current	Capital	∑
Consumption	$-C$	$+C$					0
Government Expenditures		$+G$		$-G$			0
Investment		$+I$	$-I$				0
Wages	$+W$	$-W$					0
Taxes	$-T$			$+T$			0
Interest Payments	$+r_{t-1} * B_{h\,t-1}$	$-r_{t-1} * B_{f\,t-1}$		$-r_{t-1} * B_{g\,t-1}$	$+r_{t-1} * B_{bc\,t-1}$		0
Central Bank Profits				$+r_{t-1} * B_{bc\,t-1}$	$-r_{t-1} * B_{bc\,t-1}$		0
Firms Profits	$+P_h$	$-P$	$+P_f$				0
Change in Money	$-\Delta H$					$+\Delta H$	0
Change in Bills / Corp. Notes	$-\Delta B_h$		$+\Delta B_f$	$+\Delta B_g$		$-\Delta B_{bc}$	0
∑	0	0	0	0	0	0	0

Note: Positive figures denote sources of funds, while negative ones denote uses of fund.

The government savings is the difference between government revenues and expenditures. The increasing of public debt is financed by issuing of new bills. In the opposite way, the fiscal surplus should be used to decrease public debt. By the way, the column sum of government must be zero.

We have set the central bank's net worth in zero, which implies that any profit it makes is always distributed to government. Here, the central bank certainly does make profits since it owns bills which yield interest payments, whereas its liabilities (money) pay no interest.

The household income is given by wages payments by firms, interest payments from government bills and corporate notes and profits distributed by firms. With all these revenues, households pay taxes, purchases goods and services from firms and buys government bills and corporate notes.

Goods sales are the only source of revenues to firms. Households buy a quantity C and government buy a quantity G of goods and services. The current account represents the income flows within the sector, while the capital account represents sources to finance firms. For the model be consistent, the column sum must be zero. The entire resource flows entering should be spent. The firms spend their resources paying wages to households. The difference between all flows constitutes the profit. It is assumed that a share of profits is retained to finance new investments, while the other part is distributed to the households.

3. Model Behavioral Structure

Social account is no able for forecast, by itself, the path taken by the economy. For this purpose, this section will present the behavior relations that determine the time path of endogenous variables. The behavior of firms, households, government and central bank will be modelled. After that we will calculate the short-term equilibrium value of output and remaining endogenous variables.

3.1. Firms

Equation (1) defines the level of investment in the current period. The firm must choose the desired level of investment, defining a rate of growth for the capital stock. The desired rate of capital accumulation rate is given by (2), where γ_0 is the parameter that represents the *animal spirits* of entrepreneurs, γ_1 represent the sensitivity of capital accumulation to the level of capacity utilization and the last term γ_2 is the sensitivity of capital accumulation in relation to changes in interest rate. This is a typical Kaleckian specification for investment function, and it is based on Lavoie and Godley (2012). The actual and full capacity utilization level of output is given by equations (3) and (4). Equation (5) shows that retained profits are equal to *(1-d)* times total profits, where *d* is the coefficient of profit distribution. If retained profits are not enough to finance the desired level of investment, than firms will issue corporate notes in order to get the necessary funds in capital markets. Furthermore, we will suppose that there is no limit for the level of indebtedness of firms, which implies that firms are always capable to secure the required amount of funds to finance the desired level of investment. This means that there is no financial constraint to investment.

$$I_d = \Delta K = GR_k * K_{-1} \quad (1)$$

$$GR_k = \gamma_0 + \gamma_1 * u_{-1} - \gamma_2 r \quad (2)$$

$$u = \frac{Y_s}{Y_{fc}} \quad (3)$$

$$Y_{fc} = \frac{K}{\sigma} \quad (4)$$

$$P_f = (1-d).P = (1-d).(Y - W - r_{-1}.B_{f_{t-1}}) \quad (5)$$

$$\Delta B_f = B_f - B_{f_{-1}} = I_d - P_f \quad (6)$$

3.2.Households

The households receive income form three sources. First, as stated on equations (5) and (7), all residual profits (difference between retained profit and full profit) are distributed to households. Second, as a payment for labor services, they receive wages, as can be noticed in equation (10), (10a) and (11). The last source is the interest received for holding government bills and corporate notes. This still could be noticed looking at equation (10) and (10a). Households pay taxes T_s over total income. For the sake of simplicity, we will assume that government sets a tax rate of θ over gross income, independent of the source. This means that disposable income YD is equal to $(1-\theta)Y$. Consumption expenditures depend both on the stock of wealth and disposable income as in Godley and Lavoie (2007, p. 107). Finally, households wealth's is defined in period t by the stock accumulated in $t-1$ plus the savings (difference between disposable income and consumption) in t.

$$P_h = d.P \quad (7)$$

$$V = V_{-1} + (YD - C) \quad (8)$$

$$C_d = \alpha_1 * YD + \alpha_2 * V_{-1}, \quad 0 < \alpha_2 < \alpha_1 < 1 \quad (9)$$

$$YD = W + r_{-1} * B_{h-1} + P_h - T_s \quad (10)$$

$$YD = (1 - \theta) * (W + r_{-1} * B_{h-1} + P_h) \quad (10a)$$

$$W = w * Y \quad (11)$$

3.3.Government and Central Bank

Now we turn attention to the behavior of government and central bank. Regarding government expenditures, equation (12) states that government set the level of public expenditures in order to obtain a desired ratio to the level of capital stock. This means that government expenditures are driven by capital accumulation. Equation (13) shows that all taxes revenues come from disposable income from households. The implicit idea is that only households are taxed, in other words, retained profits are tax free. Equation (14) states that government issues new bills in order to finance any budget deficit. Households will demand a share of those bills. The difference between the supply of new bills and demand by households will be bought by central bank, that acts as a residual buyer as show equation (15). Note in equation (16) that central bank also buys corporate notes. Finally, in equation (17) we notice that interest rate is an exogenous variable, determined by central bank.

$$G_t = \gamma * K_{-1} \quad (12)$$

$$T_d = \theta * (W + r_{-1} * B_{h-1} + P_h), \quad \theta < 1 \quad (13)$$

$$\Delta B_g = B_g - B_{g-1} = (G + r_{-1} * B_{s-1}) - (T + r_{-1} * B_{bc-1}) \quad (14)$$

$$\Delta H_s = H_s - H_{s-1} = \Delta B_{bc} \quad (15)$$

$$B_{bc} = B_g + B_f - B_h \quad (16)$$

$$r = \bar{r} \quad (17)$$

3.4.GDP in Short Run

A simple way to solve the model is established by the calculation of short-term GDP. We can see in equation (18) short-term output is determined by the demand side. The superior bound for the short-term GDP is given by the full capacity output

determined in equation (4). If the real effective demand is greater than full employment capacity than output is supply constrained. Once short-term GDP is determined, we can calculate all other model variables and defined their desired paths.

Equations (18), (19) and (20) shows can be used to get the equation (20).

$$Y_{sr} = \begin{cases} C + I + G & if \quad C + I + G < Y_{fc} \\ Y_{fc} & if \quad C + I + G \geq Y_{fc} \end{cases} \tag{18}$$

Where we have:

$$C_t = \alpha_1 * (1 - \theta) * [w + (1 - d) * (1 - w)] * Y_t + \propto_1 * (1 - \theta) * r * B_{h_{t-1}} + \alpha_2 * V_{t-1} \tag{19}$$

$$I_{t=}(\gamma_0 + \gamma_1 * u_{t-1} - \gamma_2 * r) * K_{t-1} \tag{20}$$

$$G_t = \gamma * K_{t-1} \tag{21}$$

After some algebraic manipulation, we found:

$$Y_{sr} = \frac{\propto_1 * (1-\theta) * r * B_{h_{t-1}} + \alpha_2 * V_{t-1} + (\gamma_0 + \gamma_1 * u_{t-1} - \gamma_2 * r) * K_{t-1} + \gamma * K_{t-1}}{1 - \alpha_1 * (1-\theta) * [w + (1-d) * (1-w)]} \tag{22}$$

3.5.Portfolio Decisions

The next equations (ADUP 1,2,3 and 4) defines portfolio restrictions. The equations of the portfolio decisions are based on the seminal approach proposed by Tobin (1969). Thus, we present the equation (23), in matrix form, that households keeps a share of their expected wealth in the form of asset i, and these proportions varies according to changes on interest rates and disposable income. Finally, as we can see in equations (24) and (25) expectations are adaptive.

$$\lambda_{10} + \lambda_{20} = 1 \tag{ADUP.1}$$

$$\lambda_{11} + \lambda_{21} = 0 \tag{ADUP.2}$$

$$\lambda_{12} + \lambda_{22} = 0 \tag{ADUP.3}$$

$$\lambda_{14} + \lambda_{24} = 0 \tag{ADUP.4}$$

$$[H_h \quad B_h] = [\lambda_{10}\ \lambda_{20}] * V^e + [\lambda_{11}\ \lambda_{12}\ \lambda_{21}\ \lambda_{22}] * [0\ r] * V^e + [\lambda_{14}\ \lambda_{24}] * YD_r^e \tag{23}$$

$$V^e = V_{-1} \tag{24}$$

$$YD^e = YD_{-1} \tag{25}$$

4. Calibration and Basic Simulation

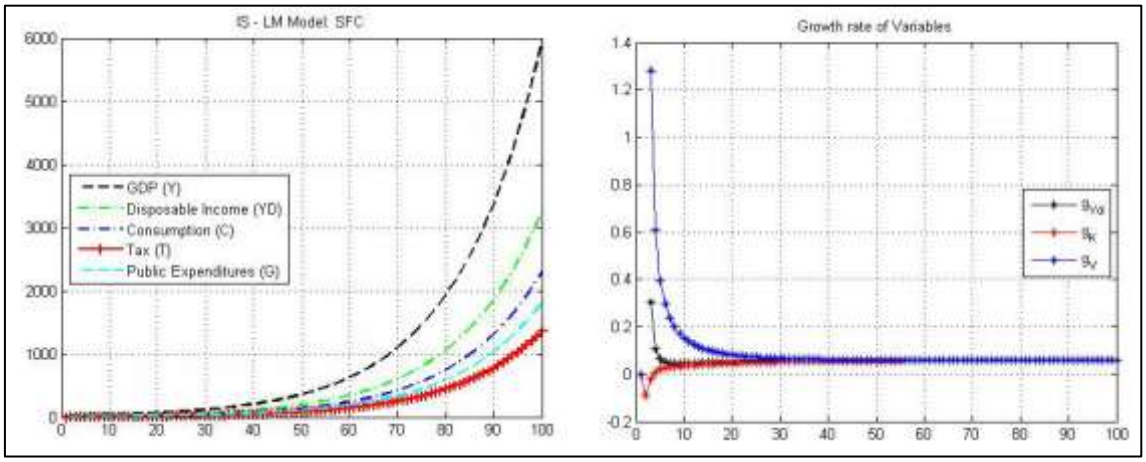
The model was simulated in *MATLAB 2013 software environment*. We calibrate our model in order to make it as close as possible to what we find in the literature. We have on the table 3 below the values used and the parameter description.

Table 3: Calibration and initial conditions of Model IS-LM

α_1	0.6
α_2	0.02
γ_0	0.02
γ_1	0.2
γ_2	0.2
σ	1.5
θ	0.3
r	0.03
φ	0.1
γ	0.15
w	0.6
λ_{10}	0.5
λ_{11}	-0.4
λ_{12}	0.3
λ_{20}	0.5
λ_{21}	0.4
λ_{22}	-0.4
$K_{Initial}$	100

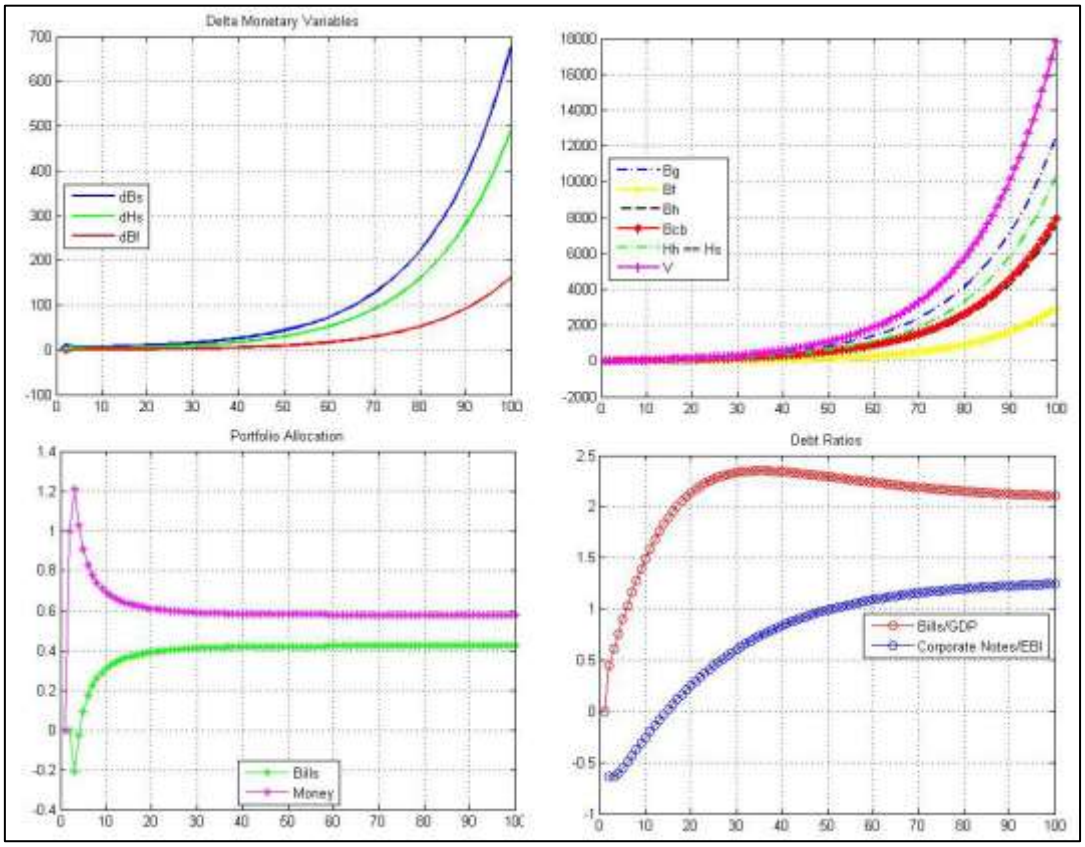
In the figure 1, we have two quadrants. The first quadrant (west) shows the path of the main aggregates of the real economy. In the second quadrant (east) we have the time path of growth rates of disposable income, capital and wealth. Since all growth rates converge to the same positive constant, than we can conclude that the model has a *balanced growth path*.

Figure 1: Main results of IS-LM simulated model.



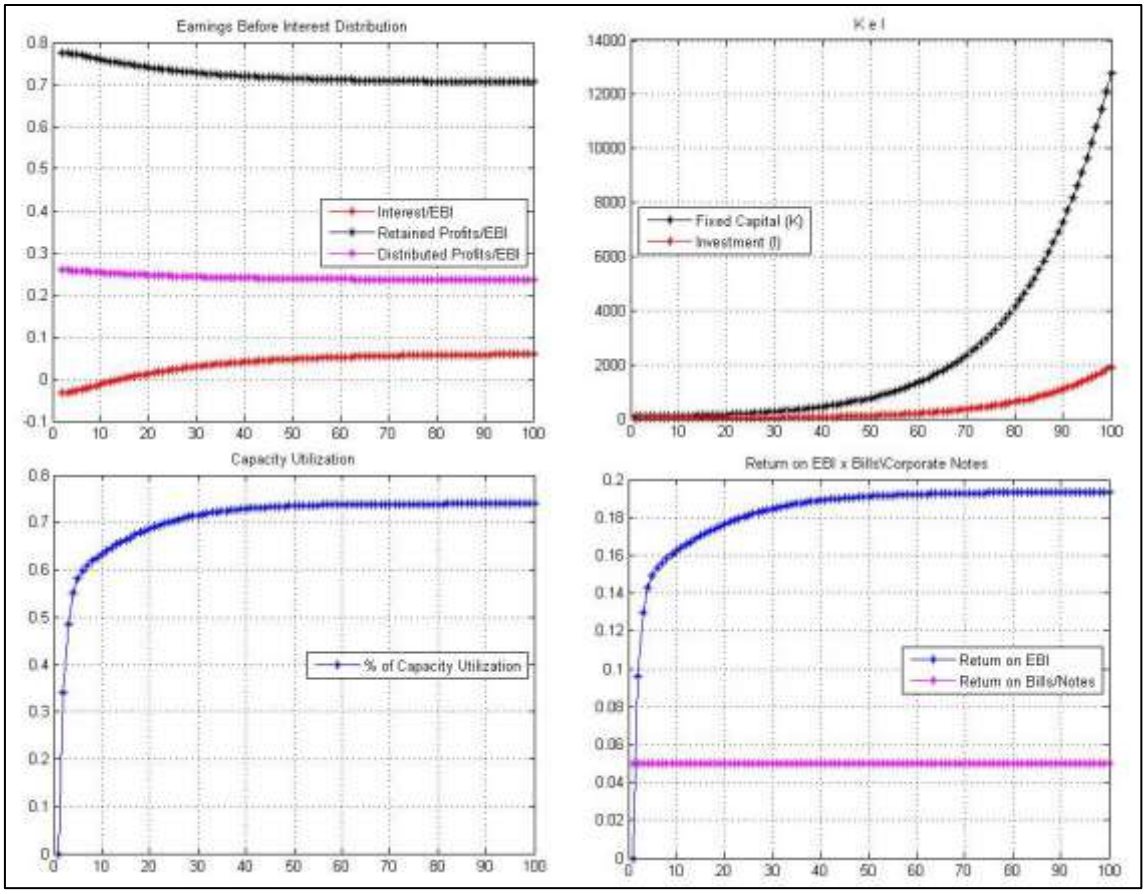
In the figure 2, we have four quadrants. The first quadrant (top/west) shows the paths of monetary variables. In the second quadrant (top/east) we have the behavior of the stocks of financial wealth. The third quadrant (bottom/west) shows the behavior of portfolio composition. As can be seen, after a certain time portfolio composition is kept constant. The fourth quadrant (bottom/east) shows the path of the ratios: Bills/GDP and Corporate Notes/EBI (Earnings before interest). An important point is that the public debt / GDP and private debt/EBI converges to a positive constant rather than keep growing indefinitely. This is an important point to be highlighted because it shows that the model doesn't have an explosive behavior in the public and private debt levels.

Figure 2: More results of IS-LM simulated model.



In figure 3, we have in the first quadrant (top/west) the path of the following ratios: Interest/EBI, Retained Profits/EBI and Distributed Profits/EBI. The basic idea in this chart is show that the interest payment doesn't crush the profit nor investment activity. On the second quadrant (top/east) we have the path of fixed capital and investment confirming what was shown in the previous graph. On the third (bottom/west), we have the path of the capacity utilization. As can be seen, capacity utilization converges to a lower than one value, meaning that the model reproduces the traditional Keynesian result of *underemployment equilibrium*. The last quadrant (bottom/east) shows the path of interest rate and of profit rate, that is, the return of the financial investment (bills and corporate notes) and the return on investment in the real economy (fixed capital invested by firms). As we can see, profit rate is consistently higher than interest rate.

Figure 3: Firms and Government results of IS-LM simulated model.



The data obtained in the steady state was summarized and can be seen in Table 4. We have that the GDP growth rate, the fixed capital growth rate and wealth growth rate converge to the same value (5.79% p.p). The return on productive capital remains above the return on financial capital and capacity utilization remains below the rate of full capacity utilization (73.95% of full capacity). The wealth/GDP ratio converges to 2.98, Bills/GDP ratio to 2.065 and Corporate Notes/EBI to 1.281.

Table 4: Main values in Steady-State

Simulation Results on Steady-State	
g_GDP	5.79%
g_K	5.79%
g_V	5.79%
r_K	19.36%
r	5.00%
u	73.95%
V/GDP	2.98
Bills/GDP	2.065
Corporate Notes/EBI	1.281

In Table 4, we can see that along the balanced growth path we have: $r_k > g_k > r$. This means that this economy is *dynamically efficient* (Blanchard and Fischer, 1989, pp.103-4) and firms had a *hedge financial posture* (Foley, 2003, pp.160-01). This last property of the balanced growth means that there is no financial fragility in the long-run equilibrium.

5. Comparative Dynamics

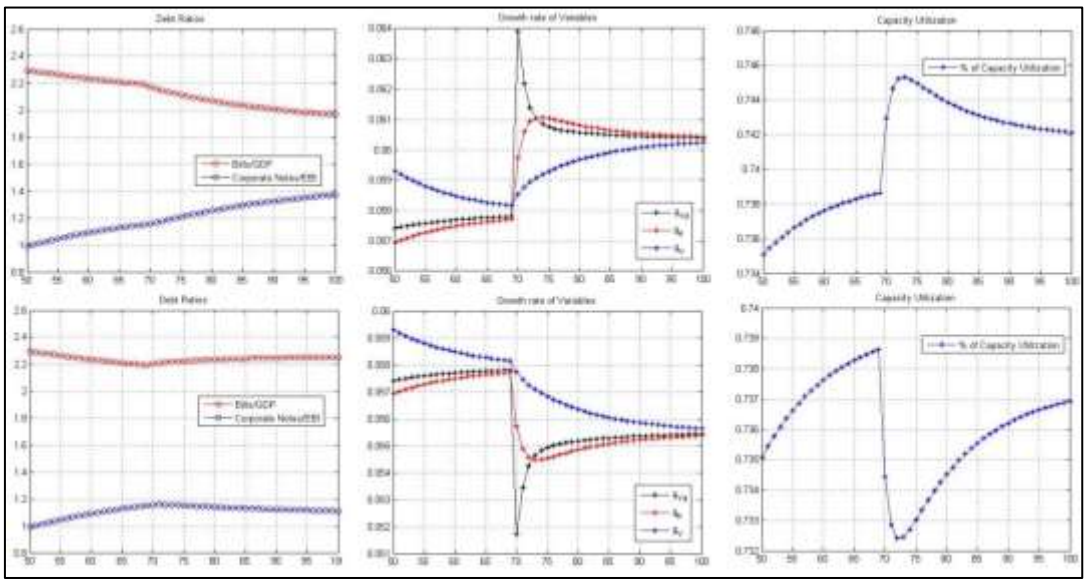
Here, we present the main effects over macroeconomic variables after some shocks. In table 5, we present these results. The shocks were given as follows: it was chosen deliver a shock at the time the model had reached its steady state, in other words, the time which the rates have converged to grow at the same value. Thus, we wait until the period 70 to add a shock. Thus, Table 5 shows the observed values of the key variables in the period 100 and the charts in period 1 to 100.

Table 5: Results of the main variables, after shocks.

Shock	g_Y_d	g_V	g_K	Bills/GDP	Notes/EBI	Capacity Utilization	Ret_EBI
Alpha1 +10%	6,34%	6,34%	6,34%	1,58	1,04	76,71%	19,88%
Alpha1 -10%	5,33%	5,33%	5,33%	2,60	1,38	71,67%	18,95%
d +10%	5,34%	5,34%	5,34%	3,32	-0,66	71,69%	17,78%
d -10%	6,27%	6,27%	6,27%	1,14	2,48	76,34%	20,76%
Ghama_0 +10%	6,04%	6,04%	6,04%	1,89	1,36	74,18%	19,48%
Ghama_0 -10%	5,55%	5,55%	5,55%	2,27	1,03	73,74%	19,24%
r +10%	5,78%	5,78%	5,78%	2,12	1,12	74,38%	19,50%
r -10%	5,81%	5,81%	5,81%	2,02	1,28	73,54%	19,23%
Theta +10%	5,31%	5,31%	5,31%	1,95	1,39	71,54%	18,93%
Theta -10%	6,30%	6,30%	6,30%	2,14	1,05	76,50%	19,84%
w +10%	6,52%	6,52%	6,52%	0,75	3,40	77,59%	18,67%
w - 10%	5,12%	5,12%	5,12%	4,10	-1,67	70,64%	20,21%

In Figure 4 we present the results of shocks in γ_0, the autonomous component of investment demand, which represents the *animal spirits of entrepreneurs*. In the upper quadrant we have the result of a 10% increase in the parameter that presents the animal spirits of entrepreneurs. As discussed earlier, the shock was given in the period 70. We had an increase in the growth rate of all the variables in its steady state. We had an increase in capacity utilization and the values related to the ratio public debt / GDP and Notes/EBI remain almost the same. Already in the lower quadrant, we have the result of a decrease of 10% over the same parameter. We can see that there is a fall in the growth rate of all the variables in its steady state. We had a decrease in capacity utilization and the ratio public debt / GDP and Notes/EBI remain almost the same.

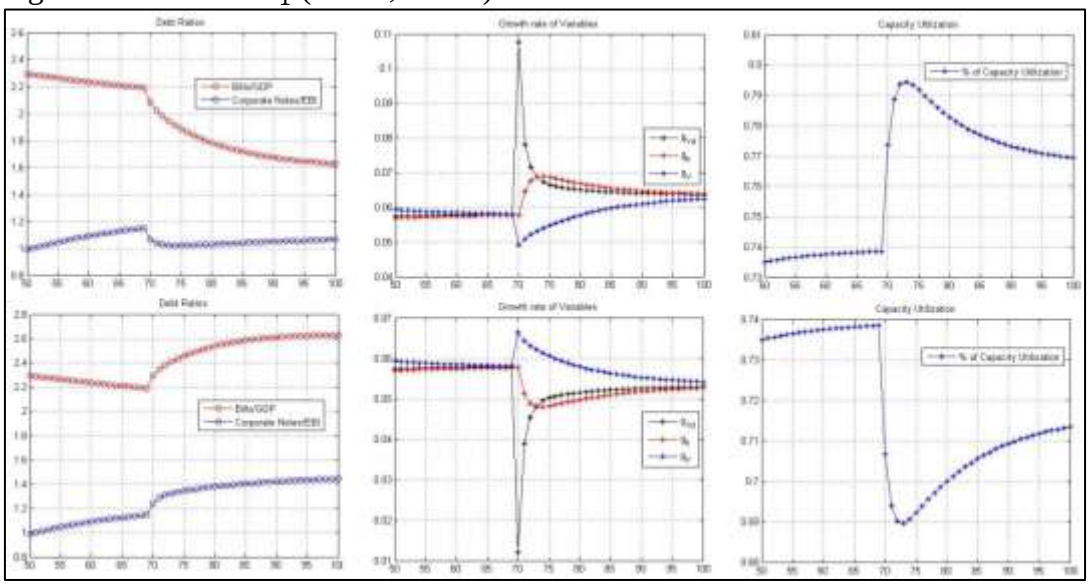
Figure 4: Shocks in γ_0 (+10%; - 10%).



Note: Positive shocks on top line and negative ones in bottom line.

In figure 5 we show the results of a positive and negative shocks in α_1, *the marginal propensity to consume out of disposable income*. Upper quadrant there was a +10% shock in the parameter and the lower quadrant -10%. We can see that the positive shock led to an increase in growth rates of the variables, an increase in the capacity utilization and a little drop in the public debt / GDP and a little increase in notes/EBI. The negative shock, led to a fall in growth rates, a drop in capacity utilization and a little increase in the public debt / GDP and in Notes/EBI. This means that the old result of "paradox of thrift" still holds in the model, since an increase/decrease in the marginal propensity to consume produces an increase/decrease in the growth rate of capital stock, wealth and disposable income, as also an increase/decrease in the level of capacity utilization.

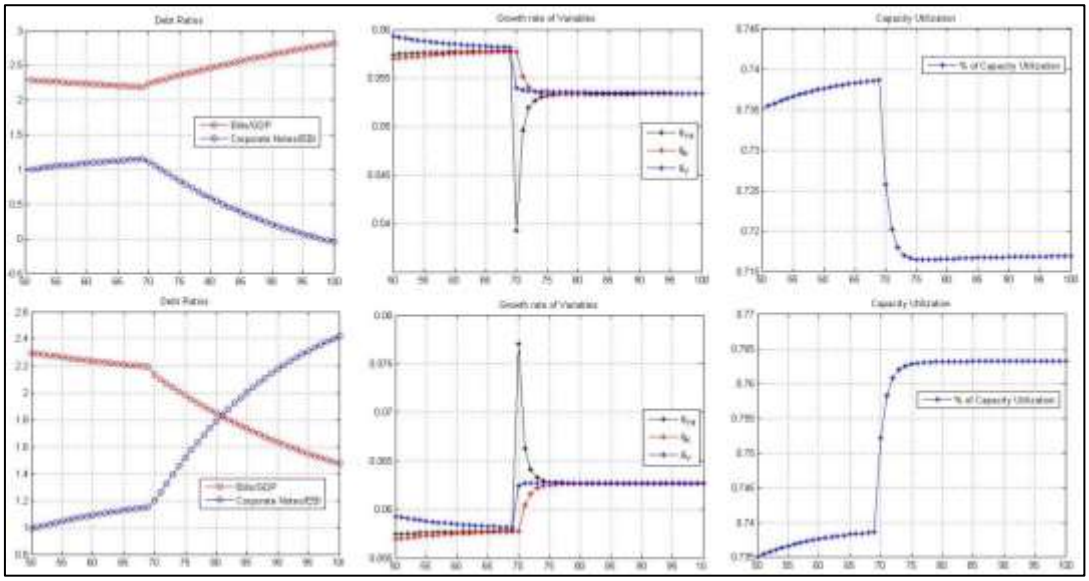
Figure 5: Shocks in α_1 (+10%; - 10%).



Note: Positive shocks on top line and negative ones in bottom line.

In Figure 6 we show the effects of a shock in *d*, the *coefficient of profit distribution*. The upper quadrant shows the results of a shock of +10% in *d* and in the lower quadrant a shock of -10% in *d*. The positive shock led to a fall in growth rates, a drop in capacity utilization and a rise in the public debt / GDP and a fall in Notes/EBI. The negative shock resulted in a rise in growth rates, an increase in capacity utilization, a fall in the ratio public debt / GDP and an increase in Notes/EBI. An interesting result can be observed in the dynamics of corporate debt. When we increase *d* by 10%, we are assuming that more profits are distributed to families and less is used to finance business investment. However, as the investment demand doesn't fall, finance is supplied by the issuance of new corporate notes which then increase the firm's debt level. Otherwise, given a shock of -10% in *d*, we have fewer notes to be issued or at worst, may even be repurchased reducing the level of indebtedness of firms.

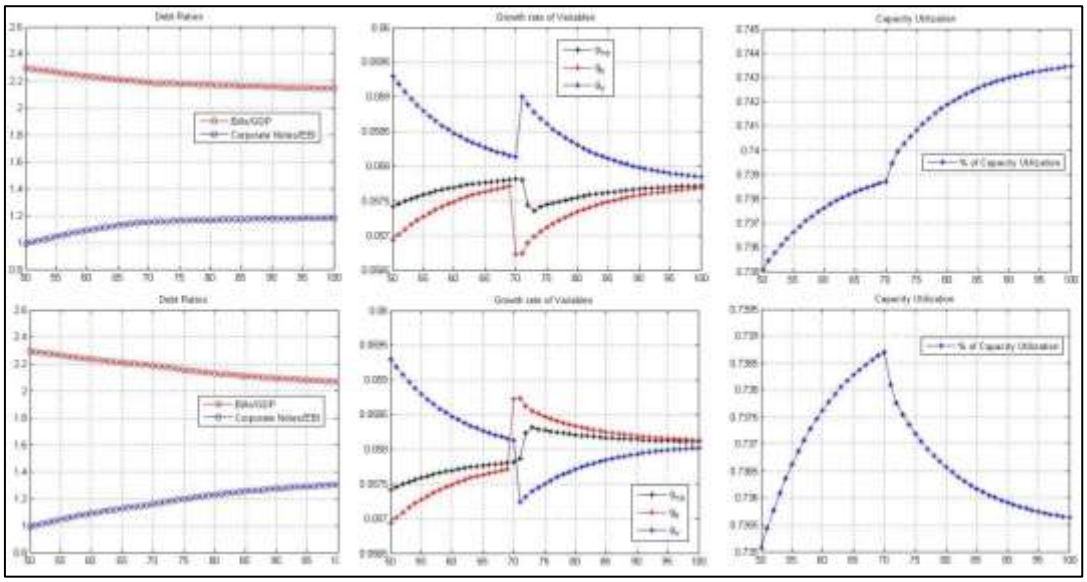
Figure 6: Shocks in d (+10%; - 10%).



Note: Positive shocks on top line and negative ones in bottom line.

In Figure 7 we show the effects of a shock in *r, the level of interest rate*. The upper quadrant shows the results of a shock of + 10% in *r* and in the lower quadrant a shock of -10% in *r*. The positive shock led to a little fall in growth rates (to a value that is almost the same before the shock), a rise in capacity utilization and no change in the value of the ratio public debt / GDP and Notes/EBI. The negative shock resulted in a little increase in growth rates (to a value that is almost the same before the shock), an increase in capacity utilization and no changes in the ratio public debt / GDP and Notes/EBI.

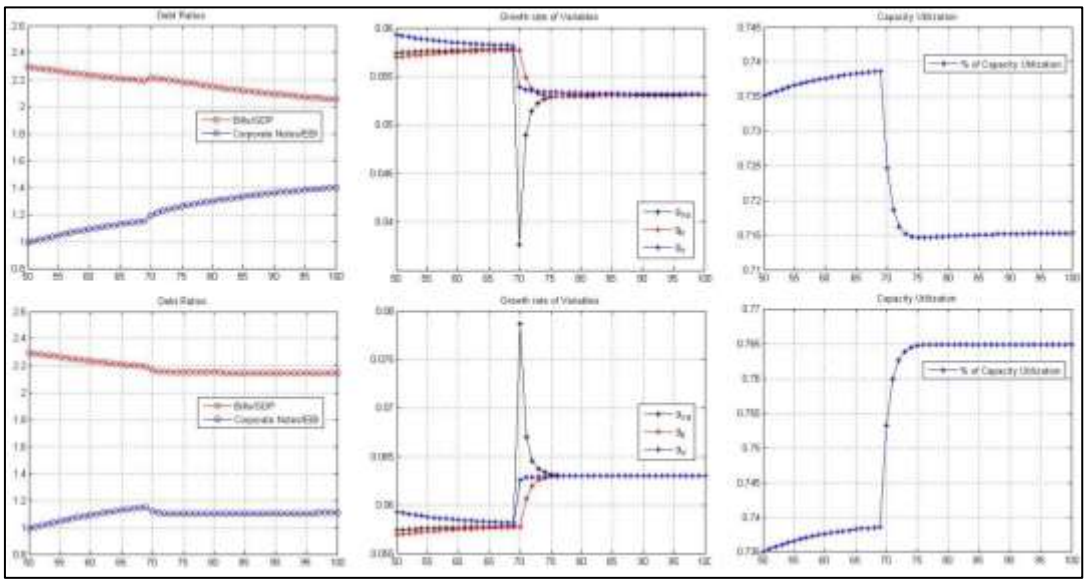
Figure 7: Shocks in r (+10%; - 10%).



Note: Positive shocks on top line and negative ones in bottom line.

In Figure 8 we show the effects of a shock in θ, *the tax rate*. The upper quadrant shows the results of a shock of + 10% in θ and in the lower quadrant a shock of -10% in θ. The positive shock led to a little fall in growth rates, a fall in capacity utilization and no changes in the ratio public debt / GDP and Notes/EBI. The negative shock resulted in an increase in growth rates (to a value that is almost the same before the shock), a fall in capacity utilization and no changes in the ratio public debt / GDP.

Figure 8: Shocks in θ (+10%; - 10%).

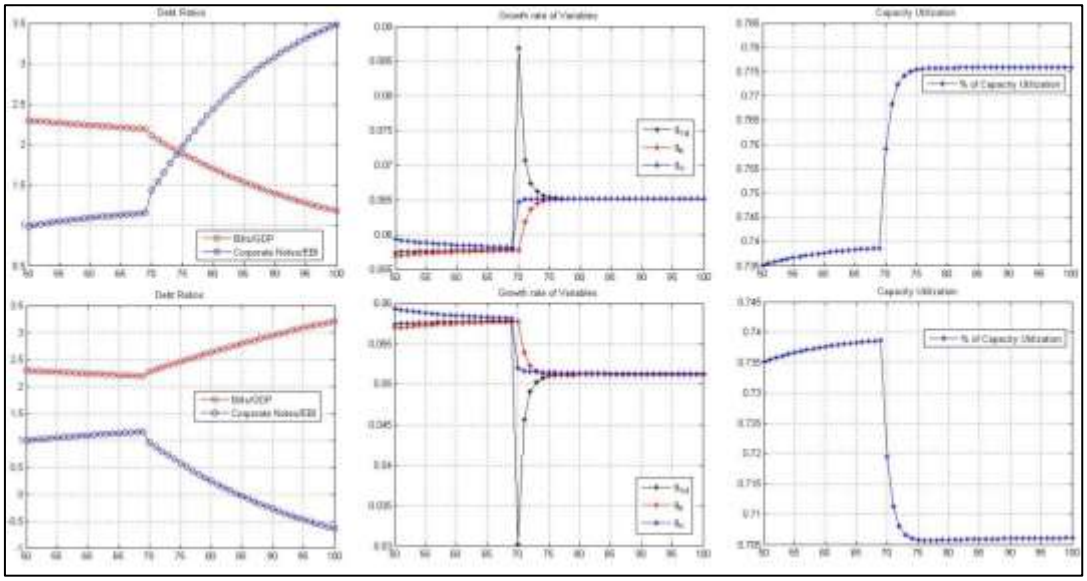


Note: Positive shocks on top line and negative ones in bottom line.

In Figure 9 we show the effects of a shock in *w*, *the wage-share*. The upper quadrant shows the results of a shock of +10% in *w* and in the lower quadrant a shock of -10% in *w*. The positive shock led to a rise in growth rates (this was the shock that led to higher growth rates), a rise in capacity utilization and a fall in the public debt / GDP with an increase on private debt. The negative shock resulted in a fall in growth rates (this was the shock that led to lower growth rates), a fall in capacity utilization and a rise in the ratio public debt / GDP with a decrease in private debt. With falling wages, more profit can be obtained to finance the investment demand and consequently there may be a drop in private debt by repurchasing the notes already issued. The opposite shock has the same symmetrical argument.

This last result shows that in the economy at hand prevails a *wage-led accumulation regime*, since an increase/decrease in wage share is followed by an increase/decrease in the growth rates of capital stock, disposable income and wealth, as well as an increase/decrease in the level of capacity utilization.

Figure 9: Shocks in *w* (+10%; - 10%).



Note: Positive shocks on top line and negative ones in bottom line.

6. Final Remarks

The idea of this paper is to build a Stock-and-Flow Consistent version of the IS-LM model for a closed economy with endogenous money supply, in order to analyze the dynamic properties of the model. We show that once the capacity effect of investment is taken into account and investment exceeds depreciation rate, the equilibrium of the model is no longer characterized by a stationary level of real output, but by a constant growth rate of real output.

The level of activity is represented in the model by the variable capacity utilization, which is constant and lower than one over the equilibrium path. Furthermore, any shock generated was unable to put the economy at full capacity utilization. This means that the SFC version of the IS-LM model reproduces the traditional Keynesian result of underemployment equilibrium. Moreover, in steady-state disposable income, capital stock and private wealth all grow at a same constant rate. We also showed that along balanced growth path, the economy is dynamically efficient and firms had a hedge financial posture.

Regarding the comparative dynamics of the model, we had performed some numerical simulations about the dynamic effects of shocks over the time path of endogenous variables. The most sensible parameter was *w*. The shocks in *w* generated the highest and lowest values in: growth rates, capacity utilization and public debt/GDP ratio. Thus, the model shows that some traditional results of Keynesian Theory as the "paradox of thrift" and a wage-led regime of accumulation.

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Table 5: List of all variables and parameters of the model

I_d	current period investment
ΔK	capital accumulation
GR_k	capital accumulation rate
K_{-1}	capital stock at period t-1
u	degree of production
r	interest rate
Y_s	product at current period
Y_{fc}	full capacity product
P	full profit
P_f	retained profit
P_h	residual profit
V	household's wealth at current period
V_{-1}	household's wealth at period t-1
V^e	expected household's wealth
YD	disposable income
YD^e	expected disposable income
C_d/C_t	consumption at current period
W	wages
w	Wage share
B_h	household's treasury bills
T_s/T_d	taxes
G_t	government expenditures
B_s	issued bills
B_{bc}	bills that are bought by Central Bank
H_s	money
γ_0	parameter that represents the animal spirit
γ_1	sensibility for level of capacity utilization
γ_2	sensibility for interest rate
σ	constant
d	retained profit rate
α_1	marginal propensity to consume
α_2	rate of household's wealth used in consumption
γ	parameter related to government expenditure
θ	tax rate
λ_{10}	Autonomous demand of money
λ_{11}	Sensitivity of money demand to interest
λ_{12}	Sensitivity of bills demand to interest
λ_{20}	Autonomous demand of bills
λ_{21}	Sensitivity of money demand to expected disposable income
λ_{22}	Sensitivity of bills demand to expected disposable income