Public Debt Management in a Dynamic Stock-Flow Consistent Model
Implications for the Brazilian case

Breno Santana Lobo[*]
José Luis Oreiro[**]

Abstract: The existence of floating-rate bonds in the composition of public debt is associated with some factors that tend to negatively affect the trajectory of the economy over time. The main objective of this article is to analyze the changes caused by a change in the public debt composition over the dynamics of a given economy. In order to do that, we built a dynamic stock-flow consistent post-keynesian model, in which the government bond market is modeled to reflect the main features of the Brazilian case. The parameters and initial conditions of the model are calibrated in order to form a baseline scenario that reflects in a satisfactory way the main stylized facts of modern economies. The simulation results indicate that the extinction of floating-rate bonds does not have negative effects on the economy in the short run. In the long run, however, uncontrolled public spending due to an increase in the debt service takes the economy to a path of instability. To stabilize the economy, government should adjust its economic policy to its debt management policy. Fiscal policy, monetary policy and income policy may be used by the government. A restrictive fiscal policy can be useful to stabilize the economy. However, it is associated with smaller growth rates. An active fiscal policy, associated with some specific objective, can reverse this result, suggesting that the fiscal policy can contribute to control inflation. Restrictive monetary policy can also be used to stabilize the economy. However, it is not the best policy to control inflation. Income policy has the best results.

Key-words: Public debt management, Economic policy, Post-Keynesian Macroeconomics, Stock-flow consistent models, Macrodynamic simulation model.

JEL: C63, E12, E61, E63, H63

October, 2012

[*] Analyst at Central Bank of Brazil and graduate student of economics at Universidade de Brasilia (UnB). E-mail: breno.lobo@bcb.gov.br.

[**] Associate professor in the Economics Department at Universidade de Brasilia (UnB), Level I Researcher of the National Council for Scientific and Technological Development (CNPq) and Vice-President of Brazilian Keynesian Association (AKB). E-mail: joreiro@unb.br. Homepage: www.joseluisoreiro.ecn.br.

1 INTRODUCTION

Brazilian National Treasury (STN) is in charge of public debt management. Its main objective is to minimize long-term financing costs, ensuring the maintenance of prudent risk levels, and contributing to the proper running of the market for the public debt. To achieve this goal, one of its guidelines is the gradual replacement of floating-rate (Selic-linked) bonds by fixed-rate and inflation-linked bonds (STN, 2011a).

In Brazil, a significant portion of the federal public debt is made up of Financial Treasury Bills (LFT)[1], which are securities linked to the overnight rate (average rate of daily operations with public bonds registered in the Selic system, or simply, the Selic rate). The Selic rate is the interest rate used by the Central Bank of Brazil (BCB) as the operational target of monetary policy, which operates under an inflation targeting framework. This rate reflects the liquidity conditions in the money market. Therefore, the same interest rate that controls the banking system liquidity is used to remunerate part of the federal public debt. As the objectives of monetary policy and public debt management are not the same, this architecture has the potential to promote undesirable effects on one or another policy depending on which objective is prevailing[2]. According to Carneiro (2007), it is hard to imagine that the interest rate can be decided by the Central Bank without considering the stock of securities in the market, or that the objective pursued by the Treasury of stretching public debt can be satisfied independently of what is designed for the trajectory of the interest rate. In practice, therefore, the separation between public debt management and monetary policy, as happens in most countries, does not mean that the two policies can be conducted without any coordination of goals.

In addition to this potential conflict of interest, LFT may limit the effectiveness of two important transmission channels of monetary policy: the interest rate channel and the asset price channel. LFT are securities of high liquidity and daily profitability. These characteristics make these bonds act as a quasi-currency that earns interest daily. This imputes a short-term logic over financial investments. As a consequence, there is no room for a yield curve that reflects the structure of risk and return of the economy. It follows that the opportunity cost of productive investments tends to be very high. This limits the ability of growth of the investment rate in the Brazilian economy.

According to Moura (2007), the low variability in LFT prices makes it difficult to form a term structure of interest rates in the country, which limits the effectiveness of the interest rate channel in the transmission of monetary policy. Carneiro (2007) states that the term structure of interest rates continues to involve a very large inaccuracy in the farthest corners, implying very high risks for the buyer of long securities. That is, the possibility of a financial investment as that of one day, with an interest rate close to the rate that would be obtained in a long-term financial investment, causes the agents to concentrate their financial investments in the very short-run. Therefore, LFT take the place of long-term bonds, causing a specialization

[1] Although the share of LFT is decreasing, they still remain as the most relevant bonds in federal public debt. In 2011, they represented 31% of federal public debt, in a stock of R$ 546.4 billion, which corresponded to about 14% of the GDP this year. These numbers indicate that LFT are indeed relevant. Thus, its existence is important for the dynamics of the Brazilian economy (STN, 2011a).

[2] Barbosa (2006) supports the hypothesis that the short-term Brazilian real interest rate is high due to the contagion effect that LFT has on monetary policy. For him, the curve representing the term structure of interest rates in Brazil becomes less steep because of the existence of the risk premium that has to be paid for LFT, which, through arbitrage operations, ends up passing to the interest rate of the reserve market.

of the money and capital market in the debt rollover and an abandonment of any other practice. According to Franco (2007), LFT radiate the overnight logic throughout the term structure of interest rates, promoting an unfair competition for the companies interested in funding long-term projects. This happens because they are forced to pay high premiums over the overnight rate as a reward for the liquidity waiver and the credit risk. Therefore, financial wealth tends to concentrate on LFT, no matter the size of the interest rates, maturities and liquidity of the investment.

Concerning the asset price channel, increases on interest rates raise LFT price. This effect is the opposite of the one observed on fixed-rate bonds like National Treasury Bills (LTN). Increases on interest rates diminish LTN price. If it is supposed that a portion of aggregate consumption is based on households' financial wealth, a restrictive monetary policy that raises interest rates increases the financial wealth of LFT owners. As a consequence, they increase their consumption and therefore, aggregate demand. On the other hand, a restrictive monetary policy decreases LTN price. As a consequence, the financial wealth of LTN owners diminishes, lowering aggregate consumption. This wealth effect associated to fixed-rate bonds spreads the right signals of monetary policy[3]. However, as LFT don´t exhibit wealth effect, the monetary policy transmission through public debt is not effective in Brazil[4].

Therefore, reducing LFT portion on public debt can be associated to beneficial effects over the dynamics of the economy. However, there are costs associated with this policy. Firstly, LTN interest rates are generally greater than Selic rate. Secondly, LTN maturity is generally smaller than LFT's. This means that this policy can increase public debt service, which can have negative effects on government budget and, consequently, the whole economy.

The main objective of this paper is to analyze the effects of a public debt management policy aimed at reducing the share of floating-rate bonds on the dynamics of a given economy.

To achieve this purpose, we build a dynamic stock-flow consistent model. Although there are several papers that use different SFC models to analyze different macroeconomic issues, Godley and Lavoie (2007) are the ones who make an effort to build a method of modeling economies in order to integrate into a single analysis the various relevant macroeconomic aspects, such as credit, money, income, production and wealth, and thus understand how the economic system functions as a whole.

This method is based on the premise that every transaction conducted by a sector implies an equivalent transaction to other sectors and that each financial transaction generates a corresponding change in the total amount of stock variables, so that all the financial assets of a sector has a counterparty liability in possession of another sector. In SFC models, everything comes from somewhere and everything goes somewhere. Therefore, the economic system is fully articulated and logically consistent in accounting terms.

---

[3] The wealth effect evaluates how changes in households' wealth induced by monetary policy affect consumption. See Pigou (1943), Ando and Modigliani (1963) and Ludvigson *et al* (2002).

[4] Some papers investigate the effects of LFT on the transmission of monetary policy through its wealth effect. See, for example, Andrade and Pires (2009), Amaral and Oreiro (2008) and Pastore (1996, 2007). These authors argue that the existence of LFT increases the ineffectiveness of monetary policy because of the lack of wealth effect of such bonds.

Specifically, SFC models consist of several behavioral equations modeling the economic system, as well as accounting matrices that provide consistency for the model. Afterwards, calibration of the initial conditions and parameters is needed in order to reflect stylized facts of modern economies. Finally, computer simulation can be used to simulate path changes of endogenous variables due to shocks introduced on relevant parameters of the model.

We build a model based on the set of models developed by Godley and Lavoie (2007). Its main innovation and contribution is the modeling of the government bond market as to reflect the fact that a certain share of public debt is made of floating-rate bonds, whose remuneration is given by the interest rate used by the monetary authority as an operational target of monetary policy in an inflation targeting regime.

SFC models advantage is that they try to understand how the economic system works as a whole. The problem of the Marshallian approach of comparative statics is that it is not consistent in the sense that stocks and flows do not satisfy accounting identities with individual and aggregate budget constraints. In this sense, these models have black holes in their structure, unlike SFC models (Godley and Lavoie, 2007).

After the set up of the model, we calibrate the parameters and the initial conditions in order to achieve a baseline scenario that can reproduce satisfactorily the main stylized facts observed in modern economies. Finally, because it is a non-linear macrodynamic model whose steady state cannot be determined analytically, it should be computationally solved.

Afterwards, we conduct simulation exercises designed to observe the changes in the dynamics of this artificial economy resulting from a change in debt management policy. A major goal of these exercises is to simulate the process of coordination that must exist between debt management policy and macroeconomic policy.

Besides this introduction, the paper has four other sections. Section 2 presents the model set up. Section 3 explains the calibration process and describes the baseline scenario. The simulation exercises, as well as their results and policy implications are reported in section 4. The last section summarizes the main conclusions.

2 MODEL SET UP

There are four sectors in our artificial economy: households, firms, government and banks. Households are divided into three groups depending on the origin of their income: workers, productive capitalists and financial capitalists. Macroeconomic policy decisions are taken independently by two entities that make up the government: the ministry of finance and the central bank. In order to simplify the model, it is a closed economy. Therefore, there are no foreign trade transactions, nor international capital flows. Firms, the government and banks are institutions that have their own life and rationality. As a consequence, they cannot be considered as mere intermediaries that act solely to serve the interests of households.

SFC models have two basic components: a set of accounting matrices that defines the consistency between asset stocks and transaction flows and a set of behavioral equations that establish relationships between the institutional agents. Before presenting these two components of the model, we should explain the set up of the public debt market.

2.1 Public bonds

As mentioned earlier, the purpose of the model is to analyze the effects of the existence of floating-rate (Selic-linked) bonds over the dynamics of a given economy. Since the existence of Selic-linked bonds is an idiosyncrasy of the Brazilian economy, the set up of the public debt market will reflect the characteristics of the Brazilian public debt market. In this sense, although federal debt is nowadays composed mostly of four different types of securities[5], we admit the existence of only two types of bonds in the economy: fixed-rate and floating-rate bonds.

The selling price of one unit of LTN is given by the following formula (STN, 2011b):

$$P_{LTN} = \frac{1000}{(1+i)^{du/252}} \tag{1}$$

where *i* is the annual interest rate of these bonds and *du* is the number of working days between the date of the sale of the bond and its maturity date.

Fixed-rate bonds pricing follows LTN pricing, such that:

$$p_{TPRE} = \frac{1000}{(1+i_{TPRE})^{T-t}} \tag{2}$$

where i_{TPRE} represents fixed-rated bonds interest rate and *t* represents the current period. It is assumed, without loss of generality, that the bonds are issued with a maturity date of four periods. Within this period, the government shall issue only this bond. These hypotheses imply that we are assuming that *T* must assume only values that are multiple of four, such that:

$$T = 4z \tag{3}$$

where $z = 1,2,3,\ldots$ is the time interval comprised of four periods. *z* is equal to 1 for the periods {0,1,2,3}, and so on.

Fixed-rate bonds price in (2) refers to their purchase price. During periods in which new bonds are issued, in periods that are multiples of four, the selling price is different. In these periods, fixed-rate bonds selling price is given by its face value, which is, as in LTN, equal to 1000. In other periods, fixed-rate bonds purchase and selling price are equal, so that there is no discount on that market:

$$p^{v}_{TPRE} = \begin{cases} 1000, if\ t\ is\ a\ multiple\ of\ 4 \\ p_{TPRE}, otherwise \end{cases} \tag{4}$$

Fixed-rate bonds interest rate is determined endogenously. It is given by a risk premium (*RP*) that is added to the overnight interest rate, such that:

[5] In addition to LFT and LTN, National Treasury Notes (NTN) series B and F are also significantly present in the composition of public debt. NTN-F are also fixed-rate bonds. Its difference from LTN is that they pay semiannual interest coupons. NTN-B are inflation-linked bonds.

$$i_{TPRE} = i_{over} + RP \quad (5)$$

The risk premium is made of two components. The first one is exogenous. The other depends on the public debt to GDP ratio. The endogenous component reflect market expectations about public debt sustainability:

$$RP = RP_0 + RP_1 \left(\frac{DP}{Y}\right)_{-1} \quad (6)$$

The selling price of one unit of LTN is given by (STN, 2011b):

$$P_{LFT} = \frac{(1000 * i_{acum})}{(1 + discount)^{du/_{252}}} \quad (7)$$

where *discount* represents the bond discount at the time of the sale[6] and i_{acum} represents the daily variation in the Selic rate between 1 July 2000 and the day that the bond is evaluated[7].

Floating-rate bonds pricing follows LFT pricing, such that:

$$p^v_{TPOS} = 1000 * FA \quad (8)$$

where FA is the accumulation factor of the overnight interest rate, given by:

$$FA = (1 + i_{over})FA_{-1} \quad (9)$$

As supposed to fixed-rate bonds, it is assumed that floating-rate bonds are always issued with a maturity date of four periods. Within this period, the government shall issue only this bond. It is also assumed that the accumulation factor returns to its initial value every four periods, corresponding to its maturity[8]. This implies that in such periods the purchase price and the selling price of these bonds are different. In its maturity, the purchase price of floating-rate bonds is always equal to 1000, such that:

$$p_{TPOS} = \begin{cases} 1000, if\ t\ is\ a\ multiple\ of\ 4 \\ p^v_{TPOS}, otherwise \end{cases} \quad (10)$$

Comparing equations (7) and (8), we can see that it was assumed that floating-rate bonds are always traded at par, i.e., with no discount. Therefore, the denominator of (7) would always

[6] LFT discount is a rate that is deducted from the Selic rate variation to assess the profitability of the bond according its demand conditions in the secondary market. Negative values imply that LFT are trading with a premium. The value of the discount defines the risks associated with LFT. In regular market conditions, LFT discount is in general close to zero. Therefore, its effect on LFT price can be disregarded. However, in times of stress when the perception of the credit quality of the Treasury deteriorates and financial markets demand higher risk premiums, LFT can be traded with a substantial discount, so that its unit price may drop significantly.

[7] STN has established that the price of one unit of LFT is R$1,000.00 on 1 July 2000. This date is, therefore, its data-base, which explains the term 1000 in the numerator of equation (7). This was done to increase the liquidity of the bonds in the secondary market, since bonds issued on different dates, but with the same data-base and maturity date, have identical flows. It turns out that the bonds become expendable (STN, 2011b).

[8] In Brazil, as already mentioned, the accumulation factor begins on 1 July 2000 and does not return to its original value. The hypothesis that the accumulation factor returns to its initial value every four periods is not crucial to the dynamics of the model. It is only used to avoid the exponential growth of floating-rate bonds nominal price. This avoids working with very high nominal values in simulations.

be equal to one. In normal market conditions, LFT are often traded with a discount of 0.02%, which is a rather low value. Thus, admitting that the denominator of equation (7) is always equal to one is not far from the observed.

Consistent with the characteristics of LTN and LFT, fixed-rate and floating-rate bonds do not yield a regular coupon. Therefore, they do not generate for their owners a regular flow of income. Capital gains arising from changes in bond prices in each period, whether as a result of changes in interest rates or due to the effect of time, are added to the financial wealth of their owners. In the maturity period, the government pays to bonds owners the amount initially invested, plus interest. It is assumed that bonds owners always reinvest the amount they receive in maturity periods on new bonds of the same characteristics. Therefore, there is never a net redemption of bonds because they come to maturity.

The model does not take into consideration two features of the Brazilian public debt market. The first one is related with the existence of multiple issuance period length. The other one is related with the existence of multiple maturity dates for each type of bond. As already mentioned it was assumed that the two types of bonds are always issued in the same period and always have the same maturity which is equal to four periods. The value assigned to the length of the issuing period is irrelevant to the dynamics of the model. We could have assigned any random value without changing the main results of the simulations. The choice that was made can be justified by the need to focus on the effects resulting from changes in interest rates on bond prices. If it were assumed that the bonds could be issued at any time, this could explain part of the observed dynamics. As already mentioned, LFT issuance period length is generally greater than LTN's. The average maturity of public debt is important to identify its sustainability. The higher the average maturity, the best is the structure of public debt. In this regard, LFT are better than LTN. Assuming that both bonds have the same issuance period length implies that the model is unable to capture this aspect of public debt.

The option to not model multiple maturity dates for each type of bond is to avoid complexity. If it was taken into account, we would have to admit the existence of *n* bonds, where *n* would be the amount of different types of existing bonds multiplied by the number of different maturities available for each bond. Public bonds of same characteristics would have to be considered as if they were different because they have different maturity dates. This choice would include a very large number of financial assets on the list of assets available in the decision process of portfolio allocation of households and banks, which would bring a level of complexity greater than desired for the model.

The problem with this choice is that the model is unable to provide any explanation of the term structure of interest rates. Therefore, the issue addressed in the previous section that LFT can be harmful to economic growth by concentrating investments in the short term and by competing with long-term investment projects cannot be logically drawn from the model. This benefit arising from the exchange of LFT by LTN will be admitted as given.

2.2 Balance sheet matrix and transactions flow matrix

The first step of every SFC model is to build a balance sheet matrix. This matrix describes the stocks of assets and liabilities of each sector and establishes logical relationships between them. Each financial asset owned by a given sector has a financial liability of the same amount owned by another or others sectors as a necessary counterpart. This characteristic is essential for the consistent accounting of the stocks of the model. This implies that the set up of the balance sheet matrix must follow a single rule: all its columns and rows must sum to zero. The only lines that do not have to follow this rule are those related with tangible assets such as fixed capital and inventories held by firms. Assets are represented by positive signals and liabilities by negative signals, as shown in Figure 1.

Some considerations about the notation must be made in advance. Capital letters represent monetary values of the assets at market prices[9], while lowercase letters represent objects that can be counted in terms of units. The exceptions to this rule are the letter *p*, which is related to the unit price of the various assets, according to its accompanying subscript[10], and the variables related to public bonds, *TPRE* and *TPOS*, which refer to the physical quantity of existing bonds. *V* and *DP* represent, respectively, the stocks of wealth and public debt. The subscript *f* will always be associated with firms. The subscripts *w*, *c*, *a*, *b* and *BC* are associated, respectively, with workers, productive capitalists, financial capitalists, banks and the central bank. The subscript *p* that follows these subscripts refers to the ownership of the asset. It is introduced to distinguish the ownership of the asset from its demand or its supply, when they will be associated, respectively, with the subscripts *d* and *s*.

The balance sheet matrix analysis points out the assumptions implicit in the model. First, we analyze the assumptions related to firms. It is assumed that the only assets owned by firms are tangible assets, corresponding to their fixed capital and inventories. Therefore, it is assumed for simplicity that firms do not allocate their wealth in financial assets. This implies that the entire amount of resources at their disposal is used to finance inventories and to purchase new fixed capital. Besides retained profits, the amount of resources available to firms refers to loans from banks. Therefore, we do not consider the possibility of financing by issuing private bonds. Likewise, bank loans are the only credit instrument available. It is worth noting that the stock of wealth of the firms, which generally must be positive, since the nominal value of inventories and fixed capital should exceed the stock of loans, has no practical significance for the model.

Households are split into three groups according to the origin of their income. It is assumed, following Kaldor (1956) and Pasinetti (1962), that the propensity to consume out of wages and profits are different. It is assumed that workers consume their entire income. Therefore their propensity to save is equal to zero. As a consequence, they do not accumulate financial wealth and thus they have no net wealth.

Capitalists’ propensity to save is greater than zero. This implies that financial and productive capitalists accumulate financial wealth. It is assumed that there are three different assets in which they can allocate their net wealth. They can allocate it in demand deposits with banks and buy from the government both types of bonds. For simplicity, households do not own tangible capital assets like real estate, for example.

[9] Except for inventories. Their pricing will be detailed later.

[10] When not accompanied by a subscript, it refers to the single unit price of goods produced by the economy. It is assumed that the price of fixed capital is also equal to that price.

Figure 1 – Balance sheet matrix

	Households			Firms	Government		Banks	$\sum$
	Workers	Productive Capitalists	Financial capitalists		Ministry of Finance	Central Bank		
Inventories (IN)				+IN				+IN
Fixed capital (K)				+K				+K
Loans (L)				-L			+L	0
Fixed-rate public bonds (TPRE)		$+TPRE_{cp} \cdot p_{TPRE}$	$+TPRE_{ap} \cdot p_{TPRE}$		$-TPRE \cdot p_{TPRE}$	$+TPRE_{BCp} \cdot p_{TPRE}$	$+TPRE_{bp} \cdot p_{TPRE}$	0
Floating-rate public bonds (TPOS)		$+TPOS_{cp} \cdot p_{TPOS}$	$+TPOS_{ap} \cdot p_{TPOS}$		$-TPOS \cdot p_{TPOS}$	$+TPOS_{BCp} \cdot p_{TPOS}$	$+TPOS_{bp} \cdot p_{TPOS}$	0
Demand deposits (M)		$+M_{cp}$	$+M_{ap}$				-M	0
Bank reserves (H)						-H	+H	0
Net Worth	0	$-V_c$	$-V_a$	$-V_f$	+DP	$-V_{BC}$	$-V_b$	-(IN+K)
$\sum$	0	0	0	0	0	0	0	0

Source: Authors

Likewise, it is assumed that households do not need to accumulate wealth in the form of high powered money. Given the current technological development and increasing electronification of payment instruments, it is assumed that all purchases of goods are made through debit cards or checks, where the movement of funds between two different agents is given by means of credits and simultaneous debt in their demand deposit accounts. Moreover, it is not assumed the existence of term deposits in banks. Assuming that the interest rate that banks pay for the issuance of term deposits is less than or equal to the interest rate paid by the government in its public bonds and assuming a direct, large and unrestricted, with no transaction costs, access to public debt market, there is no incentive for households to buy time deposits. We can think of as if there were no financial institutions such as mutual funds and pension funds, for example, to intermediate financial investments of households. These operations would be directly held by them.

Financial wealth is usually positive. The negative signal associated with net worth in Figure 1 is a convention needed to balance all the accounts so that every column of the matrix sums to zero. This is necessary to preserve the consistency of the model (Godley and Lavoie, 2007)

The central bank also holds bonds issued by the government. The bank reserves are its only liabilities. On the side of the ministry of finance, the nominal value of public bonds represents public debt.

Banks have four types of assets: loans taken by firms, the two types of bonds issued by the government and bank reserves that they hold in the central bank. Demand deposits of households are their only liability.

Besides the balance sheet matrix, every SFC model must contain a matrix that explicitly shows the flow of funds arising from all monetary transactions of the economy in each period. Figure 2 shows the transactions flow matrix. Positive signs represent sources and negative signs represent uses. Just as in the balance sheet matrix, the sum of each row and each column must be zero. This restriction ensures the logical accounting consistency of the model.

Workers receive wages from firms. With this income, they pay taxes to the government and consume all the rest, so that they have no savings and consequently no financial wealth.

Productive capitalists receive distributed profits from firms. They buy products from firms and pay taxes to the government with this income. The same behavior is observed for financial capitalists. The difference is that their source of income is the profit of banks in each period.

The flow of income and expenditure detailed between the dotted line and the first sum represents the flows relating to operations and rollover of maturing government bonds. These entries are different from zero only in periods multiples of four in which bonds come to maturity. As it was assumed that all the bonds are rolled, their presence has no influence on the level of savings of each sector. They are displayed in the matrix for illustrative purposes.

Figure 2 – Transactions flow matrix

		Households			Firms		Government		Banks		$\sum$
		Workers	Productive Capitalists	Financial Capitalists	Current	Capital	Ministry of Finance	Central Bank	Current	Capital	
Consumption		$-C_w$	$-C_c$	$-C_a$	$+C$						0
Government spending					$+G$		$-G$				0
Investment					$+I$	$-I$					0
Inventories					$+\Delta IN$	$-\Delta IN$					0
[GDP (memo)]					[Y]						
Taxes		$-T_w$	$-T_c$	$-T_a$			$+T$				0
Wages		$+WB$			$-WB$						0
Firms' profits			$+FD_f$		$-F_f$	$+FU_f$					0
Banks' profits				$+F_b$					$-F_b$		0
Fixed-rate public bonds							$-p^v_{TPRE} . TPRE^v_{bp}$		$+p^v_{TPRE} . TPRE^v_{bp}$		0
Floating-rate public bonds							$-p^v_{TPOS} . TPOS^v_{bp}$		$+p^v_{TPOS} . TPOS^v_{bp}$		0
Interest on loans					$-i_{l-1} . L_{-1}$				$+i_{l-1} . L_{-1}$		0
Fixed-rate public bonds (maturity)			$+p^v_{TPRE} . TPRE_{cp-1}$	$+p^v_{TPRE} . TPRE_{ap-1}$			$-p^v_{TPRE} . (TPRE_{-1} - TPRE^v_{bp})$	$+p^v_{TPRE} . TPRE_{BCp-1}$	$+p^v_{TPRE} . (TPRE_{bp-1} - TPRE^v_{bp})$		0
Fixed-rate public bonds (rollover)			$-p_{TPRE} . TPRE^{rol}_{cd}$	$-p_{TPRE} . TPRE^{rol}_{ad}$			$+p_{TPRE} . TPRE^{rol}_{s}$	$-p_{TPRE} . TPRE^{rol}_{BCd}$	$-p_{TPRE} . TPRE^{rol}_{bd}$		0
Floating-rate public bonds (maturity)			$+p^v_{TPOS} . TPOS_{cp-1}$	$+p^v_{TPOS} . TPOS_{ap-1}$			$-p^v_{TPOS} . (TPOS_{-1} - TPOS^v_{bp})$	$+p^v_{TPOS} . TPOS_{BCp-1}$	$+p^v_{TPOS} . (TPOS_{bp-1} - TPOS^v_{bp})$		0
Floating-rate public bonds (rollover)			$-p_{TPOS} . TPOS^{rol}_{cd}$	$-p_{TPOS} . TPOS^{rol}_{ad}$			$+p_{TPOS} . TPOS^{rol}_{s}$	$-p_{TPOS} . TPOS^{rol}_{BCd}$	$-p_{TPOS} . TPOS^{rol}_{bd}$		0
$\sum$		0	SAV_c	SAV_a	0	$-INV_f$	$-SAV_g$	0	0	0	0
Change in the stocks of	Loans					$+L_d$				$-L_s$	0
	Demand deposits		$-M_{cd}$	$-M_{ad}$						$+M_s$	0
	Floating-rate public bonds		$-TPOS_{cd} . p_{TPOS}$	$-TPOS_{ad} . p_{TPOS}$			$+TPOS_s . p_{TPOS}$	$-TPOS_{BCd} . p_{TPOS}$		$-TPOS_{bd} . p_{TPOS}$	0
	Fixed-rate public bonds		$-TPRE_{cd} . p_{TPRE}$	$-TPRE_{ad} . p_{TPRE}$			$+TPRE_s . p_{TPRE}$	$-TPRE_{BCd} . p_{TPRE}$		$-TPRE_{bd} . p_{TPRE}$	0
	Bank reserves							$+H_s$		$-H_d$	0
$\sum$		0	0	0	0	0	0	0	0	0	0

Source: Authors

The difference between the income flows and the expenditure flows of productive and financial capitalists represents their savings, which in general but not necessarily should be positive. The second part of the transactions flow matrix, after the first sum illustrates the saving use by each sector, so that each column sums to zero. This rule represents the budget constraint of each sector. This budget constraint describes how the balance between flows of expenditures, incomes of production factors and transfers necessarily generates as a counterpart changes in the stocks of assets and liabilities. As stated by Godley and Lavoie (2007, p.38):

> The accounts of the transactions flow matrix […] are comprehensive in the sense that everything comes from somewhere and everything goes somewhere. Without this armature, accounting errors may pass unnoticed and unacceptable implications may be ignored. With this framework, 'there are no black holes'.

Consistent with the balance sheet matrix, capitalists' savings in each period can be directed to three different assets. Note that it is assumed for simplicity that all taxes are paid by households[11].

The resources of firms come from their sales. Households buy an amount of *C* and the government an amount of *G* of firms' production. Besides that, firms also buy fixed capital and finance inventories. The sum of these four factors represents the total amount produced by the economy, i.e. its Gross Domestic Product (GDP). Purchases and sales of fixed capital and inventories financing are made through the same sector. Thus, we need to distinguish the current account from the capital account of the firms.

The current account represents the income flows of the firms sector. The capital account represents its sources of financing. As we can see from the current account, the whole flow of revenues must be spent. This is necessary for the model accounting consistence as the columns must sum to zero. The revenues of the firms are spent on wages for workers and on interest for banks. It is assumed that such interest shall be paid at the beginning of each production period. Therefore, the relevant interest rate and loan stock are those observed in the previous period. The subscript *-1* in Figure 2 represents this convention.

The difference between inflows and outflows of funds represents the profit of firms. In order to follow a stylized fact of modern capitalist economies, it is assumed that a portion of the profits of firms are retained to finance new investments. The other portion is distributed to productive capitalists. Usually but not necessarily the amount of retained profits is not enough to cover the whole amount invested by firms. This implies that firms should look for other sources of financing. Consistent with the balance sheet matrix, the other source of financing available to firms are bank loans. Positive variations of the stock of bank loans represent an entrance of funds to firms.

The government expenses come from the products purchased from firms and from the financial costs due to public debt. These costs refer to the amortization and interests paid to banks every time they sell bonds. It is assumed that households do not sell bonds periodically. On the other hand, banks sell a portion of their bonds in each period. Taxes paid by households are the single source of government revenues.

---

[11] As it is assumed that firms and banks profits are distributed to households, this implies that at least indirectly these sectors are also being taxed.

The difference between revenue and expenditure flows represents government savings in each period. This result is displayed on the first sum of Figure 2. Negative values represent fiscal deficits, which must be entirely financed through the issuance of public bonds. Fiscal surpluses are used to reduce public debt. This ensures that the column of the ministry of finance sums to zero.

The central bank does not have revenues or expenditures flows, which implies that bank reserves are not remunerated. Although the central bank has no transactions flows, there are variations in the stocks of the financial assets it holds. To keep the accounting of the model coherent and consistent, the variation in its stock of assets, which corresponds according to the balance sheet matrix to the two types of bonds issued by the government, must be equal to the change in its stock of liabilities which is given by bank reserves.

Banks receive revenues from the interests paid by firms and from the selling of public bonds. It is assumed that banks sell a fixed portion of its stock of bonds in each period. As the probability of default of bank loans is not modeled, the accumulation of financial wealth by banks would be an end in itself, given that the debt does not generate a regular flow of income in the form of interest that would be distributed to financial capitalists. For this reason, it is assumed that banks sell part of its bonds portfolio once one of its objectives is to distribute profits to financial capitalists.

Demand deposits and bank reserves are not remunerated. Thus, the two sources of income correspond to bank profits. It is assumed that bank profits are fully distributed to capitalists. Besides this flow of funds, the transactions of other sectors imply that the stocks of assets and liabilities of banks also vary in each period. In order to maintain the consistency of the model, the variation in the stock of demand deposits must be exactly equal to the sum of the variation in the stock of public bonds, loans and bank reserves.

Finally, in order to integrate both matrices, we need to add capital gains to the transactions flow matrix. This procedure enables us to realize how the stock of assets at the beginning of a period varies until it reaches its new value at the end of that period. The capital gains matrix is illustrated in Figure 3 and the integration matrix in Figure 4.

Capital gains refer to changes in the stock of assets arising from changes in their price between any two periods. In the model, the price of public bonds, fixed capital and inventories can change between two periods[12].

The first row of the integration matrix presents the net worth of each sector at the beginning of a period, which corresponds to the net worth at the end of the previous period, according to the balance sheet matrix[13]. Changes in the net worth of a given sector come from two components: changes arising from transactions and changes arising from capital gains. Changes in the stock of assets of the central bank and banks arising from transactions are always equal to zero. Thus, the net worth of these two sectors corresponds to the capital gains accumulated over time.

[12] In maturing periods, the public bonds price variation that is relevant is that associated with the sale price and not with the purchase price as illustrated in figure 3.

[13] The only difference is that the signals appearing in the first part of the integration matrix are opposite to the ones appearing in the balance sheet and transactions flow matrices. The purchase of any asset corresponds to an increase in the net worth of a given sector. Therefore, it must be associated with a positive sign.

Figure 3 – Capital gains matrix

	Households: Productive Capitalists	Households: Financial Capitalists	Firms	Government: Ministry of Finance	Government: Central Bank	Banks	$\sum$
Fixed capital			$+k_{-1} \cdot \Delta p$				$+k_{-1} \cdot \Delta p$
Inventories			$+in_{-1} \cdot \Delta UC$				$+in_{-1} \cdot \Delta UC$
Fixed-rate public bonds	$+TPRE_{cp-1} \cdot \Delta p_{TPRE}$	$+TPRE_{ap-1} \cdot \Delta p_{TPRE}$		$-TPRE_{-1} \cdot \Delta p_{TPRE}$	$+TPRE_{BCp-1} \cdot \Delta p_{TPRE}$	$+TPRE_{bp-1} \cdot \Delta p_{TPRE}$	0
Floating-rate public bonds	$+TPOS_{cp-1} \cdot \Delta p_{TPOS}$	$+TPOS_{ap-1} \cdot \Delta p_{TPOS}$		$-TPOS_{-1} \cdot \Delta p_{TPOS}$	$+TPOS_{BCp-1} \cdot \Delta p_{TPOS}$	$+TPOS_{bp-1} \cdot \Delta p_{TPOS}$	0

Source: Authors

Figure 4 – Integration matrix

		Households: Productive Capitalists	Households: Financial Capitalists	Firms	Government: Ministry of Finance	Government: Central Bank	Banks	$\sum$
Net worth (end of last period)		$+V_{c-1}$	$+V_{a-1}$	$+V_{f-1}$	$-DP_{-1}$	$+V_{BC-1}$	$+V_{b-1}$	$+(IN+K)_{-1}$
Change in stocks due to transactions	Loans			$-L_d$			$+L_s$	0
	Demand deposits	$+M_{cd}$	$+M_{ad}$				$-M_s$	0
	Floating-rate public bonds	$+TPOS_{cd} \cdot p_{TPOS}$	$+TPOS_{ad} \cdot p_{TPOS}$		$-TPOS_s \cdot p_{TPOS}$	$+TPOS_{BCd} \cdot p_{TPOS}$	$+TPOS_{bd} \cdot p_{TPOS}$	0
	Fixed-rate public bonds	$+TPRE_{cd} \cdot p_{TPRE}$	$+TPRE_{ad} \cdot p_{TPRE}$		$-TPRE_s \cdot p_{TPRE}$	$+TPRE_{BCd} \cdot p_{TPRE}$	$+TPRE_{bd} \cdot p_{TPRE}$	0
	Bank reserves					$-H_s$	$+H_d$	0
	Fixed capital			$+\Delta k \cdot p$				$+\Delta k \cdot p$
	Inventories			$+\Delta in \cdot UC$				$+\Delta in \cdot UC$
Change in stocks due to capital gains	Fixed capital			$+k_{-1} \cdot \Delta p$				$+k_{-1} \cdot \Delta p$
	Inventories			$+in_{-1} \cdot \Delta UC$				$+in_{-1} \cdot \Delta UC$
	Fixed-rate public bonds	$+TPRE_{cp-1} \cdot \Delta p_{TPRE}$	$+TPRE_{ap-1} \cdot \Delta p_{TPRE}$		$-TPRE_{-1} \cdot \Delta p_{TPRE}$	$+TPRE_{BCp-1} \cdot \Delta p_{TPRE}$	$+TPRE_{bp-1} \cdot \Delta p_{TPRE}$	0
	Floating-rate public bonds	$+TPOS_{cp-1} \cdot \Delta p_{TPOS}$	$+TPOS_{ap-1} \cdot \Delta p_{TPOS}$		$-TPOS_{-1} \cdot \Delta p_{TPOS}$	$+TPOS_{BCp-1} \cdot \Delta p_{TPOS}$	$+TPOS_{bp-1} \cdot \Delta p_{TPOS}$	0
Net worth (end of current period)		$+V_c$	$+V_a$	$+V_f$	$-DP$	$+V_{BC}$	$+V_b$	$+(IN+K)$

Source: Authors

The first part of the integration matrix corresponds to the second part of the transactions flow matrix. The second part corresponds to the capital gains matrix. The only difference in the first part is the addition of two further lines: one concerning the variation in the physical stock of fixed capital and the other on the variation in inventories. The counterpart to these lines can be found, respectively, in the lines referring to the fixed investment and the accumulation of inventories in the transactions flow matrix.

All these matrices represent a complete and logically consistent system of macroeconomic transactions. However, they do not characterize, besides logical constraints, the behavior of the different institutional actors. It is necessary to establish behavioral relations between the variables of the model in order to generate a plausible path for the economy. Provided they maintain consistency between stocks and flows, the establishment of these relations is free. The next subsection presents the behavioral relations assumed by the model.

2.3 Behavioral equations

2.3.1 FIRMS

We assume that firms are institutions that must take decisions on production, investment, inventories, pricing, costs and funding. These decisions need not be based on the principle of profit maximization, in which firms have perfect information about their and all their competitors demand conditions and cost structure. It is also assumed that firms operate in an imperfect competition environment, in which oligopolistic structures prevail.

We assume that firms take decisions in historical time and in an environment characterized by the existence of Keynes-Knight uncertainty. Thus, the temporal nature of economic processes matters (Davidson, 1978)[14]. This implies that decisions regarding production shall be taken prior to the knowledge of the quantity of unities sold. Therefore, firms have to decide about how many units to produce based on expectations about future demand for their products. At the same time, once there is a time lag between actual spending and firms' revenues, this process necessarily involves the accumulation of inventories. Accordingly, we assume that firms decide the quantity of units to produce in a given period based on their expected sales and on their desired variation in inventories, according to equation (11):

$$y = s^e + in^e - in_{-1} \qquad (11)$$

where *y*, *s* and *in* represent respectively output, sales and inventories measured in terms of physical quantities. The superscript *e* indicates expected values, whereas the subscript *-1* refers to quantities observed in the previous period. The subscript *t* referring to the current period is removed from the notation in order to clean it. Thus, variables that appear without subscript refer to amounts of the current period.

Equation (11) implies that the principle of effective demand is being observed. The decision

[14] A key feature of the model is that it takes into account the notion of historical time. This implies that the time is seen as irreversible. Thus, once a decision is made and implemented, it cannot be reversed without incurring non negligible costs. Accordingly, the decision making process of various economic agents must follow a logically consistent temporal sequence. As will become evident during the presentation of the behavioral equations, the decision process is formulated to be logically coherent and temporally consistent.

to produce is based on firms' expectations about their demand. Thus, the production of goods always tries to adjust to its demand.

In general, the precise way in which expectations are formed is not crucial to the development of SFC models (Godley and Lavoie, 2007). This implies that any expectation formation rule is allowed in SFC models. We assume that agents' rationality is limited or procedural in the sense used by Simon (1976). Given the difficulties faced by individuals and institutions to acquire and process information, they take decisions based not on any optimization principle, since no one knows or can know the optimal solution for any problem, but on norms based on conventions, habits and behavior rules they attach themselves. This behavior is seen as rational because it is the best answer to a complex environment that operates under uncertainty.

Compatible with the assumption of procedural rationality, we assume that expectations are adaptive. This expectation formation rule is also compatible with the concept of short-term expectations formulated by Keynes (1936), in which the revision process is seen as gradual and continuous, being based on the observation of past results. Experience guides the formation of expectations. Thus, as stated by Carvalho (1992, p.68), "here one recognizes the environment and adapts to it."

Accordingly, firms review their past sales expectations based on what occurred in the previous period. Thus, expected sales in the current period can be expressed by:

$$s^e = \beta s_{-1} + (1-\beta) s^e_{-1} \tag{12}$$

where β is a reaction parameter of the firms related to their expectations.

Firms' expectations on inventories correspond to the level of inventories they want to keep at the end of the period. We assume that firms have a target level of inventories (in^T), which is given by some fraction of their expected sales in each period. Thus, the inventory level that firms wish to keep at the end of a given period corresponds to any deviations of the inventories level in the previous period with respect to its target. We assume that firms want to correct only a portion of this deviation in a given period. Therefore, the equations that show the inventories level desired by firms and the inventories target can be written respectively as:

$$in^e = in_{-1} + \gamma_{in}(in^T - in_{-1}) \tag{13}$$

$$in^T = \sigma^T s^e \tag{14}$$

where γ_{in} is a parameter that represents the fraction that partially adjusts inventories in relation to its target and σ^T is a parameter behavior of firms that represents the target inventories to sales ratio.

Variations in the level of inventories at the end of each period are the difference between total output and total sales:

$$in = in_{-1} + (y - s) \tag{15}$$

Total sales correspond to the sum of the amount purchased by households regarding their consumption (c), the amount purchased by government related to its consumption (g) and the amount of investments made by firms (i), such that :

$$s = c + g + i \tag{16}$$

After production decisions, firms must decide about pricing. We take prices as a mechanism of income distribution under the control of firms. They are not the result of an anonymous market clearing mechanism that leads to an instantaneous equilibrium between supply and demand (Lavoie, 2006).

We adopt the Kaleckian approach in which firms set their prices based on their costs (Kalecki, 1971). In this approach, given the uncertainty on the level of aggregate demand and on the behavior of their competitors, firms use the convention of setting their prices based on a mark-up (φ) on some measure of their unit cost (UC) (Dutt, 2003):

$$p = (1 + \varphi)UC \tag{17}$$

Firms choose their mark-up based on capacity utilization (u) and on their level of indebtedness. Firms' level of indebtedness is given by their liabilities to assets ratio. According to figure 1, their liabilities refer to the stock of loans and their assets to inventories and fixed capital:

$$\varphi = \varphi_0 + \varphi_1 u_{-1} + \varphi_2 \frac{L_{-1}}{IN_{-1} + K_{-1}} \tag{18}$$

Firms' unit cost refers to their variable cost. We assume for simplicity that firms are vertically integrated. Thus we can ignore the existence of intermediate goods. This implies that firms' unit cost is given exclusively by the labor unit cost, i.e., the amount of money needed to produce one unit of output:

$$UC = \frac{WB}{y} \tag{19}$$

where WB is the wage bill. It is the result of multiplying the nominal wage rate W and the number of workers employed N:

$$WB = W * N \tag{20}$$

Substituting (20) in (19) and knowing that labor productivity (pr) is by definition the amount of goods produced per worker:

$$pr = \frac{y}{N} \tag{21}$$

We have:

$$UC = \frac{W}{pr} \tag{22}$$

We assume the existence of dynamic scale economies. Thus labor productivity growth rate is endogenous. We adopt the Kaldorian technical progress function (Kaldor, 1957a) and assume that labor productivity growth rate is an increasing function of industrial output growth (g_y).

Furthermore we introduce the growth rate of the fixed capital to worker ratio in the technical progress function. We assume that the higher is this ratio, the greater is the dynamics of a given economy, once it increases the probability of incorporation of productivity gains arising from the development of productive activities, knowledge of workers and facilities for the dissemination of new knowledge and agglomeration economies. Thus the technical progress function can be written as:

$$pr = pr_{-1}\left(1 + \tau_0 + \tau_1 g_{y_{-1}} + \tau_2 \left(\frac{\Delta\left({}^{k}/_{N}\right)}{\left({}^{k}/_{N}\right)_{-1}}\right)_{-1}\right) \tag{23}$$

where *k* represents fixed capital in terms of physical quantities.

Equation (21) can be rewritten as a function of *N*. This new equation gives the amount of workers employed by firms in each period:

$$N = \frac{y}{pr} \tag{24}$$

By (17) and (22), we can see that, besides the observed variations in labor productivity, changes in prices are determined by the level of the mark-up and variations in the nominal wage rate[15].

The evolution of the nominal wage rate over time is not given by the interaction between demand and supply of labor, whose eventual imbalance would be instantly adjusted by variations in the rate of nominal wages. We assume the existence of contracts between workers and firms . They reflect the bargaining power of workers and normative pressures arising from conventions and beliefs about what would be a fair wage (Seccareccia, 2003).

We assume that the rate of nominal wage changes only at discrete intervals of time and that these changes happen at the beginning of each period. According to the assumptions made in the previous paragraph, we assume that the nominal wage rate is an increasing function of the difference between the target real wage of workers (ω^T) and the real wage rate observed in the previous period:

$$W = W_{-1}\left(1 + \Omega_3\left(\frac{\omega^T}{\frac{W_{-1}}{p_{-1}}} - 1\right)\right) \tag{25}$$

where Ω_3 is a reaction parameter reflecting the degree of bargaining power of workers.

We assume that workers internalize productivity gains to their target real wage. Moreover this target reflects the prevailing conditions in the labor market. As higher rates of economic

---

[15] Since this is a closed economy, it does not make any sense to incorporate exogenous shocks such as variations in commodity prices, which are usual sources of inflation in developing economies. Note also that we assume that changes in interest rates of bank loans, which are part of firms' flow of expenditures, as shown in figure 2, are not passed through prices.

growth are associated in general with lower unemployment rates, we assume that workers embody a portion of the real economic growth rate (g_{gdp}) in its real wage target such that:

$$\omega^T = \left(\frac{W}{p}\right)^T = \omega^T_{-1}\left(1 + \Omega_1 * \frac{\Delta pr}{pr_{-1}} + \Omega_2 * g_{pib}{}_{-1}\right) \quad (26)$$

where Ω_1 and Ω_2 are parameters that weights workers' reaction.

The unemployment rate is by definition given by:

$$UR = \frac{PEA - N}{PEA} \quad (27)$$

where *PEA* represents the economically active population of the economy, which is assumed to grow at an exogenous and constant rate g_N in each period such that:

$$PEA = (1 + g_N)PEA_{-1} \quad (28)$$

Based on these equations, we can conclude that inflation in this economy is mainly associated with five factors: (i) technical progress dynamics, (ii) aggregate demand growth, (iii) workers' bargaining power, (iv) firms' indebtedness level and (v) workers' distributive dissatisfaction.

Besides decisions concerning output and pricing, firms must decide how much they should invest in each period and what are the sources of funding for these investments. The physical amount of fixed capital purchased in each period grows at a rate g_i, which can be interpreted as the rate of accumulation of fixed capital in this economy:

$$i = (1 + g_i)i_{-1} \quad (29)$$

The rate of accumulation of fixed capital represents the investment function of the model. The investment is a function of the capacity utilization, due to the so-called accelerator effect, and the weighted average cost of capital (*WACC*). The cost of capital is calculated as a weighted average of the two sources of funding for firms: retained earnings and bank loans. The cost of retained earnings is given by its opportunity cost, represented by the overnight interest rate. The cost of bank loans is given by their rate of interest. Thus, the weighted average cost of capital can be expressed as:

$$WACC = \frac{FU_f}{FU_f + \Delta L} i_{over} + \frac{\Delta L}{FU_f + \Delta L} i_l \quad (30)$$

Thus, the investment function can be specified as follows:

$$g_i = g_0 + \gamma_u u_{-1} - \gamma_r WAC_{-1} \quad (31)$$

As we can see, investment decision is based on three factors: g_0, which is an exogenous variable that represents the animal spirits of entrepreneurs, responsible for the autonomous investment of the economy, u, which is a proxy for capacity utilization, and *WACC*, which is the weighted average cost of capital[16]. The parameters γ_u and γ_r represent, respectively, the sensitivity of investment to variations in capacity utilization and the sensitivity of investment to variations in the cost of capital.

We use the output to capital ratio as a proxy for the degree of capacity utilization[17]:

$$u = h\frac{y}{k} \tag{32}$$

Its inclusion in the investment function is justified for two reasons: firstly, it represents the standard accelerator effect, and secondly, because it is a monotonically increasing function of the profit rate. Thus, it represents the positive effect of increasing profit rates on the propensity to invest of entrepreneurs (Godley and Lavoie, 2007).

We assume that fixed capital depreciates at a rate equal to δ. Thus, the stock of fixed capital accumulated in a given period is given by:

$$k = (1 - \delta)k_{-1} + i \tag{33}$$

Before analyzing the decision process of firms regarding investment funding, it is necessary to present some accounting identities. So far, we have presented how physical amounts of different variables are determined. Now it is time to transform them into monetary values. This is done by multiplying them by the price index p:

$$S = s * p \tag{34}$$
$$I = i * p \tag{35}$$
$$K = k * p \tag{36}$$

where S, I and K represent, respectively, the monetary values of sales, investment and fixed capital.

The monetary value of inventories is not accounted for by the selling price of products, since they were not actually sold. The accounting should be determined by their production cost. Thus:

$$IN = in * UC \tag{37}$$

According to (11), the total output of the economy is given by total sales and the variation of inventories. Thus, nominal GDP of the economy should be the sum of these two components accounted for by their appropriate prices:

$$Y = s * p + \Delta in * UC + in_{-1} * \Delta UC \tag{38}$$

[16] The weighted average cost is only known after the effective realization of investments. Thus, the relevant weighted average cost is the one observed in the previous period. The same applies to the degree of capacity utilization.

[17] As the output to capital ratio tends to be much less than one, we inserted the parameter h in the equation in order to enable that the value assumed by the degree of capacity utilization is around one in the simulation exercises. We do this in order to better reflect the empirically observed data for this variable.

We can rewrite (38) as it appears in the form of memorandum in Figure 2. Thus, nominal GDP can also be expressed by:

$$Y = C + G + I + \Delta\text{IN} \tag{39}$$

We can now examine the final decision-making process faced by firms: how to finance their investments. According to Figure 2, there are two sources of financing available to firms: retained earnings and bank loans.

Firms' profits can be determined by the flow of transactions in each period. The flow of resources of firms is given by the column of the current account of the firms in Figure 2. Thus, consistent with the required consistency between stocks and flows, firms' profit function can be expressed as:

$$F_f = Y - WB - i_{l_{-1}} L_{-1} \tag{40}$$

From this profit, firms must decide the amount to be used to finance at least part of their investment projects. Thus, the portion of profits retained by firms (FU_f) is given by:

$$FU_f = \psi_f F_f \tag{41}$$

where ψ_f is a behavioral parameter of firms, which can be interpreted as their savings rate. Therefore, the portion of profits distributed to households must be residual:

$$FD_f = F_f - FU_f \tag{42}$$

In general, the amount of retained earnings is not sufficient to finance all their investment projects. According Figure 2, this need for third parties funding must be fully met by bank loans. In this sense, the demand for loans from firms is determined residually, such that:

$$L_d = I + \Delta IN - FU_f \tag{43}$$

The behavioral equations of firms also allow the evaluation of the dynamics of the rate of profit and the profit share. The current rate of profit is defined as the ratio between firms' profits and the nominal value of fixed capital. Profits share is defined as the ratio between firms' profits and the total amount of income generated in each period. These variables can be expressed respectively as follow:

$$q = \frac{F_f}{K} \tag{44}$$

$$m = \frac{F_f}{Y} \tag{45}$$

2.3.2 HOUSEHOLDS

Households are split into three groups according to their income class.

2.3.2.1 Workers

As mentioned above, workers' propensity to save is equal to zero. Thus, they consume all their disposable income (YD_w):

$$C_w = YD_w \quad (46)$$

Workers' disposable income corresponds to their personal income minus taxes paid to the government:

$$YD_w = YP_w - T_w \quad (47)$$

Workers' personal income is given by the wages paid by firms:

$$YP_w = WB \quad (48)$$

Taxes depend on the tax rate θ determined by the government. Taxes are levied on their personal income:

$$T_w = \theta * YP_w \quad (49)$$

Workers' consumption, in physical amount, is given by the deflation of their consumption in nominal terms:

$$c_w = \frac{C_w}{p} \quad (50)$$

2.3.2.2 Productive and financial capitalists

We assume that productive and financial capitalists have the same behavioral parameters. It is hardly necessary, in general, to write two equations to discriminate the same relationship just because the two groups are distinct. When this is the case, to distinguish the notation, we will use the subscript *c* to refer to production capitalists and the subscript *a* to refer to financial capitalists. The subscript *j* is used in equations common to both groups. We have that *j = c, a*.

Capitalists must make two fundamental decisions, which are distinct and temporally sequential. First, they must decide about consumption. After that, they must decide how to allocate their savings, understood as non-consumed income and therefore residual, between the various assets in the economy (Keynes, 1936).

Capitalists decide the physical amount of goods they will consume. Compatible with Modigliani's (1986) life cycle theory capitalists' consumption function depends on their expected real disposable income in the current period and their real stock of wealth accumulated by the previous period, according to (51)[18]:

$$c_j = \alpha_1 yd_j^e + \alpha_2 v_{j_{-1}}\ ;\ j = c, a \quad (51)$$

where c_j represents the physical amount of goods purchased by capitalists in each period, yd^e_j represents the expected real disposable income, v_{j-1} represents the wealth of capitalists in the previous period in real terms, α_1 is a parameter that represents the propensity to consume out

[18] We omit the expression *j=c,a* in next equations in which the subscript *j* appears in order to keep them cleaner.

of income and α_2 is a parameter that represents the propensity to consume out of financial wealth.

The amount of wealth relevant to consumption decision is the one of the preceding period because we assume that consumption decision temporally precedes portfolio allocation decision. Therefore, households know their amount of wealth in the current period only after the production period.

Capitalists' nominal amount of consumption in each period is given by:

$$C_j = c_j * p \tag{52}$$

Following firms' expectation rule, we assume that capitalists' expectation about their real disposable income in each period also follows an adaptive rule. This expectation is a weighted average of the real disposable income in previous period and the real disposable income expectation in previous period. The behavioral parameter ε serves as the weight of the equation:

$$yd_j^e = \varepsilon yd_{j_{-1}} + (1-\varepsilon)yd_{j_{-1}}^e \tag{53}$$

Actual real disposable income *yd* is not simply given by the deflation of nominal disposable income *YD*. Real disposable income is defined as the income that can be consumed without changing real wealth. This implies that wealth depreciation arising from inflation shall be deducted from income. That is, we assume that households do not suffer from monetary illusion. Thus, the effects of inflation over financial wealth matters for their expectations about real disposable income:

$$yd_j = \frac{YD_j}{p} - \pi \frac{V_{j_{-1}}}{p} \tag{54}$$

The first term of equation (54) represents the regular flow of income deflated by current price level. The second term represents inflation loss of the nominal stock of wealth accumulated by the previous period properly deflated by current price level[19].

Nominal disposable income can be obtained from the transactions flow matrix shown in Figure 2. Disposable income of capitalists is given by their personal income YP_j minus the amount of taxes T_j they must pay to the government, such that:

$$YD_j = YP_j - T_j \tag{55}$$

Productive capitalists' and financial capitalists' personal income are given respectively by the portion of firms' profits that is distributed and the profits of banks, such that:

$$YP_c = FD_f \tag{56}$$
$$YP_a = F_b \tag{57}$$

[19] Godley and Lavoie (2007) use this definition of real disposable income. In chapter 9, they give a formal proof that real disposable income does not correspond to deflated nominal disposable income in the absence of monetary illusion.

The amount of taxes paid by capitalists in each period is given by the tax rate θ determined by the government, which levies on the amount of their personal income, such that:

$$T_j = \theta * YP_j \tag{58}$$

Capitalists' real wealth is the deflated value of their nominal wealth:

$$v_j = \frac{V_j}{p} \tag{59}$$

According to Figure 4, the variation of nominal wealth in each period has two components: one relating to changes in stocks arising from transactions and other relating to changes in stocks resulting from capital gains.

The first component corresponds to the savings (SAV_j) of capitalists' households in each period, which is given by their non-consumed disposable income, as shown in Figure 2:

$$SAV_j = YD_j - C_j \tag{60}$$

The second component corresponds to the capital gains (CG_j) detailed in Figure 3. This amount is given by:

$$CG_j = TPRE_{jp_{-1}} * \Delta p_{TPRE} + TPOS_{jp_{-1}} * \Delta p_{TPOS} \tag{61}$$

Therefore, the amount of financial wealth that capitalists hold at the end of each period, as shown in Figure 4, is given by:

$$V_j = V_{j_{-1}} + SAV_j + CG_j \tag{62}$$

We can now analyze the decision making process of households relating to their savings allocation. The behavioral equations that determine households' portfolio allocation are based on Tobin (1969, 1982). We assume that households decide how to allocate their savings between the three assets available in the economy based on two factors: their liquidity preference and their expected return of each asset.

We follow Godley and Lavoie (2007) and take as a proxy for liquidity preference the disposable income to stock of wealth ratio. The greater the disposable income to stock of wealth ratio the higher the liquidity preference of households, due to the fact that a greater amount of wealth should be kept in the form of deposits in order to meet their transactional needs. This implies that deposits are seen as the most liquid asset, while other assets have equal levels of liquidity.

Demand deposits are not remunerated. Thus, their profitability is zero. Government bonds' actual profitability in each period is only known ex-post. Thus, households' portfolio decisions are based on their expectations about the profitability of these assets. The expected return on fixed-rate bonds is given by the expected change in the price of these securities, which follows, like other expectations in the model, an adaptive rule, such that:

$$\Delta p^e_{TPRE} = \xi \Delta p_{TPRE_{-1}} + (1 - \xi) \Delta p^e_{TPRE_{-1}} \tag{63}$$

Floating-rate bonds profitability is directly given by the level of the overnight interest rate. Thus, the expected return on these bonds is given by the expected overnight interest rate, which also follows an adaptive rule:

$$i_{over}^{e} = \xi i_{over_{-1}} + (1-\xi) i_{over_{-1}}^{e} \tag{64}$$

Households purchase demand deposits, fixed-rate government bonds and floating-rate government bonds in each period in order to maintain a certain proportion of their wealth allocated in each of these assets. The parameters λ_{20}^{j}, λ_{30}^{j} and λ_{40}^{j} represent respectively these proportions, such that:

$$M_{jp} = \lambda_{20}^{j} V_j \tag{65}$$
$$TPRE_{jp} * p_{TPRE} = \lambda_{30}^{j} V_j \tag{66}$$
$$TPOS_{jp} * p_{TPOS} = \lambda_{40}^{j} V_j \tag{67}$$

Liquidity preference and the expected profitability of each asset determine the proportion of wealth that households want to maintain in each one of them, such that:

$$\lambda_{20}^{j} = \lambda_1 + \lambda_2 \frac{YD_j}{V_j} + \lambda_3 \Delta p_{TPRE}^{e} + \lambda_4 i_{over}^{e} \tag{68}$$
$$\lambda_{30}^{j} = \lambda_5 + \lambda_6 \frac{YD_j}{V_j} + \lambda_7 \Delta p_{TPRE}^{e} + \lambda_8 i_{over}^{e} \tag{69}$$
$$\lambda_{40}^{j} = \lambda_9 + \lambda_{10} \frac{YD_j}{V_j} + \lambda_{11} \Delta p_{TPRE}^{e} + \lambda_{12} i_{over}^{e} \tag{70}$$

The increase in the profitability of an asset increases the demand for that asset. Thus, the increase in the profitability of any asset will necessarily reduce the demand for other assets. This implies that the profitability of each asset should enter all equations[20].

We need to specify the evolution of the stock of each asset in order to get to their demand function. We assume, as will be discussed later, that households' demand for financial assets is always met. Thus, the stock of such assets in the current period is given by:

$$M_{jp} = M_{jp_{-1}} + M_{jd} \tag{71}$$
$$TPRE_{jp} = TPRE_{jp_{-1}} + TPRE_{jd} \tag{72}$$
$$TPOS_{jp} = TPOS_{jp_{-1}} + TPOS_{jd} \tag{73}$$

In periods where there is maturity of government bonds, their stock comes to zero and the whole amount of the resources arising from this process is used to buy new bonds of same characteristics. Once the selling price of old bonds and the purchase price of the new ones are different, we must distinguish between the demand for government bonds arising from the rollover process from regular demand. Accordingly, in multiples of four periods, equations (72) and (73) should be rewritten as:

[20] The λ parameters of equations (68)-(70) are subject to adding-up constraints (Tobin, 1969). Godley and Lavoie (2007) explain and detail these constraints. All these constraints were observed in the model calibration.

$$TPRE_{jp} = TPRE_{jd}^{rol} + TPRE_{jd} \quad (72a)$$
$$TPOS_{jp} = TPOS_{jd}^{rol} + TPOS_{jd} \quad (73a)$$

Accordingly Figure 2, the demand for government bonds arising from the rollover process is given by:

$$TPRE_{jd}^{rol} = \begin{cases} \dfrac{p_{TPRE}^{v} * TPRE_{jp_{-1}}}{p_{TPRE}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ 0 \quad , caso\ contrário \end{cases} \quad (74)$$

$$TPOS_{jd}^{rol} = \begin{cases} \dfrac{p_{TPOS}^{v} * TPOS_{jp_{-1}}}{p_{TPOS}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ 0 \quad , caso\ contrário \end{cases} \quad (75)$$

Households' regular demand for each of the three assets depends on the level of their savings. The proportion of savings allocated on each of the assets, which must meet the restrictions imposed by equations (65)-(67) is given respectively by:

$$M_{jd} = \lambda_{M}^{j} SAV_{j} \quad (76)$$
$$TPRE_{jd} * p_{TPRE} = \lambda_{TPRE}^{j} SAV_{j} \quad (77)$$
$$TPOS_{jd} * p_{TPOS} = \lambda_{TPOS}^{j} SAV_{j} \quad (78)$$

Replacing (65)-(67) and (76)-(78) in (71)-(73)[21], we get the values of the parameters λ_{M}^{j}, λ_{TPRE}^{j} and λ_{TPOS}^{j} that are compatible with portfolio allocation:

$$\lambda_{M}^{j} = \frac{\lambda_{20}^{j} V_{j} - M_{jp_{-1}}}{SAV_{j}} \quad (79)$$

$$\lambda_{TPRE}^{j} = \begin{cases} \dfrac{\lambda_{30}^{j} V_{j} - TPRE_{jp_{-1}} * p_{TPRE}^{v}}{SAV_{j}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ \dfrac{\lambda_{30}^{j} V_{j} - TPRE_{jp_{-1}} * p_{TPRE}}{SAV_{j}} & , otherwise \end{cases} \quad (80)$$

$$\lambda_{TPOS}^{j} = \begin{cases} \dfrac{\lambda_{40}^{j} V_{j} - TPOS_{jp_{-1}} * p_{TPOS}^{v}}{SAV_{j}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ \dfrac{\lambda_{40}^{j} V_{j} - TPOS_{jp_{-1}} * p_{TPOS}}{SAV_{j}} & , otherwise \end{cases} \quad (81)$$

Finally, we have to specify the total demand for demand deposits:

$$M_{d} = M_{cd} + M_{ad} \quad (82)$$

And also total households' consumption, both in physical and in nominal amounts:

[21] In periods where there is maturity of bonds, equations (66)-(67) and (77)-(78) should be replaced in equations (72a)-(73a).

$$c = c_w + c_c + c_a \quad (83)$$
$$C = c * p \quad (84)$$

2.3.3 GOVERNMENT

2.3.3.1 Ministry of Finance

Taxes paid by households are the only source of government resources. Taxes total amount is given by:

$$T = T_w + T_c + T_a \quad (85)$$

We assume that government spending in real terms grows at an exogenous rate g_g in each period, such that:

$$g = (1 + g_g)g_{-1} \quad (86)$$

Government spending in nominal terms is given by:

$$G = g * p \quad (87)$$

Besides these expenditures, the government has regular expenses with the debt service, both amortization and interest, arising from sales of public bonds by banks in each period. Thus, government savings, defined as the difference between revenue flows and expenditure flows, is given according Figure 2 by:

$$SAV_g = T - G - TPRE_{bp}^{v} * p_{TPRE}^{v} - TPOS_{bp}^{v} * p_{TPOS}^{v} \quad (88)$$

Public sector borrowing requirement (*PSBR*) is the symmetric of the amount of government savings, such that:

$$PSBR = -SAV_g \quad (89)$$

A positive *PSBR* corresponds to a fiscal deficit. In the model, fiscal deficits must be fully financed in each period through the issuance of public bonds. The Ministry of Finance should decide the quantities of each type of security to be issued in order to finance these deficits. The parameter λ_g represents the portion of the deficit that is financed through the issuance of floating-rate bonds, such that:

$$TPOS_s * p_{TPOS} = \lambda_g * NFSP \quad (90)$$
$$TPRE_s * p_{TPRE} = (1 - \lambda_g) * NFSP \quad (91)$$

We assume that the variation in the stock of public bonds corresponds to the quantity of securities issued in each period, such that:

$$TPOS = TPOS_{-1} + TPOS_s \quad (92)$$
$$TPRE = TPRE_{-1} + TPRE_s \quad (93)$$

In the maturity of the bonds, their stock is given by the sum of the usual issuance of bonds and the bonds issued due to the rollover process. Thus, equations (92)-(93) must be rewritten as:

$$TPOS = TPOS_s + TPOS_s^{rol} \tag{92a}$$
$$TPRE = TPRE_s + TPRE_s^{rol} \tag{93a}$$

The issuance of bonds due to the rollover process is given by:

$$TPOS_s^{rol} = \begin{cases} \dfrac{p_{TPOS}^{v}\left(TPOS_{-1} - TPOS_{bp}^{v}\right)}{p_{TPOS}} & ,if\ t\ is\ a\ multiple\ of\ 4 \\ 0 & ,otherwise \end{cases} \tag{94}$$

$$TPRE_s^{rol} = \begin{cases} \dfrac{p_{TPRE}^{v}\left(TPRE_{-1} - TPRE_{bp}^{v}\right)}{p_{TPRE}} & ,if\ t\ is\ a\ multiple\ of\ 4 \\ 0 & ,otherwise \end{cases} \tag{95}$$

Public debt (*DP*) in nominal terms in each period can be expressed according Figure 1 by:

$$DP = TPOS * p_{TPOS} + TPRE * p_{TPRE} \tag{96}$$

The parameter λ_g is determined by a public debt management rule that reflects the government preference to issue one or other type of public bond. Due to the characteristics of floating-rate bonds, we assume that the government has a target of the portion of these bonds in public debt, such that:

$$TPOS * p_{TPOS} = \Phi^T * DP \tag{97}$$

The parameter Φ^T can be interpreted as a public debt management policy parameter. Its value is determined by the preferences of the Ministry of Finance. Substituting (92), (93) and (96) in (97)[22], we have after some algebraic manipulation the value of the parameter λ_g for which the public debt management rule is satisfied:

$$\lambda_g = \frac{\Phi^T * TPRE_{-1} * p_{TPRE} - (1 - \Phi^T)TPOS_{-1} * p_{TPOS}}{NFSP} + \Phi^T \tag{98}$$

This parameter takes the following value in maturity periods:

$$\lambda_g = \frac{\Phi^T * TPRE_s^{rol} * p_{TPRE} - (1 - \Phi^T)TPOS_s^{rol} * p_{TPOS}}{NFSP} + \Phi^T \tag{98a}$$

The implicit assumption in these equations is that all the bonds issued by the government are bought, which implies that the demand for bonds must adjust to their supply. Therefore, there is no excess or shortage of public bonds supply. Thus, bonds prices are not an adjustment mechanism between supply and demand. Bonds prices are previously determined by the government. They do not reflect supply and demand conditions in bonds market, as previously explained.

[22] Equations (92a) and (93a) should respectively replace equations (92) and (93) in maturity periods.

2.3.3.2 Central bank

Although central bank has no regular flows of income and expenditure, their stocks of assets and liabilities vary in each period as a result of transactions made by other sectors of the economy. As mentioned earlier, the issuance of bonds in each period is constrained by the size of the fiscal deficit. Therefore, there is a constraint on the supply side. Central bank role in bonds market is to equalize eventual distortions between supply and demand. That is, central bank acts as a market maker in bonds market. In this sense, it either buys the entire amount of bonds that did not match households and banks demand or sell any amount of bonds demanded by households and banks that was not supplied by the government. These cases reflect respectively positive and negative demands of government bonds by central bank. This demand can be expressed as follows:

$$TPRE_{BCd} = TPRE_s - TPRE_{cd} - TPRE_{ad} - TPRE_{bd} \tag{99}$$
$$TPOS_{BCd} = TPOS_s - TPOS_{cd} - TPOS_{ad} - TPOS_{bd} \tag{100}$$

The variation in the stock of public bonds held by central bank corresponds to its demand, such that:

$$TPRE_{BCp} = TPRE_{BCp_{-1}} + TPRE_{BCd} \tag{101}$$
$$TPOS_{BCp} = TPOS_{BCp_{-1}} + TPOS_{BCd} \tag{102}$$

Equations (101) and (102) should be replaced in maturity periods by:

$$TPRE_{BCp} = TPRE_{BCd} + TPRE_{BCd}^{rol} \tag{101a}$$
$$TPOS_{BCp} = TPOS_{BCd} + TPOS_{BCd}^{rol} \tag{102a}$$

Central bank demand for bonds due to the rollover process is given, according to Figure 2, by:

$$TPRE_{BCd}^{rol} = \begin{cases} \dfrac{p_{TPRE}^{v} * TPRE_{BCp_{-1}}}{p_{TPRE}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ 0 & , otherwise \end{cases} \tag{103}$$
$$TPOS_{BCd}^{rol} = \begin{cases} \dfrac{p_{TPOS}^{v} * TPOS_{BCp_{-1}}}{p_{TPOS}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ 0 & , otherwise \end{cases} \tag{104}$$

The availability of bank reserves must meet the budget constraint given in Figure 2, such that:

$$H_s = TPRE_{BCd} * p_{TPRE} + TPOS_{BCd} * p_{TPOS} \tag{105}$$

This implies, as shown in Figure 4, that the amount added to central bank's net worth in each period corresponds to its capital gains, such that:

$$V_{BC} = V_{BC_{-1}} + CG_{BC} \tag{106}$$

Central bank's capital gains are shown in Figure 3:

$$CG_{BC} = TPRE_{BCp_{-1}} * \Delta p_{TPRE} + TPOS_{BCp_{-1}} * \Delta p_{TPOS} \tag{107}$$

Now we must define the way in which the overnight interest rate is determined in order to complete the analysis of the role of central bank in the economy. Central bank determines this rate by means of a reaction function based on Taylor rule (Taylor, 1993). This implies that the overnight interest rate is determined endogenously. We assume that central bank adopts an inflation targeting regime in which it sets the interest rate as a reaction to deviations of inflation relative to its target and output relative to its potential, such that:

$$i_{over} = \kappa * i_{over_{-1}} + (1-\kappa)\left[(r+\pi^*) + \mu_1(\pi_{-1} - \pi^*) + \mu_2\left(g_{gdp_{-1}} - g^*_{gdp}\right)\right] \quad (108)$$

We have that $\pi*$ represents inflation target, which is a monetary policy parameter set by the government. We assume that potential output is a technical data and that it grows at a real rate g^*_{gdp} exogenously determined. Parameters μ_1 and μ_2 are set by central bank. They represent respectively the sensitivity to deviations of inflation relative to its target and the sensitivity to deviations of output relative to its potential. The parameter κ is introduced to smooth the variations of nominal interest rate. It weights the interest rate prevailing in the previous period and the nominal interest rate of equilibrium. This rate is composed of the equilibrium real interest rate r, the inflation target and central bank reaction to deviations of inflation relative to its target and of output relative to its potential. The equilibrium real interest rate is determined by social convention, i.e., it is the real interest rate that agents believe that should prevail in the long run[23]. In the model, it is exogenously determined. Inflation π is by definition calculated as the percentage change in price levels, such that:

$$\pi = \frac{p - p_{-1}}{p_{-1}} \quad (109)$$

2.3.4 BANKS

According to Figure 2, we assume that banks have two sources of revenue: interests paid by firms over their borrowing and the debt service paid by the government for bonds that are sold in each period. We also assume that banks have no flow of expenditures. Therefore their profit, which is fully distributed to financial capitalists' households, is given by the sum of these two sources of revenue:

$$F_b = i_{l_{-1}} L_{-1} + TPRE^v_{bp} * p^v_{TPRE} + TPOS^v_{bp} * p^v_{TPOS} \quad (110)$$

Besides their transactions flow, we should analyze the variation of assets and liabilities in each period. Their only liability is demand deposits of households. We assume, consistent with the real world, that banks are depository institutions. Therefore they receive any households' demand deposit. This implies that the "supply" of deposits should always be equal to its demand. Thus:

$$M_s = M_d \quad (111)$$

[23] The term "convention" is used in the sense used by Keynes (1936). It can be defined as a collective behavior rule which is widely used by economic agents.

Equation (111) implies that the variation in the stock of demand deposits is given by its demand, such that:

$$\Delta M = M_d \tag{112}$$

Banks have four assets: bank reserves, loans, floating-rate public bonds and fixed-rate public bonds. The demand for bank reserves has a compulsory component and a volunteer component. The compulsory component is given by the reserve requirements rate ρ_1 imposed by central bank on their stock of demand deposits. The volunteer component is given by the liquidity preference of banks ρ_0. Therefore this demand can be expressed as follows:

$$H_d = (\rho_0 + \rho_1)M_s \tag{113}$$

We follow Moore's (1988) horizontalist approach and assume that the supply of credit is infinitely elastic, i.e., banks lend money on demand[24]. Accordingly, we have that:

$$L_s = L_d \tag{114}$$

This hypothesis implies that the variation in the stock of loans in each period is given by its demand, such that:

$$\Delta L = L_d \tag{115}$$

Now we have to define the demand for floating-rate and fixed-rate public bonds. Defining a method of determining the demand for one of these assets, the other must be residually determined. Therefore banks' demand for public bonds is given in nominal terms by:

$$TPOS_{bd} * p_{TPOS} + TPRE_{bd} * p_{TPRE} = M_s - L_s - H_d \tag{116}$$

The demand for floating-rate bonds represents a fraction λ_b of that amount, while the demand for fixed-rate bonds represents a fraction $(1 - \lambda_b)$, such that:

$$TPOS_{bd} * p_{TPOS} = \lambda_b(M_s - L_s - H_d) \tag{117}$$
$$TPRE_{bd} * p_{TPRE} = (1 - \lambda_b)(M_s - L_s - H_d) \tag{118}$$

We assume that banks choose this ratio based on the expected profitability of each bond, just like households:

$$\lambda_b = \lambda_{13} + \lambda_{14}\Delta p^e_{TPRE} + \lambda_{15} i^e_{over} \tag{119}$$

Once banks sell part of their stock of bonds in each period, the stock of floating-rate and fixed-rate government bonds they hold in the end of each period is given respectively by:

$$TPRE_{bp} = TPRE_{bp_{-1}} - TPRE^v_{bp} + TPRE_{bd} \tag{120}$$

[24] This assumption implies that, unlike observed in modern economies, firms do not face financial constraints and that therefore there is no credit rationing in the economy. Since the goal of the model does not require theoretical constructs about these restrictions, we exclude this issue in order to simplify it and keep the focus on government bonds. An approach that takes into account aspects related to financial constraints faced by firms can be found in Sarquis and Oreiro (2011). Taking this aspect into consideration also have direct consequences on the specification of the investment function of the economy.

$$TPOS_{bp} = TPOS_{bp_{-1}} - TPOS_{bp}^{v} + TPOS_{bd} \tag{121}$$

In maturity periods, equations (120) and (121) shall be expressed by:

$$TPRE_{bp} = TPRE_{bd} + TPRE_{bd}^{rol} \tag{120a}$$
$$TPOS_{bp} = TPOS_{bd} + TPOS_{bd}^{rol} \tag{121a}$$

The amount of bonds demanded due to the debt rollover process is given by:

$$TPRE_{bd}^{rol} = \begin{cases} \dfrac{p_{TPRE}^{v}\left(TPRE_{bp_{-1}} - TPRE_{bp}^{v}\right)}{p_{TPRE}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ 0 \quad , otherwise \end{cases} \tag{122}$$

$$TPOS_{bd}^{rol} = \begin{cases} \dfrac{p_{TPOS}^{v}\left(TPOS_{bp_{-1}} - TPOS_{bp}^{v}\right)}{p_{TPOS}} & , if\ t\ is\ a\ multiple\ of\ 4 \\ 0 \quad , otherwise \end{cases} \tag{123}$$

The amount of bonds sold in each period is given by a fraction η of the stock of each type of bond in banks' portfolio in the previous period. This fraction can be interpreted as a behavioral parameter chosen by banks:

$$TPRE_{bp}^{v} = \eta * TPRE_{bp_{-1}} \tag{124}$$
$$TPOS_{bp}^{v} = \eta * TPOS_{bp_{-1}} \tag{125}$$

Just like the central bank, these equations imply according Figure 4 that the amount added to banks’ net worth in each period corresponds to their capital gains, such that:

$$V_{b} = V_{b_{-1}} + CG_{b} \tag{126}$$

Where:

$$CG_{b} = TPRE_{bp_{-1}} * \Delta p_{TPRE} + TPOS_{bp_{-1}} * \Delta p_{TPOS} \tag{127}$$

To complete the specification of behavioral equations, we must define how the interest rate on loans is determined. We assume that banks set the interest rate on loans based on a mark-up φ_b over the overnight interest rate set by the central bank, such that:

$$i_{l} = (1 + \varphi_{b}) i_{over} \tag{128}$$

Once the level of the interest rate on loans is not a limiting factor for the amount of loans demanded, banks have complete freedom to set this mark-up. We assume that banks want to maintain a stable interest to income ratio. Thus they have a target of the share of interests on GDP (ζ^T), such that:

$$\frac{i_{l_{-1}} L_{-1}}{Y} = \zeta^{T} \tag{129}$$

Forwarding (129) in one period in time and substituting (128) in the resulting, we get the banking mark-up compatible with this target:

$$\varphi_b = \frac{\zeta^T * Y_{+1}}{i_{over} * L} - 1 \quad (130)$$

Since we cannot know a priori the value of next period GDP, banks use their expectations. Thus (130) can be rewritten as:

$$\varphi_b = \frac{\zeta^T * Y_{+1}^e}{i_{over} * L} - 1 \quad (130a)$$

Just like all model expectations, banks also follow an adaptive rule. The only difference is that this expectation is formed to a variable of a later period. We assume that banks use the same potential output growth rate used by the central bank to form their expectation, such that:

$$Y_{+1}^e = \left(1 + g_{pib}^*\right)(\upsilon Y_{-1} + (1 - \upsilon)Y_{-1}^e) \quad (131)$$

Where υ is banks' reaction parameter related to expectations.

2.3.5 THE REDUNDANT EQUATION

Bank reserves' demand and supply are independent processes. At first, nothing could guarantee that these two variables settle. However, in SFC approach, the flow of transactions in a given sector is entirely determined by the flow of transactions in other sectors. Consequently, this consistency requirement always implies a redundant equality, i.e. an equation that do not need to be introduced into the model as it is logically implied by all other equations taken together. The equality between demand and supply of bank reserves is the redundant equation of this model:

$$H_s = H_d \quad (132)$$

3 CALIBRATION AND BASE SCENARIO

There are 54 parameters and 147 endogenous variables in the model. It is necessary to assign values to all parameters and to establish initial conditions for the endogenous variables in order to form the base scenario of the economy. All these values were established using the calibration method[25] and following the usual method used by SFC models as shown in Godley and Lavoie (2007).

This method is supported on Samuelson's (1947) correspondence principle. According to him, during calibration the researcher may be faced with the absence of any precise quantification of parameter values and initial conditions of a dynamic system. However, he needs to analytically infer the motion of a complex system. In this situation, the researcher must set the parameter values in order to establish a realistic correspondence between the static variables (parameters) and the dynamic variables (dependent variables).

[25] The method of calibration can be defined as a process of handling the initial conditions and the parameters of a given model in order to obtain a plausible combination between the data empirically observed and the simulated results (Hansen and Heckman, 1996).

Accordingly, we established the following calibration methodology: (i) we assigned an initial set of parameter values and initial conditions trying whenever possible to use empirically plausible estimates, (ii) we simulated the model computationally to obtain the dynamic path of the endogenous variables, (iii) we checked whether the dynamic path replicated some general properties or stylized facts observed in modern economies, (iv) if the dynamic path generated by the initial set of parameters is not empirically plausible, i.e. if they are not in accordance with the stylized facts of capitalist dynamics, we must choose a new set of values and repeat the experiment.

This methodology is subject to two fundamental questions. The first refers to the moment in which the researcher must end its search for a plausible set of parameters. The second issue is related to the stylized facts of capitalist dynamics which are selected as benchmarks for the dynamic paths generated by the theoretical model.

Regarding the first question, it should be noted that there is no objective criterion to determine what time the researcher must end its search for a plausible set of parameters. In this context, the researcher has no other option but to use Simon’s (1976) bounded rationality. Even if there are better parameters, the researcher must end his quest when he founds a set of parameters that are able to generate plausible paths that are adherent to those observed in the real world.

A common criticism to this procedure is that the complexity of simulation models in conjunction with the existence of free parameters, i.e. parameters whose values cannot be empirically estimated on a precise basis, provide the model builder with an almost infinite degree of freedom in obtaining the desired results. In other words, the researcher could get virtually any results from his/her theoretical model provided he/she has the time and patience needed to test several sets of parameter values until he obtains a set of values that gives the desired result.

However, this criticism is not correct. Indeed, obtaining a “good” set of parameters involves a trial and error process in which the results obtained from a certain numerical model specification are contrasted with a series of stylized facts. However, the degrees of freedom of the researcher can be substantially reduced if the number of stylized facts to be explained is large enough. In this context, a “bad” model, i.e. a model that abstracts some essential aspects of economic reality, is not able to explain a good amount of these stylized facts, whatever set of values chosen for the parameters and initial conditions.

Therefore, selecting a fairly large number of stylized facts about the dynamics of capitalist economies becomes an essential element not only to perform a good calibration process of the model parameters, but also as a criterion for judging the relevance and the plausibility of the model being presented.

Table 1 shows the final value assigned to all model parameters defined on the basis of this method.

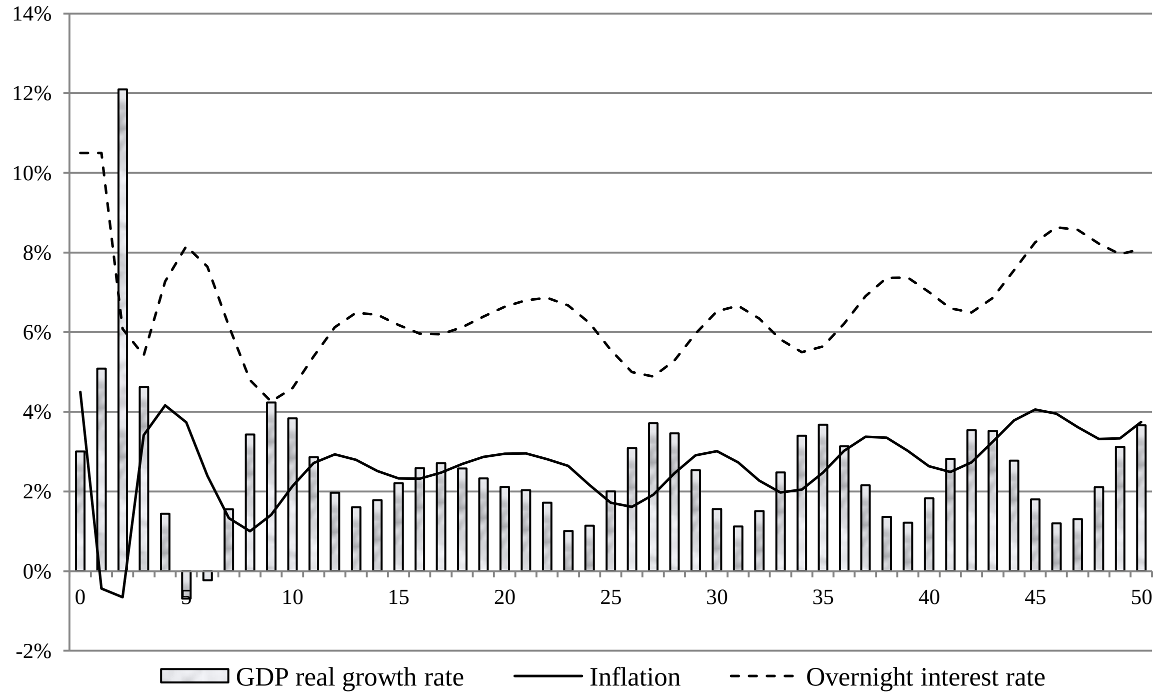
Table 1 – Values assigned to model parameters

α_1	0.94	μ_1	2	Ω_2	0.15	λ_{10}	-0.1
α_2	0.05	μ_2	0.5	Ω_3	1	λ_{11}	-0.1
β	0.8	$\pi*$	0.045	ξ	0.8	λ_{12}	0.2
δ	0.035	θ	0.1175	ψ_f	0.35	λ_{13}	0.5
ε	0.8	ρ_0	0.02	ζ^T	0.05	λ_{14}	-0.5
Φ^T	0.5	ρ_1	0.2	λ_1	0.4	λ_{15}	0.5
γ_{in}	1	RP_0	0.01	λ_2	0.2	$g*_{gdp}$	0.03
γ_r	0.3	RP_1	0.02	λ_3	-0.1	g_0	0.03
γ_u	0.015	σ^T	0.2	λ_4	-0.1	g_g	0.025
η	0.1	τ_0	0	λ_5	0.3	g_N	0.01
φ_0	0.53	τ_1	0.6	λ_6	-0.1	r	0.06
φ_1	0.5	τ_2	0.1	λ_7	0.2	h	3
φ_2	0.8	υ	0.8	λ_8	-0.1		
κ	0.5	Ω_1	0.9	λ_9	0.3		

Source: Authors

From these values and the definition of the initial conditions, we simulated in computer the behavior of this economy over fifty periods. The behavior of GDP real growth rate, inflation and overnight interest rate can be seen in Figure 5.

Figure 5 – Baseline scenario: GDP real growth rate, inflation and overnight interest rate



Source: Authors

The macrodynamics of the model is as follows: the process of economic growth is accompanied by a rise in inflation. Following a Taylor rule, central bank raises the overnight interest rate in response to rising inflation. Higher inflation and higher interest rates have two effects on economic growth. Firstly, the increase in the overnight interest rate increases the weighted average cost of capital, which decreases the growth rate of investment in the economy. Secondly, the rise in inflation causes a decrease in the real disposable income of households. Once they do not suffer from monetary illusion, consumption growth rate decreases. These two factors explain the periods of decline in the growth rate of the economy. Once this lower growth rate impacts on inflation and hence on the overnight interest rate, a new cycle of accelerating GDP growth rate restarts.

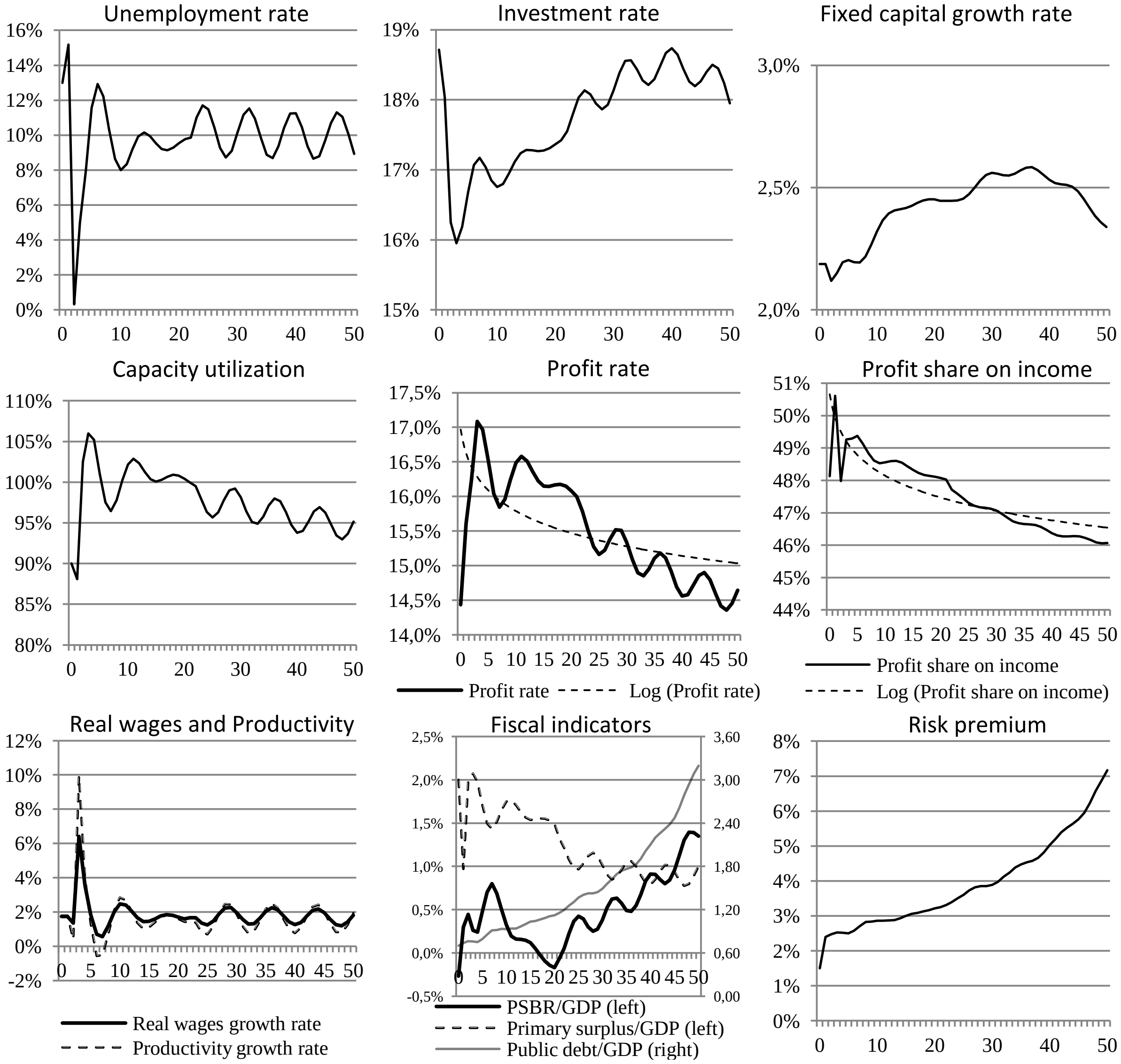
The model is able to reflect a reasonable number of empirical regularities. Befitting the most important stylized fact of modern economies according to Blanchard and Fischer (1989), this artificial economy has persistent economic growth with large fluctuations. Associated with this more general dynamic, we can observe fluctuations in the unemployment rate over the simulation period.

Moreover, with this dynamic in which the interest rate remains in a relatively high level, the investment rate of the economy cannot achieve a level in which it can sustain and induce a more robust economic growth. The rate of investment, in turn, causes the growth rate of fixed capital to remain more or less stable. As the growth rate of fixed capital is close to GDP growth rate, the degree of capacity utilization can remain close to its full capacity. Nevertheless, operating with underutilization of productive capacity is the characteristic of this economy. Given these features, both the rate of profit and the share of profits on income are decreasing over the simulated period. However, the two values tend to stabilize in the long run. This result corroborates stylized facts pointed by Kaldor (1957a).

Another stylized fact of modern economies emphasized by Kaldor (1957b) is that real wages tend to grow at the same rate of growth of labor productivity. This stylized fact is also reproduced by the model dynamics.

Relatively to the fiscal policy, once we assumed that government expenditures are exogenous, there is no endogenous mechanism to control public spending. This implies that primary surpluses tend to be smaller and nominal deficits larger, causing an upward trend of the government debt to GDP ratio over time. This deterioration in fiscal indicators translates into a rise in the risk premium associated with the issuance of fixed-rate bonds. However, due to the existence of floating-rate bonds, such deterioration is unable to destabilize the economy over the simulated period. The behavior of all these variables can be seen in Figure 6.

Figure 6 – Dynamics of the main variables of the model



Source: Authors

It seems reasonable to assume that the model can reproduce fairly well most of the stylized facts observed in modern economies. Given the methodology proposed here, this reasonable number of empirical regularities that the model can reproduce enables us to qualify it as a good model. Now we can present the computer simulations, which main objective is to observe the dynamics of the economy resulting from a change in the public debt management policy.

4 SIMULATIONS

4.1 Simulation: end of floating-rate public bonds

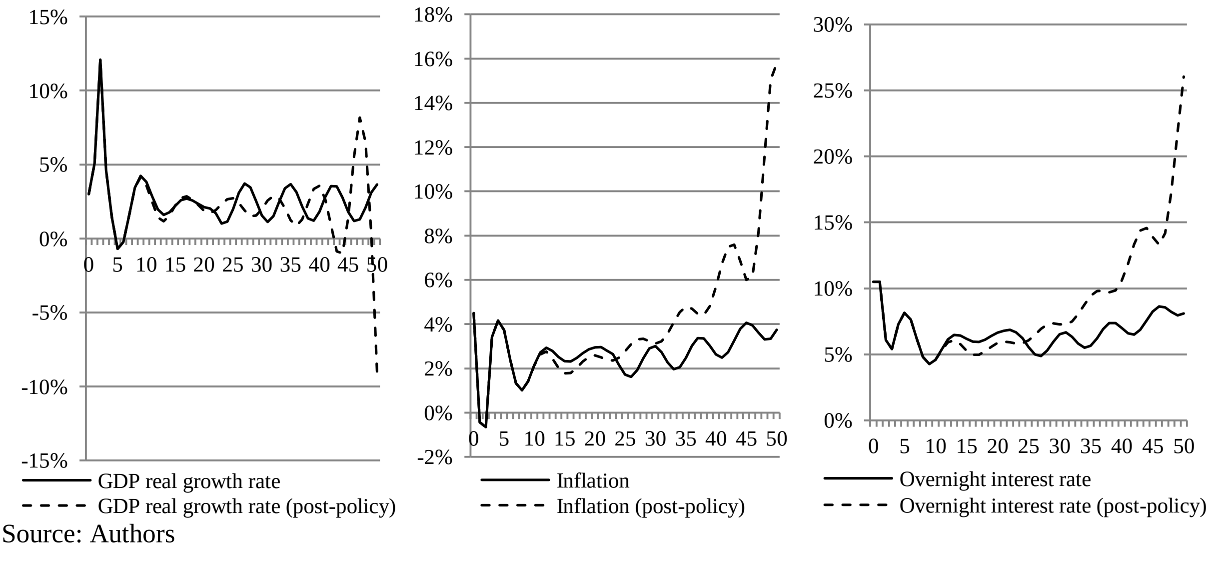
We discussed throughout the paper the possible harm to the dynamics of the Brazilian economy associated with the existence of public bonds remunerated by the overnight interest rate, which is used as an operational target of monetary policy by the central bank in an inflation targeting regime. In this sense, a lower share of such bonds in public debt – or even their extinction – would benefit the Brazilian economy. However, there are potential costs associated with an eventual change in government’s debt management policy.

The simulations were designed to observe the behavior of the economy modeled in section 2 arising from changes in some policy parameters. This behavior is compared to the dynamics of the economy in the baseline scenario, which was described in the previous section.

The aim of the simulation was to analyze the effects on the dynamics of the economy arising from a shock in public debt management. Given the potential benefits associated with a public debt composed solely by fixed-rate bonds, we assumed that the government stops issuing floating-rate bonds in the eighth period. Besides that, the government also replaces all pre-existing floating-rate bonds by fixed-rate bonds. That is, the stock of floating-rate bonds comes to zero in the eighth period. This policy change means that the parameter Φ^T comes abruptly from 0.5 in period seven to 0 in period eight[26].

The change in the dynamics of GDP real growth rate, inflation and overnight interest rate arising from this policy change can be observed in Figure 7. Two considerations regarding the changes in the dynamics of these three variables can be highlighted. The first consideration is that a debt management policy that aims to reset the stock of floating-rate bonds while keeping all other policy parameters constant leads to a process of destabilization of the economy. GDP real growth rate becomes more unstable, alternating periods of high economic growth with periods of low growth or even recession. Inflation tends to go out of control and the monetary policy based on a Taylor rule, keeping all its parameters constant, is not able to contain this process. On the contrary, the maintenance of higher interest rates can explain the greater instability of economic growth cycles, once it influences the behavior of the investment rate of the economy, which becomes increasingly smaller.

Figure 7 – Simulation: GDP real growth rate, inflation and overnight interest rate

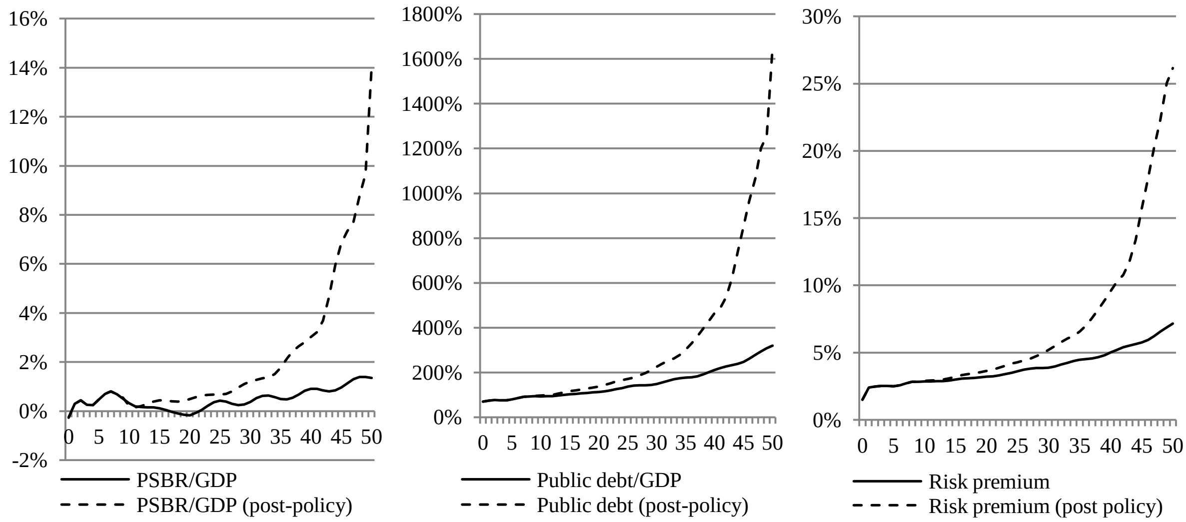


Source: Authors

[26] Some adjustments must be made in some parameters in order to make this policy compatible with the structure of the model. As the government stops issuing these bonds, we must adjust all parameters relating to floating-rate bonds' demand in order to make this demand also equal to zero. We assume that the entire floating-rate bonds' demand of households and banks is transferred to fixed-rate bonds. Furthermore, the weighted average cost of capital must have its calculation modified. With the availability of floating-rate bonds, the opportunity cost of firms was based on the overnight interest rate. Without this availability, the opportunity cost of firms should be based on the interest rate of fixed-rate bonds.

The main factor responsible for this destabilization of the economy is the fiscal deterioration of the government resulting from the policy change. Once fixed-rate bonds have, as a rule, higher costs than floating-rate bonds, increasing expenditures lead to increasing fiscal deficits. The greater need to issue bonds to finance the government increases public debt to GDP ratio, which, in turn, increases risk premium and as a consequence fixed-rate bonds interest rate. Once they represent now the entire debt, public debt's service cost becomes even higher, which increasingly contributes to growing fiscal deficits, restarting the cycle. The changes in the dynamics of these variables after the policy change can be observed in Figure 8.

Figure 8 – Simulation: fiscal indicators



Source: Authors

The second consideration refers to the time required, after the policy change, for destabilizing the economy. As can be seen in Figure 7, after the eighth period, in which the policy change is realized, the peaks of the cycles of economic growth become smaller and both inflation and the overnight interest rate stay at levels below the ones in the baseline scenario. This situation only begins to destabilize between periods 20 and 25, i.e. at least twelve periods after the policy change, when increasing public deficits begin to have destabilizing effects on the economy. Only when the vicious cycle described above begins, the economy destabilizes and is not able to endogenously return to a balanced path.

Therefore, the change in the debt management policy brings no immediate negative results for the dynamics of the system. In fact, in the short run, this policy is associated with more stable growth cycles, lower inflation and lower overnight interest rates. However, due to the rising costs associated with fixed-rate bonds, this policy is not sustainable in the long run. It is necessary that the government adjusts its macroeconomic policy in order to keep the economy on a balanced growth path and at the same time get the benefits associated with maintaining a public debt formed only by fixed-rate bonds. The next subsection analyzes the macroeconomic policy mix available to the government to stabilize the economy in the long run[27].

[27] Besides this simulation, we also simulated changes in the dynamics of this economy resulting from a debt management policy whose target was to gradually reduce the share of floating-rate bonds in public debt. Qualitatively, the same results described in this section were observed. The difference is that the economy takes a longer period to get on an instability path and that the levels of inflation and interest rates are somewhat smaller.

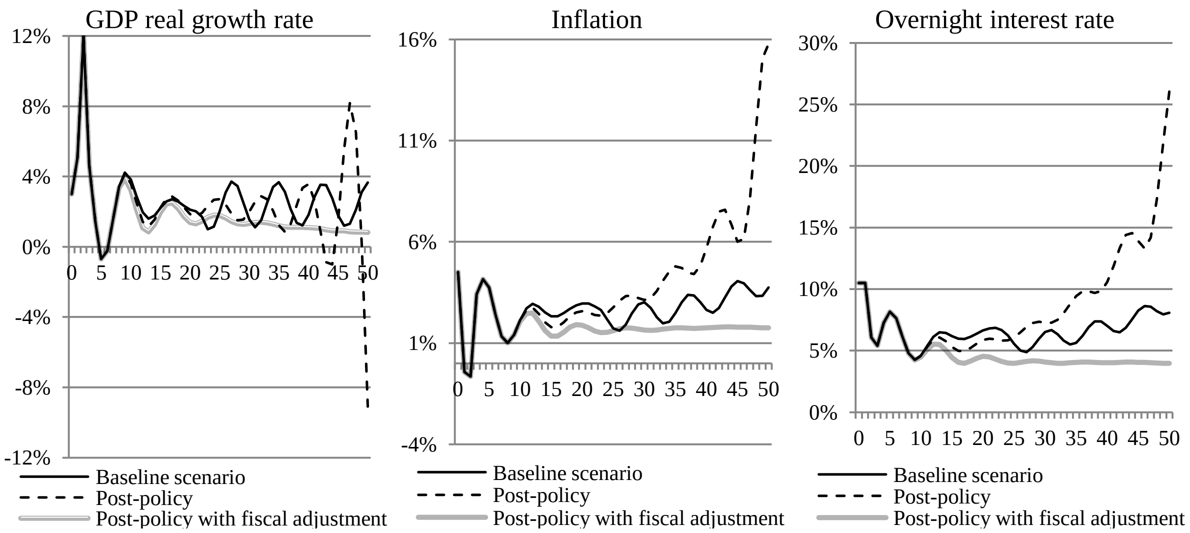
4.2 Adjustments in economic policy

Although the government can follow a mix of macroeconomic policy, we choose to present the effect of each policy separately in order to isolate its effects. Attempts to stabilize the economy can come from three different macroeconomic policies: fiscal policy, monetary policy and income policy[28].

4.2.1 FISCAL POLICY

Firstly, we assume that the government carries out a fiscal adjustment through a reduction in the growth rate of their spending from the eighth period[29]. A contractionary fiscal policy can stabilize inflation and the level of the overnight interest rate. The downside of this policy is that, despite achieving the stabilization of business cycles, it is associated with low GDP real growth rates, as can be seen in Figure 9.

Figure 9 – Debt management policy change with fiscal adjustment: GDP real growth rate, inflation and overnight interest rate



Source: Authors

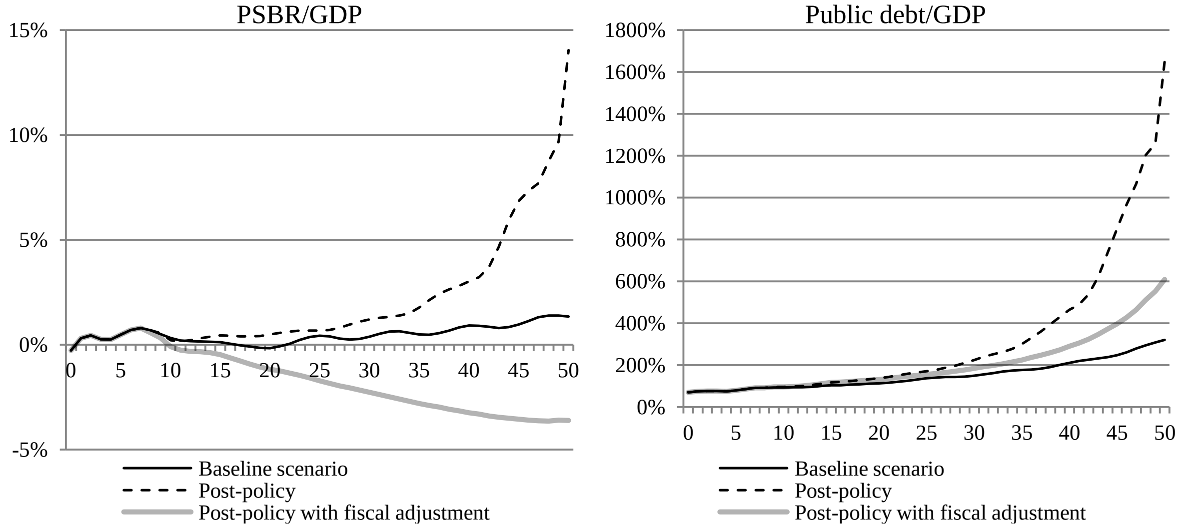
The slowdown of government spending allows the government to achieve fiscal surpluses. However, the higher cost of public debt and the low economic growth lead the public debt to GDP ratio to a path of growth higher than in the baseline scenario, as can be seen in Figure 10.

---

28 In order to assess the robustness of the results of these simulations, we also simulated the model using some other parameter sets. Despite these other sets slightly modified the system dynamics, the effects of changes in economic policy remained in qualitative terms. Therefore, simulation results do not depend on the set of calibrated parameters, but rather stem from the dynamics of the model.

29 In the simulation, the government decreases the parameter g_g to 0.01.

Figure 10 – Debt management policy change with fiscal adjustment: fiscal indicators

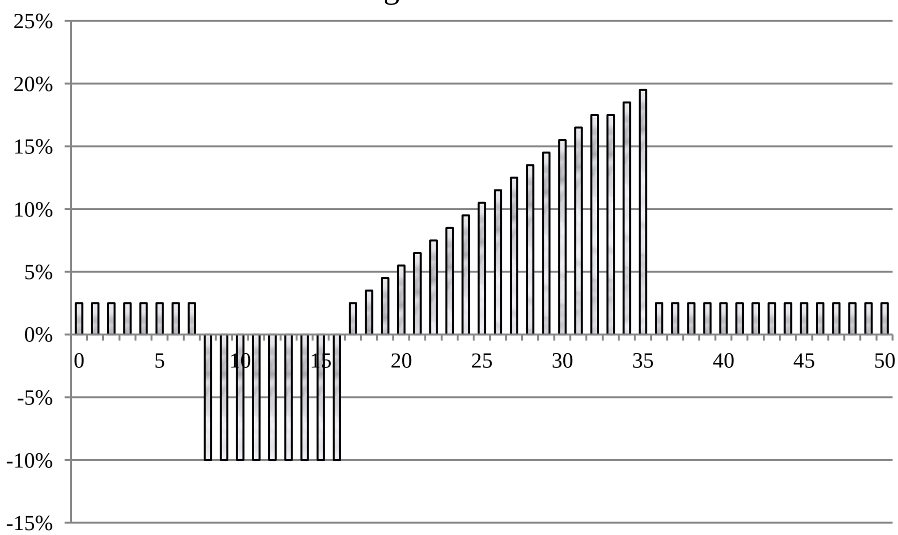


Source: Authors

This behavior suggests the need for a more restrictive fiscal policy, whose target should be to stabilize the public debt to GDP ratio. Policies of this kind, however, are associated with lower rates of economic growth and higher unemployment rates.

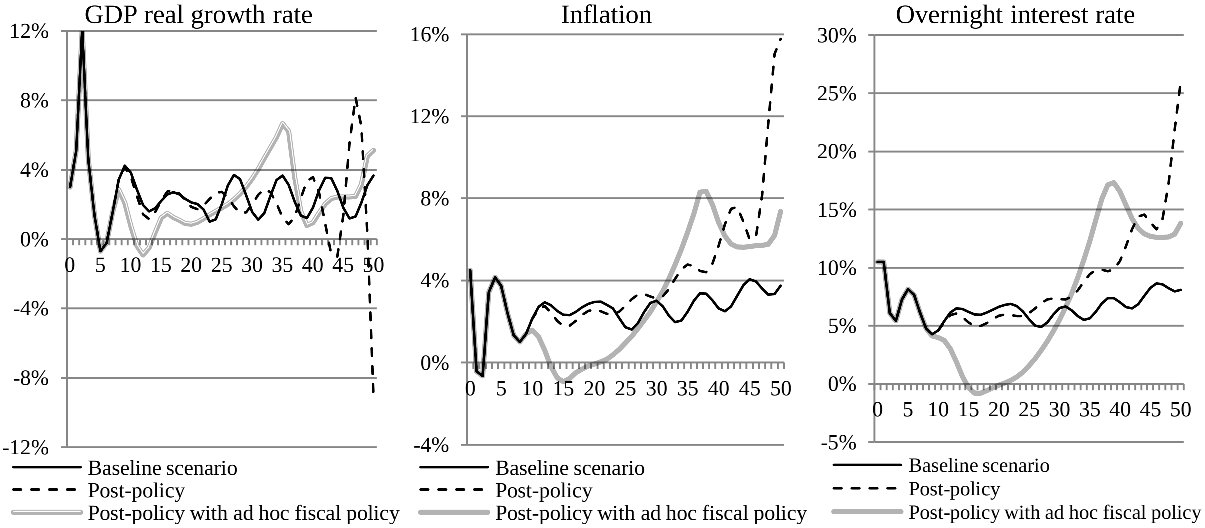
We performed a simulation of this type, in which the government sets an *ad hoc* rule for fiscal policy in order to stabilize the public debt to GDP ratio and thus lower the cost of its debt. At first, right after the policy change in debt management, the government reduces its spending significantly over a relatively long period of time. Once controlled the public debt to GDP ratio, the government must return to spend more, with increasing rates, in order to prevent the economy to enter on a recession path. The growth rate of government spending is fixed in a completely *ad hoc* way in order to illustrate how it is possible to use fiscal policy in a timely manner to stabilize the economy, as well as to show the costs associated with a more intense fiscal adjustment in the short run. The growth rate of government spending in this simulation can be seen in Figure 11, while the GDP real growth rate, inflation and overnight interest rate can be seen in Figure 12.

Figure 11 – Debt management policy change with *ad hoc* fiscal policy: public spending growth rate



Source: Authors

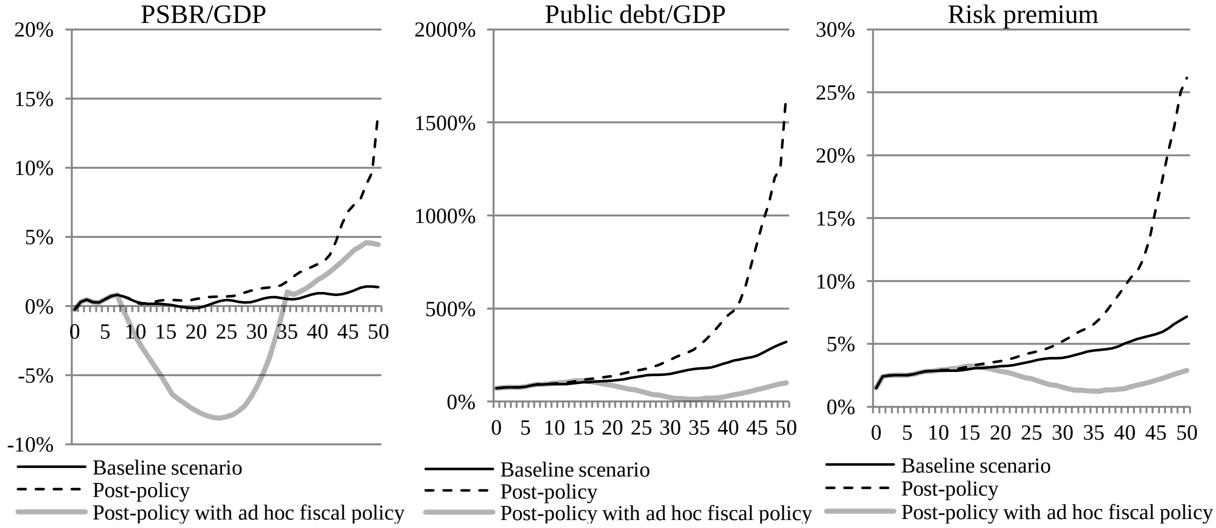
Figure 12 – Debt management policy change with *ad hoc* fiscal policy: GDP real growth rate, inflation and overnight interest rate



Source: Authors

The reduction of government spending over nine periods causes a downturn in the economy, which is accompanied by a reduction in inflation and overnight interest rate. After this period, the government returns to increasingly spend, assuming the role of main inductor of economic growth. After some periods, in order to prevent a greater inflation, the government should reduce once again the growth rate of its spending. It is clear from Figure 13 that this policy is successful in controlling the public debt to GDP ratio and the risk premium for fixed-rate bonds.

Figure 13 – Debt management policy change with *ad hoc* fiscal policy: fiscal indicators



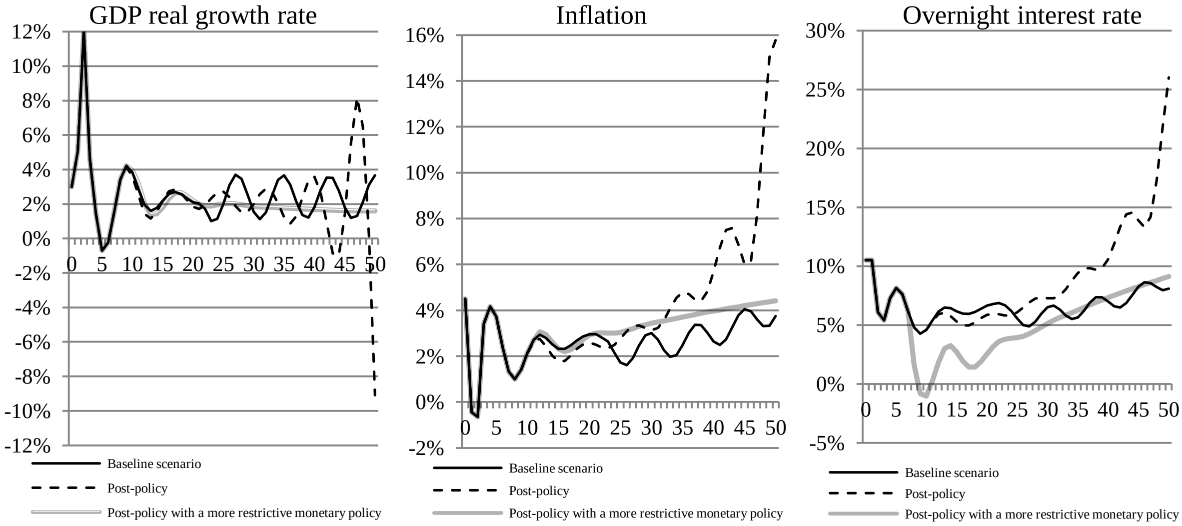
Source: Authors

4.2.2 MONETARY POLICY

Rather than a fiscal adjustment, the government may prefer to use a more restrictive monetary policy in order to avoid the uncontrolled inflation that emerges with the change on debt management policy, as illustrated in Figure 7. We assume in this experiment that the central bank reacts more strongly to deviations of inflation from its target[30].

As in the case of fiscal adjustment, the more restrictive monetary policy can control inflation and keep it below its target throughout the simulation period, albeit at higher levels than that observed in the baseline scenario. The overnight interest rate, however, remains below the level observed in the baseline scenario for most of the simulation period. The advantage of the use of monetary policy in this scenario is that it keeps the growth rate of government spending at the same level. As a result, we can observe higher rates of economic growth, as shown in Figure 14. These results suggest that the use of an inflation targeting regime as a monetary policy framework, in which the overnight interest rate is determined by a Taylor rule, is compatible with the functioning of an economy whose dynamics operate according with post-Keynesian concepts[31].

Figure 14 – Debt management policy change with a more restrictive monetary policy: GDP real growth rate, inflation and overnight interest rate

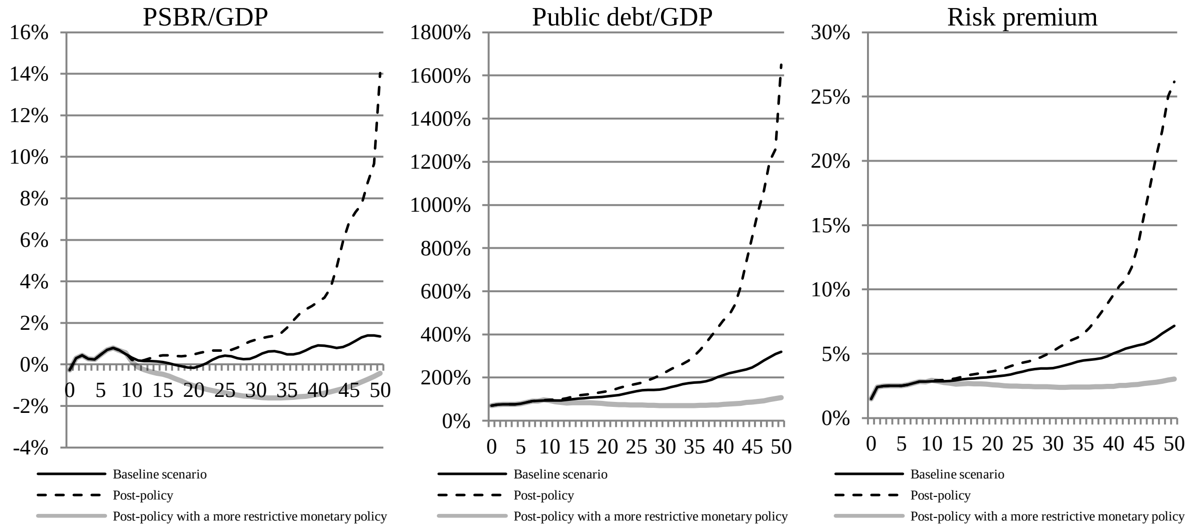


Source: Authors

The fall in the overnight interest rate that comes from a tighter monetary policy allows the reduction of the interest rate associated with fixed-rate bonds. Therefore, there is a decrease in the cost of debt, leading consequently to fiscal surpluses that are able to stabilize the public debt to GDP ratio, which in turn, avoids an overgrowth of the risk premium associated with fixed-rate bonds. Higher economic growth due to the use of monetary policy also helps to explain the stabilization of the public debt to GDP ratio. The behavior of key fiscal indicators resulting from this policy can be seen in Figure 15.

[30] In the simulation the central bank increases the parameter μ_1 to *4* from the eighth period.
[31] See Setterfield (2006).

Figure 15 – Debt management policy change with a more restrictive monetary policy: fiscal indicators



Source: Authors

4.2.3 INCOME POLICY

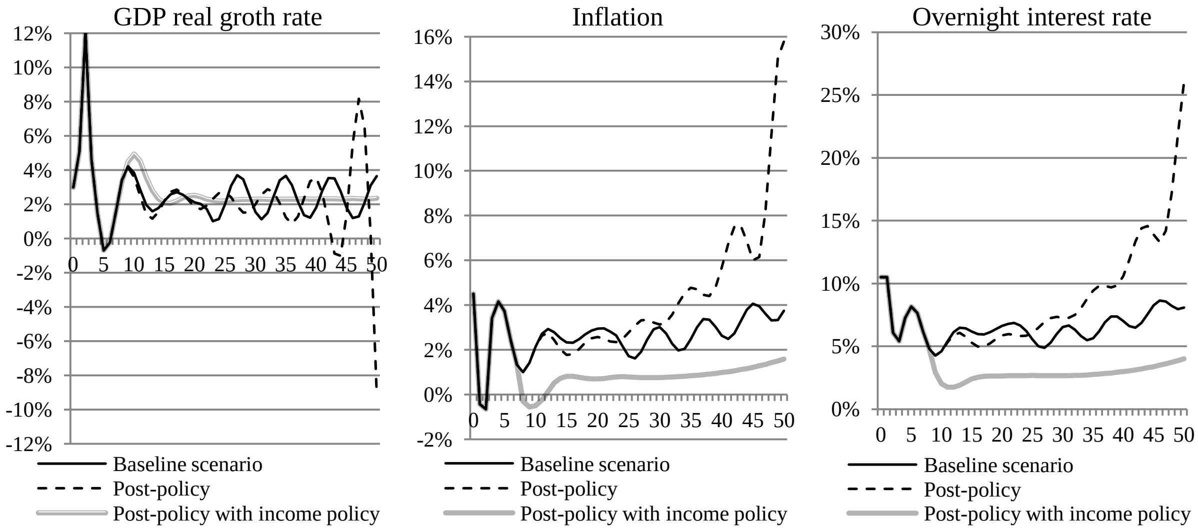
In this artificial economy, the change in the policy of public debt management leads to a destabilization of the economy in the long run. The main factor associated with this destabilization is the uncontrolled inflation derived from the deterioration of the fiscal accounts of the government, whose origins lie in the rising cost of debt. In this scenario, public spending cuts are palliative if the cost of debt is not controlled. Therefore, the fiscal adjustment must be able to stabilize the debt to GDP ratio and thus control the cost of debt. The problem with deeper fiscal adjustment measures is that they are associated, at least in the short run, with low rates of economic growth and, consequently, higher rates of unemployment.

A tighter monetary policy in turn can control this cost by maintaining the overnight interest rate at very low levels in the early periods after the change in debt management policy.

However, the Taylor rule used by the monetary authority only reacts to movements in inflation, not being able to combat its causes. In this economy, the main cause of inflation is the distributive conflict between capitalists and workers. As seen in section 2, workers increase their target real wage due to the growth of productivity and economic growth. Depending on the bargaining power of workers, real wages will grow concomitantly with their target, which tends to increase the nominal wage rate and hence the unit cost of firms. Firms will pass this cost increase to prices, which make the workers demanding higher nominal wages, feeding back the process. One way to break this inflationary cycle is to limit the bargaining power of workers. If we admit that the government can limit that power by running some sort of income policy, it is possible to control inflation in that economy acting on its causes. This is the experiment performed in this subsection.

We assume that the government has the power to influence wage negotiations between firms and workers, so that it can control workers' bargaining power. The effects of an income policy that limits the bargaining power of workers on the dynamics of the economy can be seen in Figure 16[32].

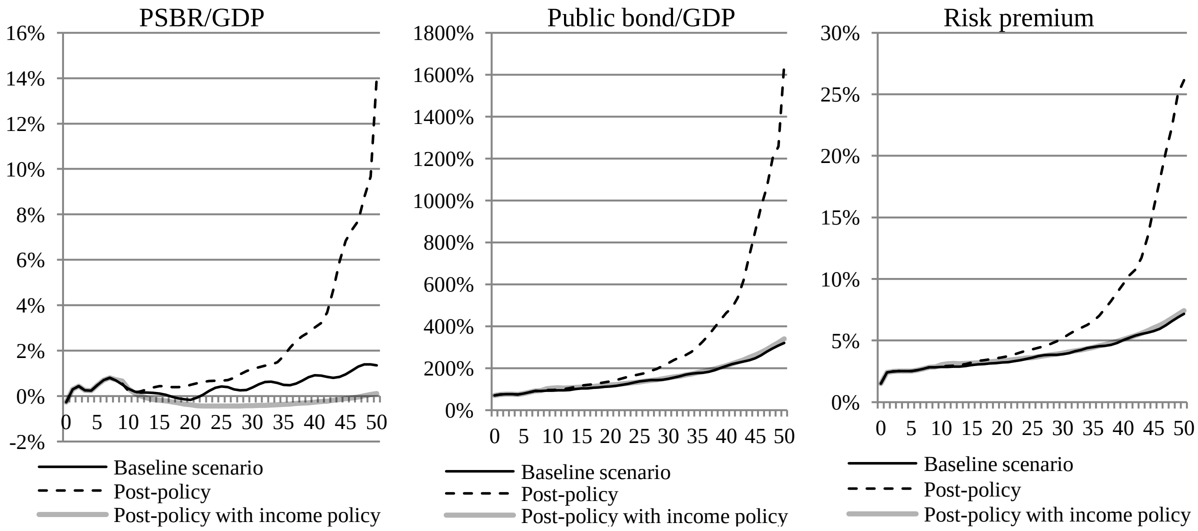
Figure 16 – Debt management policy change with income policy: GDP real growth rate, inflation and overnight interest rate



Source: Authors

We can notice that the income policy is very effective in controlling inflation, which enables the achievement of GDP real growth rates that are higher in the short run and stabilized in the long run. From a fiscal perspective, the public debt to GDP ratio and the risk premium of fixed-rate bonds follow the same trend observed in the baseline scenario. Thus it is not necessary to obtain high fiscal surpluses to maintain the sustainability of economic growth, as can be seen in Figure 17.

Figure 17 – Debt management policy change with income policy: fiscal indicators



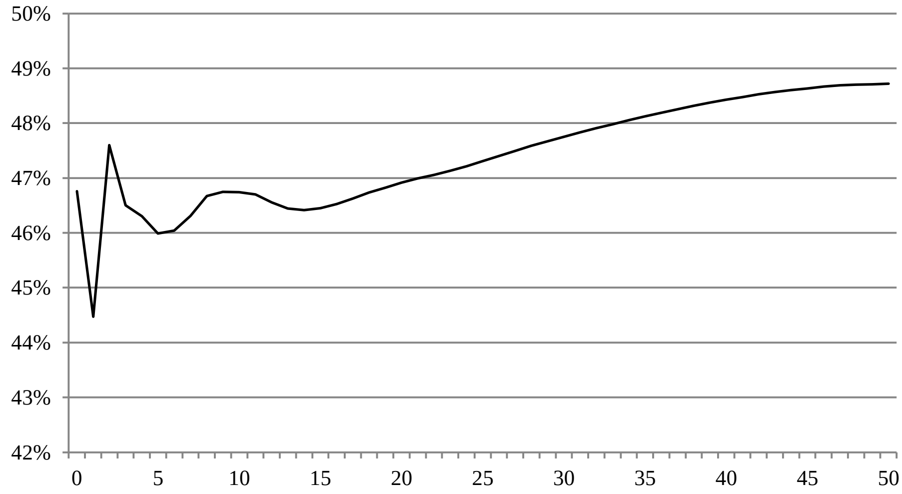
Source: Authors

An important point about the income policy is that reducing the bargaining power of workers is not associated with a reduction in the share of wages in income. Actually, we can observe a small growth after the introduction of the income policy, as shown in Figure 18. Higher

[32] In the simulation, the parameter Ω_3 is reduced to 0.4.

economic growth allows an increase in the wage bill due to the reduction of the unemployment rate. Therefore, the income policy does not redistribute power in favor of capitalists, but break the inflationary cycle largely responsible for the raise of prices in this economy.

Figure 18 – Debt management policy change with income policy: share of wages in income



Source: Authors

4.3 Wealth effect

The correct signaling of monetary policy to the consumer through public debt would be one of the benefits associated with the extinguishment of floating-rate bonds. By raising the interest rate, the price of fixed-rate bonds decreases, which in turn, reduces the financial wealth of households, reducing the consumption linked to it. This is the wealth effect transmitted by monetary policy through public debt, whose effectiveness is diminished by the existence of floating-rate bonds, once increases in interest rates increase the price of these bonds (Andrade and Pires, 2009; Amaral and Oreiro, 2008; Pastore, 1996, 2007).

Once the model calculates capital gains arising from changes in interest rates, it is possible to measure the wealth effect resulting from the change in public debt management policy. For illustration purposes, we chose two periods in which there is a positive change in the interest rate to observe if the existence of floating-rate bonds limit in the model the potential wealth effect associated with fixed-rate bonds.

In the baseline scenario, the overnight interest rate moves from 0.0460 in period ten to 0.0539 in period eleven, while the interest rate of fixed-rate bonds goes from 0.0746 to 0.0825. The price of floating-rate bonds, as expected, rises from $1090.50 to $1149.28, in a variation of $58.78. However, the increase in the interest rate is not enough to decrease the price of fixed-rate bonds, which varies $57.82, up from $865.96 to $923.78. This happens because the effect of time bringing the bonds closer to their maturity, which has the effect of increasing the price of bonds, is greater than the negative effect exerted by rising interest rates. These values generate capital gains for households[33] in the amount of $18,009,399.07 in period eleven, which are added to their financial wealth. Despite the passage of time always leads to capital

[33] Although banks and the central bank also get capital gains, these institutions do not consume part of their financial wealth. So, the wealth effect arises exclusively from capital gains added to the financial wealth of households.

gains between any two periods, there is the possibility of lower capital gains in the case where all public debt is comprised of fixed-rate bonds.

In the simulation scenario, the interest rate of fixed-rate bonds moves from 0.0749 to 0.0824 between periods ten and eleven. Their price has a variation of $58.34, rising from $865.55 to $923.89. This increase generates capital gains for households in the amount of $22,109,391.91. That is, in reality, the extinction of floating-rate bonds increased capital gains of households. This follows from the fact that in the process of exchange of floating-rate bonds for fixed-rate bonds, it is possible to buy a larger amount of fixed-rate bonds due to the difference in prices between the two types of bonds. Thus, increases in interest rates are not associated, in the model, with a decrease in consumption[34]. This result is consistent with Loyo (2007) and Arida (2007), who argue that this transmission channel is not likely to have significant empirical relevance in Brazil.

Some data help to support this hypothesis. According to Bittencourt (2009), only 6.6% of federal securities debt was owned by individuals in December 2008. Most of the debt (46.5%) was owned by financial corporations, like treasure houses and investment funds, and by institutional investors (30%), like pension funds and insurance companies. The remainder of the debt was held by non-financial corporations (10.4%) and non-resident investors (6.5%).

Thus, changes in the prices of government bonds due to changes in interest rates will change the financial wealth of households only through this small portion of the debt they hold. Furthermore, the perception of changes in households' financial wealth is limited by the fact that they hold public bonds mostly through investment funds, so that this professional and outsourced management of the financial wealth may react to changes in interest rates to prevent the fall of its value by changing its composition. In this sense, it does not seem that households significantly decrease their consumption in response to a rise in interest rates due to the decrease in the prices of fixed-rate bonds that are part of their financial wealth. Moreover, even assuming that households do take their consumption decision based on their disposable income and on their financial wealth, the share of consumption that comes from financial wealth tends to be much smaller than the share of consumption based on disposable income. This implies that changes in financial wealth tend to have no significant effect on the level of household consumption.

Therefore, despite increases in interest rates exert a negative effect on the price of fixed-rate bonds, the dynamics of prices due to the influence of time is of a growth trend, so that daily capital gains is the norm even for fixed-rate bonds. Therefore, it seems important to consider the dynamics of time over bonds prices in analysis that attempt to estimate the wealth effect associated with fixed-rate bonds. The analysis undertaken here suggests that this factor tends to have more significant effects on bonds prices than the interest rate.

5 CONCLUSION

In Brazil, a significant share of public debt consists of bonds whose yield is given by the basic interest rate of the economy, which is determined by the BCB and prevails in the market for bank reserves. This interest rate – the Selic rate – is used as the operational target of the inflation targeting regime used by the BCB in order to ensure the stability of the purchasing power of the currency. That is, the Selic rate is at the same time an operational instrument of

[34] Obviously, if the interest rate remained constant between the two periods, capital gains would have been even greater.

monetary policy and the rate of return of a share of public debt. As the objectives of monetary policy and public debt management are distinct, there is a possibility that the Selic rate, determined for monetary policy purposes, has negative effects on public debt dynamics.

Besides this potential contagion effect, the existence of LFTs, which are the bonds remunerated by the Selic rate, reduce the effectiveness of monetary policy in two ways. Firstly, LFTs restrict the effectiveness of the interest channel of the monetary policy transmission by preventing the formation of a term structure of interest rate based on a relationship between risk and return based on the maturity of the bonds. Due to their daily remuneration and high liquidity, LFTs generate incentives for agents to maintain a significant portion of their financial wealth in the short run, which decreases the flow of funds available to finance long run investment projects whose remuneration is not much above that offered by LFTs. As a consequence, the existence of LFTs limits the capacity of growth of the investment rate in Brazilian economy.

Secondly, LFTs restrict the effectiveness of the asset price channel of monetary policy transmission. Rises in interest rates lead to decreases on fixed-rate bonds prices. This decrease implies capital losses for bonds’ holders and therefore reductions in their financial wealth. Assuming that part of households’ consumption is based on their level of wealth, increases in interest rates would have a negative effect on the level of consumption in the economy. Thus fixed-rate bonds exhibit wealth effect. LFTs, in contrast, do not exhibit wealth effect. They have their prices increased as a result of a rise in interest rates. Thus these bonds do not correctly signalize the desired direction of monetary policy.

Given these characteristics, it is expected that the extinction of LFTs increase the effectiveness of monetary policy transmission and make the goals of monetary policy and public debt management less conflicting. However, there are costs associated with a possible extinction of LFTs. These bonds generally have longer maturities and pay less than LTNs, which are fixed-rate bonds. Therefore, the public debt management should consider the potential benefits and costs involved in reducing the portion of LFTs on public debt.

In this sense, the main objective of this paper was to analyze the changes in the dynamics of a given economy resulting from a change in public debt management policy. For that, we built a dynamic stock-flow consistent model. Its set-up was based on post-Keynesian theory, in which (i) the economy is composed of institutions with own goals and rationales, (ii) propensities to save are different depending on the social class of households, (iii) institutions' decisions are taken in an non-ergodic environment of uncertainty in which prevails the notion of historical and dynamic time, (iv) the principle of effective demand is taken into consideration, (v) households’ savings are residual; (vi) agents have procedural rationality, (vii) prices act as a mechanism of income distribution under the control of firms, (viii) the technical progress function follows Kaldor-Verdoorn law, (ix) the portfolio allocation is Tobinian; (x) the money supply is endogenous, and (xi) the main cause of the inflationary process comes from the distributive conflict between workers and capitalists.

All these features were introduced into a SFC model, in which the flow of transactions in a given sector is entirely determined by the flow of transactions in other sectors. Furthermore, all sectors of the economy have budget constraints that allow the whole flow of resources to generate changes in the stocks of assets and liabilities, generating a coherent and consistent artificial economic system. Government bond market was modeled to reflect the main features of the Brazilian market.

We calibrated the parameters and the initial conditions of the model, generating an economic system whose dynamics could reflect satisfactorily the main stylized facts observed in modern economies. Forming a base scenario, it was possible to perform the simulations required to analyze the behavior of this economy due to policy changes in public debt management. The experiment simulated the extinction of floating-rate bonds, so the debt becomes exclusively composed by fixed-rate bonds for once.

The change in debt management policy resulted, over at least twelve periods, in more stable economic growth rates and lower levels of inflation and overnight interest rate. In the long run, however, as the government's fiscal situation was deteriorating, the economy entered into an unstable path with uncontrolled inflation and very high overnight interest rate, which in turn, helped to further destabilize the system.

These results suggest that a policy of debt management that aims at reducing the share of floating-rate bonds in the composition of public debt must be accompanied by macroeconomic policies that are able to stabilize the dynamics of the economic system in the long run. Although it is possible and even desirable to use a mix of macroeconomic policies, we decided to isolate potential policies in the simulation exercises in order to observe the individual effect of each one of them. In the model, the government can manipulate three different policies: fiscal policy, monetary policy and income policy.

Both restrictive fiscal and monetary policies were able to control the inflationary process and stabilize the business cycle. However, the fiscal adjustment by restricting public spending is associated with lower growth rates and hence higher unemployment rates. This result, however, is associated with the maintenance of a fiscal policy in which government expenditures grow at fixed and constant rates, i.e. there is no discretion. Should the government use the fiscal policy actively associated with some goal, such as stabilizing the public debt to GDP ratio in the case of the model, it is possible to control inflation and stabilize growth cycles. This simulation suggests that the use of an active fiscal policy can help in the process of controlling inflation.

The simulations on the use of monetary policy revealed that the use of an inflation targeting regime as a framework of macroeconomic policy is compatible with post-Keynesian theory. At the same time, however, the simulations regarding the use of income policy suggest that a tighter monetary policy is not the most effective policy to control inflation.

In an economy where the inflation process arises mainly from the distributive conflict between workers and capitalists, the use of an active income policy tends to have a more stabilizing effect on the dynamics of the system. This result suggests that policies that target the causes of inflation, and not just react to its variation, are more efficient.

The paper further indicated that it is relevant to take into account all the factors that impact on the price of government bonds. In a dynamic model, it is not possible to isolate the effects of changes in interest rates and the effect of time over bonds' prices. Considering these two factors at the same time, the results of the simulation suggest that the wealth effect associated with fixed-rate bonds does not tend to be relevant.

In short, the benefits associated with a debt management policy that aims to reduce the share of floating-rate bonds appear to be higher than its cost. While the main benefit of this policy, that is, allowing the formation of a yield curve compatible with the structure of risk and return of the economy and thus end the financial logic of overnight investment into the financial system, stimulating the growth of the rate of investment in the economy, is not modeled, simulation exercises conducted suggest that, in the short run, this policy should be associated with lower interest rates and inflation with no harm effects over economic growth. However, this policy, by raising the cost of public debt, leads the economy, as the government's fiscal accounts deteriorates, to an unstable path. This dynamics suggests that government's economic policy should adjust to its debt management policy in order to control inflation and achieve high economic growth rates. That is, the debt management policy and the macroeconomic policy should be conducted in a coordinated way.

REFERENCES

AMARAL, R. Q.; OREIRO, J. L. A relação entre o mercado de dívida pública e a política monetária no Brasil. *Revista de Economia Contemporânea*, Rio de Janeiro, v.12, n.3, p.491-517, 2008.

ANDO, A.; MODIGLIANI, F. The life cycle hypothesis of savings: aggregate implications and tests. *American Economic Review*, v.53, p.55-84, 1963.

ANDRADE, J. P.; PIRES, M. C. C. A transmissão da política monetária pelo canal de crédito do efeito riqueza no Brasil. In: OREIRO, J. L.; de PAULA, L. F.; SOBREIRA, R. (orgs.) *Política monetária, bancos centrais e metas de inflação*: teoria e experiência brasileira. Rio de Janeiro: Editora FGV, 2009, chap.9, p.209-236.

ARIDA, P. As Letras Financeiras do Tesouro em seu vigésimo aniversário. In: BACHA, E. L.; OLIVEIRA FILHO, L. C. (orgs.). *Mercado de capitais e dívida pública*: tributação, indexação, alongamento. Rio de Janeiro e São Paulo: Contra Capa Livraria e Anbid, 2007, p.229-244.

BARBOSA, F. H. The contagion effect of public debt on monetary policy: the Brazilian experience. *Brazilian Journal of Political Economy*, v.26, n.2, p.231-238, 2006.

BITTENCOURT, J. L. A base de investidores da dívida pública federal no Brasil. In: SILVA, A. C.; CARVALHO, L. O.; MEDEIROS, O. L. (orgs.). *Dívida pública*: a experiência brasileira. Brasília: Secretaria do Tesouro Nacional e Banco Mundial, 2009, p.383-414.

BLANCHARD, O.; FISHER, S. *Lectures on Macroeconomics*. Cambridge: MIT Press, 1989.

CARNEIRO, D. D. Letras Financeiras do Tesouro e normalidade financeira: haverá um “peso problem”? In: BACHA, E. L.; OLIVEIRA FILHO, L. C. (orgs.). *Mercado de capitais e dívida pública*: tributação, indexação, alongamento. Rio de Janeiro e São Paulo: Contra Capa Livraria e Anbid, 2007, p.197-218.

CARVALHO, F.C. *Mr Keynes and the Post Keynesians:* principles of macroeconomics for a monetary production economy. Brookfield: Edward Elgar, 1992.

DAVIDSON, P. *Money and the Real World*, 2. ed. London: Macmillan, 1978.

DUTT, A. K. Production. In: KING, J. E. (ed.). *The Elgar companion to post keynesian economics*. Cheltenham, Northampton: Edward Elgar, 2003, p.289-293.

FRANCO, G. H. B. Notas sobre crowding out, juros altos e Letras Financeiras do Tesouro. In: BACHA, E. L.; OLIVEIRA FILHO, L. C. (orgs.). *Mercado de capitais e dívida pública*: tributação, indexação, alongamento. Rio de Janeiro e São Paulo: Contra Capa Livraria e Anbid, 2007, p.273-298.

GODLEY, W.; LAVOIE, M. *Monetary economics*: an integrated approach to credit, money, income, production and wealth. New York: Palgrave MacMillan, 2007.

HANSEN, L; HECKMAN, J. The Empirical Foundations of Calibration. *Journal of Economic Perspectives*, v.10, n.1, 1996.

KALDOR, N. Alternative Theories of Distribution. *Review of Economic Studies*, v. 23, p.83-100, 1956.

KALDOR, N. A Model of Economic Growth. *The Economic Journal*, v. 67, n. 268, p.591-624, 1957a.

KALDOR, N. Capitalist Evolution in the Light of Keynesian Economics. *Sankhyã*, Maio 1957b.

KALECKI, M. *Selected Essays on the Dynamics of the Capitalist Economy.* Cambridge: Cambridge University Press, 1971.

KEYNES, J. M. *The General Theory of Employment, Interest and Money*. London: Macmillan., 1936.

LAVOIE, M. *Introduction to post-keynesian economics*. New York: Palgrave Macmillan, 2006.

LOYO, E. Política monetária e alongamento da dívida pública. In: BACHA, E. L.; OLIVEIRA FILHO, L. C. (orgs.). *Mercado de capitais e dívida pública*: tributação, indexação, alongamento. Rio de Janeiro e São Paulo: Contra Capa Livraria e Anbid, 2007, p.297-328.

LUDVIGSON, S.; STEINDEL, C.; LETTAU, M. *Monetary policy transmission through the consumption-wealth channel*. New York: Federal Reserve Bank of New York, 2002.

MODIGLIANI, F. Life cycle, individual thrift, and the Wealth of Nations. *American Economic Review*, v.76, n.3, p.297–313, jun. 1986.

MOORE, B.J. *Horizontalists and verticalists:* the macroeconomics of credit money. Cambridge: Cambridge University Press, 1988.

MOURA, A. R. Letras Financeiras do Tesouro: quousque tandem? In: BACHA, E. L.; OLIVEIRA FILHO, L. C. (orgs.). *Mercado de capitais e dívida pública*: tributação, indexação, alongamento. Rio de Janeiro e São Paulo: Contra Capa Livraria e Anbid, 2007, p.245-254.

PASINETTI, L. Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth**.** *Review of Economic Studies*, v. 29, p. 267-279, 1962.

PASTORE, A. C. Por que a política monetária perde a eficácia? *Revista Brasileira de Economia*, v.50, n.3, 1996.

PASTORE, A. C. As Letras Financeiras do Tesouro e a eficácia da política monetária. In: BACHA, E. L.; OLIVEIRA FILHO, L. C. (orgs.). *Mercado de capitais e dívida pública*: tributação, indexação, alongamento. Rio de Janeiro e São Paulo: Contra Capa Livraria e Anbid, 2007, p.261-272.

PIGOU, A. C. The classical stationary state. *Economic Journal*, v.53, p.343-351, 1943.

SAMUELSON, P. *Foundations of Economic Analysis*. Cambridge: Harvard University Press, 1947.

SARQUIS, A.; OREIRO, J. L. Um modelo macrodinâmico pós-keynesiano de consistência entre estoques e fluxos para uma economia aberta. In: 39º ENCONTRO NACIONAL DE ECONOMIA, 2011, Foz do Iguaçu. *Anais*... Foz do Iguaçu: Anpec, 2011.

SECCARECCIA, M. Wages and labour markets. In: KING, J. E. (ed.). *The Elgar companion to post keynesian economics*. Cheltenham, Northampton: Edward Elgar, 2003, p.380-383.

SECRETARIA DO TESOURO NACIONAL. *Dívida Pública Federal*: Plano Anual de Financiamento 2011. Brasília: Secretaria do Tesouro Nacional, 2011a.

SECRETARIA DO TESOURO NACIONAL. *Metodologia de Cálculo dos Títulos Públicos Federais Ofertados nos Leilões Primários*. Available at: <http://www.stn.fazenda.gov.br/legislacao/download/divida/decretos/Bonds_Versao_portugues_atualizado_Revisado.pdf>. Access: 15 sep. 2011b.

SETTERFIELD, M. Is Inflation Targeting Compatible with Post Keynesian Economics? *Journal of Post Keynesian Economics*, v.28, 2006.

SIMON, H. A. From Substantial to Procedural Rationality. In: LATSIS, S. J. (ed.). *Method and Appraisal in Economics*. Cambridge: Cambridge University Press, 1976, p.129-148.

TAYLOR, J. B. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, v.39, p.195-214, 1993.

TOBIN, J. A general equilibrium approach to monetary theory. *Journal of Money, Credit, and Banking*, v.1, n.1, p.15–29, feb. 1969.

TOBIN, J. Money and finance in the macroeconomic process. *Journal of Money, Credit, and Banking*, v.14, n.2, p.171–204, may 1982.